# MECHANICS

## An Elementary Text-Book

MERCHANT · CHANT · CLINE

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# PREFACE

The present work is a revised edition of "Mechanics for the Upper School" which was published in 1919 and which has been widely used as an introductory text-book. The changes made have been suggested by those who have had extended experience in teaching the subject. Some sections have been re-arranged, some omitted, and others have been introduced. A large number of new diagrams and other illustrations have been added.

As will be seen, the subject is approached from the experimental and observational side, and considerable space is devoted to practical applications. Throughout the book there are references to the automobile, the aeroplane and other subjects of special interest to our modern youth. Indeed in some instances perhaps the criticism may be offered that too much attention has been given to details of some machines, but such information is just what our alert young people desire, though it is not intended for the purpose of written examinations.

Valuable assistance was given by Professor F. B. Kenrick of the University of Toronto and Professor G. A. Cornish of the Ontario College of Education in the preparation of the chapter on surface tension. Very important help has also been rendered by many teachers, but special thanks are due to C. G. Fraser, Ph.D., W. J. Hocking, B.A., E. P. Winhold, B.A., W. A. Jennings, B.A., E. Pugsley, B.A., and Wm. Smeaton, B.A.

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THE AUTHORS

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# MECHANICS

## CHAPTER I

### Measurement of Length and of Time

**1. Introduction—Divisions of Mechanics.** Mechanics is that branch of physical science which deals with the behaviour of bodies under the action of forces. Mechanical contrivances were invented before the dawn of recorded history. It is certain that the builders of the mighty monuments of ancient times could not have accomplished their work without much help from mechanical devices. Undoubtedly at first men lifted heavy objects by main strength and transported them by carrying or by dragging them over the ground. Then someone discovered the action of the lever and also found that it was easier to drag a sledge if it were mounted on rollers. From such simple expedients the knowledge of mechanical principles and the way to apply them have grown throughout the centuries until now we throw steel bridges across wide raging rivers, harness waterfalls to light our cities hundreds of miles away, and construct aeroplanes which defy even the perils of the polar regions.

There are two main divisions of Mechanics, namely, Statics and Dynamics. The former treats of bodies in a state of rest or equilibrium; the latter deals with bodies in motion.

**2. The Need for Accurate Measurement.** It is often remarked that this is the age of science. The small beginnings of the railway and the steamboat can be traced back almost a century, but their great development has taken place during the last fifty years. The ordinary telephone, the wireless telegraph and telephone, the electric dynamo and transformer,

the phonograph, the aeroplane, the automobile and many other mechanical conveniences which are common to-day, were entirely unknown sixty years ago.

These are what we call *practical applications* of science and it is evident that we cannot have the application until the principles, or laws, of science on which it is based have been discovered and enunciated. Again, the discovery of these principles is made in the physical or chemical laboratory, usually by people who have no thought that their work will have practical application, though few scientific discoveries fail to be utilized at some time.

Now in making a scientific investigation into any problem we cannot make much progress unless we are able to measure accurately the various quantities with which we have to deal. In astronomy the methods of making accurate measurements of time and angle and length were devised at an early date, and that branch of science reached mature development long before any other branch. But in later times physics and chemistry have enormously increased their boundaries, through the development of accurate methods of measurement. We have learned to measure, with great precision, the various effects produced by heat or electricity or sound, and have thus been able to state the exact laws according to which they act. Let us consider briefly some of the simpler kinds of measurements.

**3. Measurement of Length—The Metric System.** The commonest of all measurements is that of length. Whether we design a bridge, pile a cord of wood, purchase cloth or construct a watch, we must measure various lengths, sometimes with great accuracy. It is very necessary to have accurately fixed standards.

There are two systems of units in common use,—the Metric and the English. In the former the fundamental unit of length is the **metre.** This was intended to be one ten-millionth of the distance from the north pole to the equator, measured

on the meridian through Paris, and years were consumed in trying to make a metal bar which should be of exactly this length. The task was completed in 1799. But since then further measurements of the earth have been made and it has been shown that the bar is a little shorter—perhaps a hair's-breadth—than it was intended to be. So now we define the metre without reference to the earth at all; it is the distance between two lines on a metal rod which is preserved in the International Bureau of Weights and Measures at Sèvres, near Paris. The measurements are to be taken when the rod is at the temperature of melting ice. Many copies of this standard have been made and supplied to various nations. The bars are made of a hard and durable alloy composed of platinum 90 per cent. and iridium 10 per cent. and their form is shown in Fig. 1.

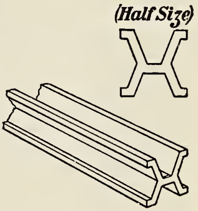



FIG. 1.—View of end and cross-section of the standard metre bars. The line defining the end of the metre is a short mark on the surface midway between the top and the bottom of the bar.

The metric system is almost universally used in scientific experiments, and it has often been proposed that the British Empire and the United States should use it in ordinary life, as is done in almost all other nations; but little progress has been made to this end in the last fifty years.

**4. Divisions and Multiples of the Metre.** The metre is divided decimally, thus:

$\frac{1}{10}$ metre = 1 decimetre (dm.)
$\frac{1}{10}$ dm. = 1 centimetre (cm.)
$\frac{1}{10}$ cm. = 1 millimetre (mm.)
1 m. = 10 dm. = 100 cm. = 1000 mm.

For greater lengths, multiples of ten are used, thus:

| | | | |
|---|---|---|---|
| 10 metres | = 1 decametre. | 10 hectometres | = 1 kilometre (km.) |
| 10 decametres | = 1 hectometre. | 1 km. | = 1000 m. |

The *decametre* and the *hectometre* are not often met with.

**5. The English System.** In this system the fundamental unit of length is the **yard.** It is said to have represented originally the length of the arm of King Henry I (1100-1135), but such a definition is not accurate enough for present-day requirements. The crude manner in which this unit was specified at that time, compared with the precise way in which it is fixed and reproduced now, may serve to illustrate the growth in the appreciation of science in the last 800 years.

The yard is now defined as the distance, at 62° F., between the centres of two transverse lines ruled on two gold plugs in a bronze bar, which is preserved in London, England, in the Standards Office of the Board of Trade of Great Britain. The bronze bar is 38 inches long and has a cross section one inch square (Fig. 2). At *a*, *a*, wells are sunk to the mid-depth of the bar, and at the bottom of each well is the gold plug or pin, about $\frac{1}{10}$ inch in diameter, on which the line defining the yard is engraved.

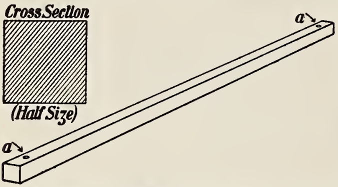


FIG. 2.—Bronze yard, 38 in. long, 1 in. sq. in section; *a*, *a*, are small wells in the bar, sunk to mid-depth.

The other units of length in ordinary use, such as the inch, the foot, the rod, the mile, are derived from the yard. Unfortunately, however, they are not obtained by dividing into tenths or by multiplying by tens, and so calculations in the English system are much longer and more tedious than in the metric system.

**6. Relations between the Two Systems.** In Great Britain the relation between the metre and the inch is officially stated to be,

1 metre = 39.370113 inches, or 1 yard = 0.914399 metre;

in the United States the metre is taken as the fundamental standard and other lengths are referred to it. By law,

1 metre = 39.37 inches, and hence 1 yard = 0.914402 metre.

Thus the U.S. yard differs from the Imperial yard by only 3 parts in 900,000, and they may be considered identical.

The following relations hold:

| | |
|---|---|
| 1 cm. = 0.3937 in. | 1 in. = 2.54 cm. |
| 1 m. = 39.37 in. = 1.094 yd. | 1 ft. = 30.48 cm. |
| 1 km. = 0.6214 mi. | 1 mi. = 1.609 km. |

Approximately 10 cm. = 4 in.
30 cm. = 1 ft.
8 km. = 5 mi.

In Fig. 3 is shown a comparison of centimetres and inches.

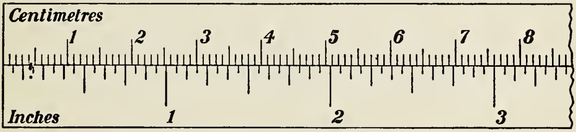



FIG. 3.—Comparison of inches and centimetres.

**7. Derived Units.** The ordinary units of surface and of volume are at once derived from the lineal units. The imperial gallon is defined as the volume of 10 pounds of water at 62° F., or is equal to 277.274 cu. in. (The U.S. or Winchester gallon = 231 cu. in.). The litre contains 1000 c.c.

The following relations hold:

| | | | |
|---|---|---|---|
| 1 sq. yd. | = 0.836 sq. m. | 1 hectare | = 100 ares |
| 1 acre | = 4840 sq. yd. | | = 2.47 acres |
| | = 4046.87 sq. m. | 1 cu. in. | = 16.387 c.c. |
| 1 sq. m. | = 10.764 sq. ft. | 1 c.dm. | = 61.024 cu. in. |
| 1 are | = 100 sq. m. | 1 gal. | = 4.546 l. |
| | = 119.60 sq. yd. | 1 l. | = 1.76 pt. |

**8. Measurement of Lengths.** Having fixed our units of length let us consider how to measure lengths with them and with what degree of accuracy we need to work. Suppose you wish to purchase a certain quantity of cloth at a dry-goods store. The clerk unrolls the cloth, and, placing it alongside his yard-stick (which is a reproduction, more or less accurate,

of the original standard yard) measures off the amount ordered. In this case the measurement is not very accurate, each yard of the cloth might easily be in error by half an inch. The skilful cabinet-maker must be much more accurate in the use of his 2-foot rule when he makes a piece of fine furniture. But in some mechanical operations a still greater degree of accuracy is demanded. In the manufacture of steel balls for ball bearings, they should not differ by $\frac{1}{100000}$ inch and they should not vary from a perfect sphere by $\frac{1}{1000000}$ inch. How shall we make these accurate measurements?

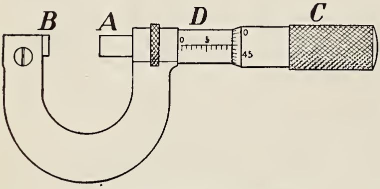


Fig. 4.—Micrometer wire gauge.

**9. Micrometer Screw Gauge.** Suppose we wish to determine accurately the thickness of a wire or of a metal plate. A suitable instrument to use is the screw gauge, illustrated in Fig. 4. $A$ is the end of an accurately made screw which works in a nut inside $D$, and can be moved back and forth by turning the cap or thimble $C$ which is attached to it and which slips over $D$. Upon $D$ is a scale, while the bevelled end of $C$ is divided into a number of equal parts, by which the fractions of a revolution are measured. By turning the cap the end $A$ moves forward until it reaches the stop $B$, and then the edge of the cap should be even with zero on the main scale and the zero mark on the circular scale should be opposite the longitudinal line of the main scale.

In order to measure the diameter of a wire we turn the screw back until the wire can just pass between $A$ and $B$, and then from the graduations on $D$ and $C$ we find the diameter required.

Suppose the pitch of the screw to be $\frac{1}{2}$ mm. and that each division of the main scale is 1 mm. Then with one revolution

of $C$ the end $A$ moves through $\frac{1}{2}$ mm. Now if there are 50 divisions on the bevelled end of $C$ it is evident that when the screw turns through one division the end $A$ moves through $\frac{1}{50} \times \frac{1}{2} = \frac{1}{100}$ mm. Such an instrument will measure to $\frac{1}{100}$ mm.

As an example let us consider the reading shown in Fig. 4. The distance between $B$ and $A$ is evidently 9 mm. + a fraction of a millimetre. Since 47 on the circular scale is opposite the longitudinal line of the millimetre scale, the cap has been turned a certain number of complete revolutions plus $\frac{47}{100}$ of a millimetre. The reading is therefore either 9.47 or 9.97 mm. and a close inspection shows that the latter is the correct reading.

Sometimes the pitch of the screw is $\frac{1}{40}$ inch and there are 25 divisions on the head $C$, in which case one division $= \frac{1}{25} \times \frac{1}{40} = \frac{1}{1000}$ inch.

**10. Vernier Calipers.** But it may happen that the object which we have to measure is too large for our gauge,—for

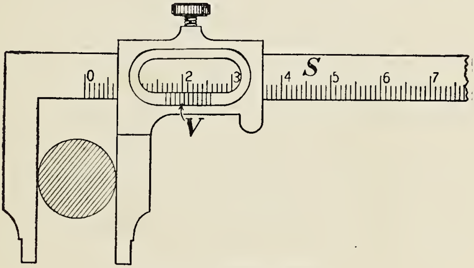



FIG. 5.—Vernier caliper.

example, a cylinder or a sphere an inch or more in diameter (through gauges have been made which will measure several inches); or perhaps the degree of accuracy demanded is not so great. In this case we might use the vernier caliper, one pattern of which is illustrated in Fig. 5. As will be seen, there are two graduated scales, the main scale $S$, on the bar of the instrument, and the vernier scale $V$, on a jaw which

slides upon the bar. When the two jaws are in contact zero on the vernier scale should coincide with zero on the main scale.

The vernier scale is used to measure fractions of a division of the scale $S$, and is usually constructed so that $n$ of its divisions are equal to $n-1$ divisions of the main scale. In taking a measurement the vernier is pushed along until the object to be measured will just pass between the two jaws.

Let the reading be that shown in the figure and suppose 10 vernier divisions are equal to 9 scale divisions and that the latter are millimetres. Then 1 division on the vernier is clearly 0.9 mm., and the difference between one scale division and one vernier division is 0.1 mm.

Now consider the enlarged image of the scale and vernier (Fig. 6). It is clear that the length $AB$, which represents the

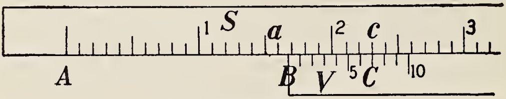



Fig. 6.—Scale and vernier.

diameter of the circular block in Fig. 5, is equal to 16 mm. + a fraction of a millimetre.

To find this fraction, look along the vernier and see where a line on it coincides with a line on the scale. It is seen that division 7 on the vernier coincides with the line $c$ on the scale. Then the fraction to be measured, namely the distance $aB$, is equal to the difference between the 7 divisions of the scale in the space $ac$ and the seven divisions of the vernier in the space $BC$. But the difference between one scale division and one vernier division is 0.1 mm. Hence the fractional part is $7 \times 0.1$ or 0.7 mm., and the entire space $AB$ is therefore 16.7 mm. or 1.67 cm.

For any other vernier the calculation is similar.

Of course there are other devices for the accurate measurement of lengths, each being designed for the special purpose in view, but in every case the screw or the scale or whatever

is the essential part of the instrument must be carefully compared with a good standard before our measurements can be of real value.

**11. Unit of Time.** The earth is our great time-measurer, the period of a rotation being denoted a **day.** Imagine a plane to be drawn through the point where one stands and also through the axis of the earth. This is the observer's **meridian plane,** and as the earth turns on its axis this plane turns with it. During every rotation this meridian plane will come to the sun (and, in succession, to every other body in the sky) though to all appearances the sun comes to the meridian, not the meridian to the sun.

The interval from the moment when the centre of the sun is on the meridian until it next arrives there is called an **apparent solar day.** Unfortunately, however, these apparent solar days are not all of equal length, the reason why being fully given in works on astronomy. Taking the average of the lengths of all the apparent solar days, we obtain a **mean solar day,** and this is chosen as the fundamental unit of time.

It is divided into 24 equal parts, each being an hour; the hour is divided into 60 equal parts, each being a minute; the minute is divided into 60 equal parts, each being a second. Thus the day contains $24 \times 60 \times 60 = 86{,}400$ seconds.

Mean solar time is the kind which is measured off by our ordinary clocks and watches. In the chapters which follow the second will be more frequently used than the day.

## PROBLEMS AND EXERCISES

(For table of values see section 6)

1. How many millimetres in $2\frac{1}{2}$ kilometres?
2. Change 186,284 miles to kilometres.
3. Change 760 mm. into inches.
4. Lake Superior is 602 feet above sea level. Express this in metres.
5. Express, correct to a hundredth of a millimetre, the difference between 12 inches and 30 centimetres.

6. Find in inches the diameter of the bore of the 42-centimetre gun.

7. The distance from Toronto to Montreal is 333 miles. Express this distance in kilometres.

8. Fig. 7 shows a four-inch scale divided into tenths of an inch and an attached vernier scale. Find as exactly as you can the reading of the zero of the vernier, and give reasons for your answer.

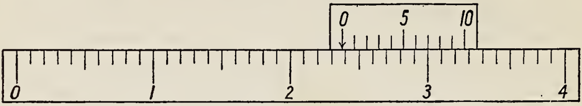



Fig. 7.

9. Read the barometer vernier shown in Fig. 8 in inches and in centimetres, explaining how you reach your results.

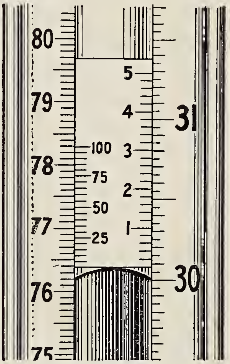



Fig. 8.

10. Give the two vernier readings shown in Fig. 9, explaining how you obtain your results.

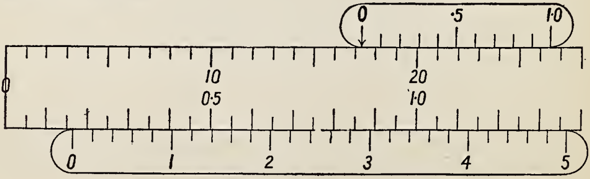



Fig. 9.

11. In the circular scale vernier shown in Fig. 10, twenty divisions on the vernier $A$, are equal to nineteen divisions on the main scale. The numbers on the main scale represent degrees. Find the reading in degrees and minutes.

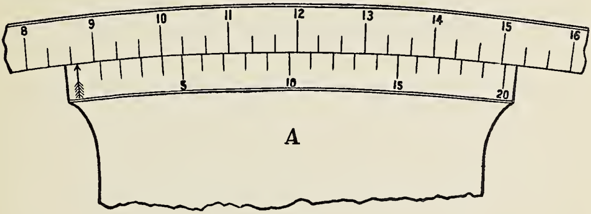



Fig. 10.

12. What is the reading of the micrometer gauge in Fig. 11? (Each division of the main scale represents 1 mm. and there are 100 divisions on the circular scale; 1 revolution of the screw = 1 mm.)

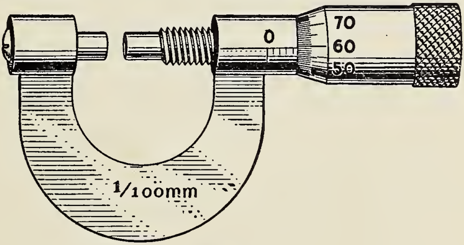



Fig. 11.

13. How would you use an ordinary rule to measure the thickness of a page of this book?

14. Devise a method for measuring the length of a curved line.

## CHAPTER II

### Velocity

**12. Bodies in Motion and at Rest.** Let us look out of the window as we travel in a railway train. The fences and the telegraph poles seem to be continually displaced backward, but our experience leads us to consider the earth, with these objects fixed in it, to be at rest while the train moves forward over it. Perhaps an automobile comes along upon a road parallel to the railway and keeps abreast with the train. We can see its wheels spinning round and we say that it, also, is moving over the earth. But let us fix our attention only on the upper part of the motor-car and not look at the ground at all; would we say that the motor-car is moving? No, it appears to be at rest. Or look at the other people travelling with you; are they at rest? You agree that both they and you are moving over the ground, but with regard to you they seem to be at rest. Thus an object may be in motion with respect to one body and, at the same time, at rest with respect to another.

**13. Definition of Motion.** When, then, is a body said to be in motion? Consider a line joining the rear of the train to a point on the track. As we travel forward this line continually increases in length, and so we may say that if the length of the line joining one body with another is changing, one body is moving with respect to the other. As to the motor-car, the line drawn from it to the train remains of constant length and, as far as the above definition is concerned, each is at rest with respect to the other.

Next, look at two children on a merry-go-round or "teetering" on a plank over a log. The line joining them is not changing its length, and yet each child will say that the other

is in motion. In this case the length of the line does not change but the **direction** does, and so we finally reach the following definition:

**Motion.—One point is in motion with respect to another point when the line joining the two points changes in length or direction.**

We have spoken of the earth as being at rest, but a moment's thought assures us that it is not absolutely at rest. It rotates on its axis and so every particle of it is in motion with respect to the sun and the stars. In addition it revolves about the sun, and, still further, the sun with the entire solar system is moving through space with respect to the stars. Indeed the motion of any particle of the earth is extremely complicated when we consider its motion with respect to the stars in the sky. It is quite evident that we cannot consider any point as absolutely at rest and so must consider the motion of one point with respect to another. Usually, however, in dealing with the motion of bodies we consider the earth to be at rest.

**14. Velocity,—Average, Uniform, Variable.** The road from Toronto to Hamilton is a very good one and a trip by automobile is very pleasant. Let us take one, and we can make a study of velocity on the way. Starting from Toronto at 10.00 a.m., we pass Port Credit (13 miles) at 10.45, Oakville (21 miles) at 11.10, and reach Hamilton (40 miles) at 12 noon. Thus we have passed over 40 miles in 2 hours, and we say our average velocity or speed during the entire trip was 20 miles per hour.

$$\textbf{Average velocity} = \frac{\textbf{Distance}}{\textbf{Time}}.$$

Of course we kept watching the speedometer all the way and we saw that its reading changed very often. Sometimes it said 5, then 10, 20, and perhaps 35 miles an hour, or, when the motor had to stop, it fell to 0. Now, while the reading on the speedometer was constant we realized that we were

travelling at a **uniform velocity,** by which we mean that we were **passing over equal distances in equal times (no matter how short the intervals of time).**

Suppose that over a 5-mile stretch of level road we kept the speedometer perfectly steady, and we found that it required 12 minutes to go this distance. Then, since the velocity was uniform, the rate was

$$\begin{aligned} &\text{5 miles in 12 minutes,} \\ \text{which} \quad &= \text{25 miles in 60 minutes, or 1 hour.} \end{aligned}$$

But generally the speedometer did not remain constant for many seconds at a time, and we realized that we were travelling with **variable velocity,** in other words, the **distances passed over during successive seconds of time were not the same.**

**15. Graphical Representation of Velocity.** If a body is moving with a uniform velocity of 6 feet per second it will traverse 6 feet in 1 second, 12 feet in 2 seconds, etc. This

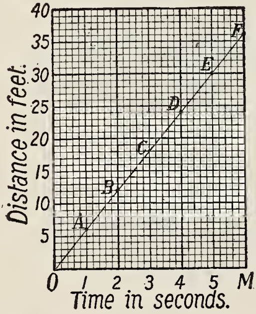



FIG. 12*a*.—Graph for uniform velocity.

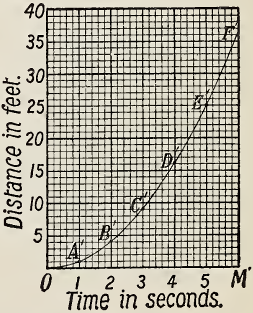



FIG. 12*b*.—Graph for variable velocity.

state of affairs is shown graphically in Fig. 12*a*, in which horizontal distances (abscissas) represent time and vertical distances (ordinates) represent spaces traversed. The dots *A*, *B*, *C*, . . . . . . represent the distances covered in 1, 2, 3, . . . . . . seconds and we see that they lie in a straight line.

The space-time graph for a body moving with uniform velocity is therefore a straight line.

Suppose, however, that the body moves 1, 3, 5, 7, 9, 11 feet in successive seconds, and that we represent this condition graphically as in Fig. 12*b*. The points $A'$, $B'$, $C'$ . . . . . . are not in a straight line and we conclude that a curved or broken line represents non-uniform or variable velocity.

The average velocity for the 6 seconds is $F'M'$ (36 ft.) divided by $OM'$ (6 sec.), or 6 ft. per sec.

Considering the separate seconds we see that the greater the velocity the steeper is the slope of the line for that particular second.

**16. Velocity at a Point.** We have still more to learn from our motor trip. There was a hill which we decided to 'take' on high gear. Having rapidly descended the other hill we began the up-grade with the speedometer indicating 30 miles per hour, which gradually fell until at the top it indicated 10 miles per hour. The entire time required to go up was 20 seconds and the distance was 200 yards.

Now the speed was changing all the time and yet we know that at every point of the 200 yards the car had a definite rate of motion or velocity. Also, to each point of the 200 yards corresponded a definite moment of time in the 20 seconds. Hence, we can say that at every point of the course and at every moment of the time the body had a definite velocity. It is difficult to explain this statement in any simpler terms, but the meaning becomes more definite when we discuss how to measure the velocity at a point.

Let us consider how a traffic officer computes the velocity of an automobile which he suspects of breaking the speed limit. He marks two points, say 220 yards apart, and starts his stop-watch as the car passes the first point and stops the watch when the car arrives at the second. If he finds the time to be 10 seconds, the motorist is likely to appear in court

to answer a charge of driving at the rate of 45 miles per hour. This rate, however, is not necessarily the velocity at either of the two marked points; it is the average velocity for the interval. The driver might have been travelling at the rate of 60 miles per hour at the first point and still have secured the calculated average velocity by a judicious application of the brakes.

It is evident then that in measuring the velocity at a point the space traversed should be as small as possible. In taking the time for a very short distance some more accurate timing device than a stop-watch must be used.

**17. Measure of Velocity at a Point.** Let $AB$ (Fig. 13)

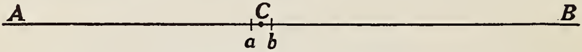


Fig. 13.—Finding velocity at $C$.

represent the 220 yards traversed at varying speed. We wish to measure the velocity at the point $C$.

We can arrange two electrical contacts 1 foot apart, one 6 inches in front of $C$, the other 6 inches back of $C$, such that as the car passes over them it will make, by means of an electrical device, a record on a moving strip of smoked paper upon which a tuning fork is writing a wavy line as shown in Fig. 14. Let the fork make 200 complete vibrations per second, that is, each complete wave represents $\frac{1}{200}$ sec., and let $c$, $d$ be the marks recorded as the car passed over $a$, $b$ (Fig. 13) respectively. These are just 3 full waves apart, and consequently the car passed over 1 foot from $a$ to $b$ in $\frac{3}{200}$ sec. We easily calculate that 1 foot in $\frac{3}{200}$ sec. is equivalent to $45\frac{5}{11}$ miles per hour.

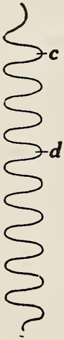


Fig. 14.—Trace of a tuning fork used in measuring a short interval of time.

This is the average velocity during the $\frac{3}{200}$ sec. and does not express exactly the velocity at $C$. If

next we take a space of $\frac{1}{2}$ foot with $C$ in the middle of it and determine the average velocity when passing over it, we shall be still nearer the desired result. It is evident that the shorter the space we take, the nearer we approach the velocity desired. Hence we say that if $s$ is an **infinitely small** length containing $C$ and $t$ is the **infinitely short** time taken to pass over $s$, the

$$\text{velocity at } C = \frac{s}{t}.$$

Some writers make a distinction between velocity and speed, the latter term being used to mean simply **rate of motion,** the former including direction as well as rate. Thus two velocities would not be considered equal unless they were equal in absolute amount and the motion was in the same direction. This distinction is very useful in a full treatment of motion, but in this book we shall deal almost entirely with motion in a straight line and no sharp distinction between the terms will be made.

## QUESTIONS AND PROBLEMS

1. Define motion, uniform velocity, variable velocity, average velocity.

2. Explain what is meant by "velocity at a point."

3. Find the equivalent, in feet per second, of a speed of 60 miles per hour.

4. An eagle flies at the rate of 30 metres per second; find the speed in kilometres per hour.

5. Express in miles per hour a velocity of (1) 40 feet per second, (2) 100 yards per minute.

6. A point moves at the rate of 50 miles in $1\frac{1}{2}$ hours. What is its velocity in feet per second?

7. Find the ratio of velocities of (1) 60 miles per hour and 44 feet per second, (2) 5 miles per 6 minutes and 10 feet per $\frac{1}{4}$ second.

8. One body moves over 30 yards in 7 minutes, and another over 12 feet in 5 seconds. If their velocities are uniform, compare them.

9. A velocity of 20 miles per hour is $v$ times a velocity of 30 feet per second. What is $v$?

10. A body has a uniform velocity of 8 feet per second. What is its displacement in 11 hours?

11. A body is moving with a uniform velocity of 20 cm. per second. What is its displacement in metres in 10 hours?

12. A body moves uniformly in a straight line at the rate of $a$ feet per second. What is its displacement in miles in $b$ hours?

13. A body is moving uniformly at the rate of $c$ cm. in $s$ seconds. How far does it go in $h$ hours?

14. The velocity of a train is 15 miles per hour. Find (1) how many minutes it will take to go 50 yards, (2) how many seconds it will take to go 25 feet.

15. The velocity of a point is $a$ feet per $b$ seconds. How long does it take it to go $c$ miles?

16. A sledge party in the Arctic regions travels northward, for ten successive days, 10, 12, 9, 16, 4, 15, 8, 16, 13, 7 miles, respectively. Find the average velocity.

17. If at the same time the ice is drifting southward at the rate of 10 yards per minute, find the average velocity northward.

18. A point has displacements of 9 cm., 10 cm., 11 cm., and 12 cm. in four consecutive seconds. Find its average velocity (1) for four seconds, (2) for the first three seconds, (3) for the last three seconds.

19. A point is displaced 5 cm., 3 cm., 1 cm., $-1$ cm., $-3$ cm. in five consecutive seconds. What is its average velocity (1) for the five seconds, (2) for the first three seconds, (3) for the last three seconds, (4) for the middle three seconds?

**18. Composition of Velocities.** Suppose a passenger to be travelling on a railway train which is moving on a straight track at the rate of 15 miles per hour, or 22 feet per second. While sitting quietly in his seat he has a velocity relative to the ground of 22 feet per second.

Next let the passenger rise and move along the corridor of the train at the rate of 4 feet per second. If he moves towards the engine his **resultant** velocity with respect to the ground is evidently 26 feet per second while if he moves towards the rear of the train the resultant velocity is 18 feet per second.

If, however, the motion of the passenger is not in the direction of the motion of the train, the calculation of the

resultant velocity becomes more complicated. It may be illustrated by the following example:

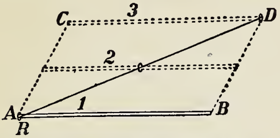



FIG. 15$a$.—Showing how to add together two motions of a ring on a rod.

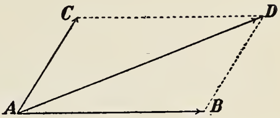



FIG. 15$b$.—The parallelogram of velocities. Velocities $AC$, $AB$ together = velocity $AD$.

Let a ring $R$ (Fig. 15$a$) slide with uniform velocity along a smooth rod $AB$, moving from $A$ to $B$ in 1 second. At the same time let the rod be moved in the direction $AC$ with a uniform velocity, reaching the position $CD$ in a second. The ring will be at $D$ at the end of a second.

At the end of half a second from the beginning the ring will be half-way along the rod, and the rod will be in position 2, half-way between $AB$ and $CD$. It is evident that between the two motions the ring will move uniformly along the line $AD$, travelling this distance in 1 second.

From this illustration we can at once deduce the law of composition of velocities.

Let a particle possess two velocities simultaneously, one represented in direction and magnitude by the line $AB$, the other by $AC$. (Fig. 15$b$.)

Complete the parallelogram $ABDC$. Then the diagonal $AD$ will represent in magnitude and direction the resultant velocity.

If the direction of $AC$ is at right angles to that of $AB$, the magnitude of the resultant velocity is easily obtained since $AD^2 = AB^2 + AC^2$. When the velocities are not at right angles, the calculation of the resultant velocity requires a simple application of trigonometry which will be found in Sec. 109. In the meantime students will find it instructive to solve a few problems graphically using a rule and protractor.

## PROBLEMS

1. Suppose a vessel to steam directly east at a velocity of 12 miles per hour, while a north wind drifts it southward at a velocity of 5 miles an hour. Find the resultant velocity. (Draw a line $AB$ in an easterly direction 12 cm. long, to represent the first component velocity; $AC$, in a southerly direction 5 cm. long, to represent the second. Complete the parallelogram, $ABDC$ which in this case is a rectangle, and find the length of the diagonal $AD$).

2. A ship moves east at the rate of $7\frac{1}{2}$ miles per hour, and a passenger walks on the deck at the rate of 3 feet per second. Find his velocity relative to the earth in the following three cases: (1) when he walks toward the bow, (2) toward the stern, (3) across the deck.

3. A ship sails east at the rate of 10 miles per hour, and a north-west wind drives it south-east at the rate of 3 miles per hour. Find the resultant velocity.

(Draw a line in the easterly direction 10 inches long, and lay off from this, by means of a protractor, a line in the south-east direction, 3 inches long. Complete the parallelogram and measure carefully the length of the diagonal.)

4. Find the resultant of two velocities, 20 cm. per second and 50 cm. per second, ($a$) at an angle of 60°, ($b$) at an angle of 30°. (Carefully draw diagrams, and measure the diagonals.)

5. A particle has three velocities given to it, namely, 3 feet per second in the north direction, 4 feet per second in the east direction, and 5 feet per second in the south-east direction. Find the resultant. (Carefully draw a diagram.)

# CHAPTER III

## Inertia and Force

**19. Newton's First Law of Motion.** Walking along the road day after day, we see a stone beside the path, but one morning it is gone! Now if some person should tell you that the stone *of itself* moved away you would consider him not in his right mind. Such an occurrence is entirely contrary to all our experience.

Sometimes we hear stories of how, in a dimly-lighted room, when several people had laid their hands upon a table, it began to move and to give certain mysterious "rappings." Whatever truth there may be in some of these reports, we may be sure that the table did not *of itself* get up on its legs and walk about. We know that such things do not happen!

Lifeless bodies at rest, when left to themselves remain at rest.

Again, in playing base-ball or cricket, when the batter strikes a 'hot grounder' the ball rolls for a long distance before it comes to rest, and the smoother the field the farther it rolls. If the ball is driven along a cement-paved street it goes still farther; and if we try the experiment on a long stretch of smooth ice it seems almost as if the ball will never stop. The friction of the surface brings it to rest at last, but it is easy to believe that if we could construct a flat level surface which would be entirely without friction, a body started upon it would go on in a straight line at the same rate forever.

If one could travel far out into space, away beyond the influence of any celestial body, and could there launch an object, large or small, it would continue to move in a straight line with the velocity initially given to it for all time—unless it should come under the influence of some other body.

In the preceding paragraphs statements have been made which everyone recognizes to be in accordance with his experience, and which may be taken as axioms, that is, self-evident truths. Now, in 1687, Sir Isaac Newton published his great book entitled, "The Mathematical Principles of Natural Philosophy," which is generally considered to be the greatest scientific book ever published. At the beginning of this he states his famous three laws of motion. The **First Law** is as follows:

**Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled by external *Force* to change that state.**

This is at once seen to be a concise summing-up of the universal experience of man.

**20. Force.** In the above examples the stone and the table were put in motion; we conclude, then, that a force entirely external to them acted upon them. Similarly with the rolling ball; it gradually travelled more slowly, and the change in its motion was due to the **force of friction,** which was quite external to it.

If a body is allowed to fall freely, we know that it moves in a straight line but with **increasing** velocity, and we say therefore that a force acts upon it. In this case it is the force of gravity or the attraction of the earth. If a body is projected outwards there is a change both in the velocity and in the direction of motion—both produced by the force of gravity.

It is well, then, to have clearly in mind:

(i) If a body changes from being at rest to being in motion;

(ii) if the speed of a body is changed; or

(iii) if the direction of motion is changed, even without change in speed; then **force** is acting on the body.

**21. Inertia.** It used to be a familiar trick on April Fool's day to place an old hat near the path to tempt the unsuspecting passer-by to kick it. Now a vigorous kick will change the

hat's condition of rest into motion with a considerable velocity; but if there happens to be a brick under the hat,—well, it is quite a different matter! It is much more difficult, or requires a much greater muscular effort, to set in motion the hat-and-brick than the hat alone. We say the former has much greater **inertia.**

A body possessing great inertia requires a great effort, that is, a great force, to put it in motion; and an equally great force is needed to stop the body if it is in motion. Moreover, the more rapidly we change the motion of the body the greater is the required force.

There is no danger in stopping a football going down the field, but a cannon ball (cannon balls were formerly spherical) of the same size would simply plough through all the players on a field and would do great damage.

An empty barrel has little inertia, and when rolling down an incline can easily be stopped, but look out for it if it is filled with flour or oil or other heavy substance.

**22. Resistance to Change of Motion.** Many simple experiments illustrate the inertia of bodies.

Lay a book upon a sheet of paper on a table. By a quick jerk the paper can be pulled out, leaving the book practically where it was before.

Pile a number of blocks as in Fig. 16 and attach a cord to one near the bottom. A vigorous pull on the string will remove the block to which it is attached and leave the others in the pile as before. It required a considerable force to pull out the block, on account of its inertia; and the other blocks remained behind on account of their inertia.

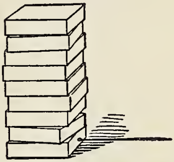


Fig. 16.—Illustrating inertia.

These experiments illustrate the inertia shown by a body when it is at rest. It might be remarked that inertia is not confined to inanimate objects. Human beings very frequently

exhibit it, but in this case it is usually called 'laziness.' Indeed 'inertia' is a Latin word, and 'laziness' is its English translation. But the inertia of a lifeless body differs from the laziness of a living person in the fact that it requires as great a force to stop the former when it is in motion as to start it from rest, but not so in the case of the latter!

As illustrations of the inertia of a body in motion the following may be mentioned:

When a locomotive leaves the rails and is quickly brought to rest the cars behind still continue their motion forward and usually do great damage.

If one is standing up in a street car when it is turning a corner it is well to hold to a strap or other support, as one's body tends to continue in the original direction of motion.

In jumping over a ditch you take a run, leap into the air, and the inertia of your body carries it forward.

When shovelling coal or snow you start its motion and its inertia causes it to continue until it reaches where you want it to go.

We see then that **the *Inertia* of a body is that property by which the body opposes any change in its condition of rest or of uniform motion in a straight line.**

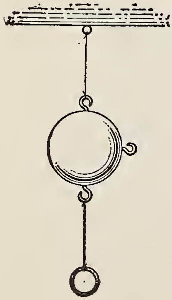


Fig. 17

## EXERCISES

1. Suspend a heavy ball (Fig. 17) by a thread not much stronger than will sustain the load. By means of a similar thread, attach a light ring or handle to the lower part of the ball. Grasp the ring and pull steadily downwards until one of the threads breaks.

Which thread breaks? Why should this thread rather than the other break?

Now suspend the ball and ring as before. Again grasp the ring, and, with a quick jerk, pull suddenly downward.

Which thread now breaks? Why?

2. Suspend a heavy weight, say 10 pounds, by a stout cord 15 or 20 inches long. Tie a fine thread around the middle of the weight and give it a sudden pull sideways.

What change takes place in the condition of (*a*) the thread, (*b*) the weight?

Tie the thread again around the weight, and, by means of a series of well-timed, gentle pulls, set the weight swinging to-and-fro. When it is going through a fairly wide arc, try to stop the weight at its lowest position by suddenly tightening the thread when it reaches this point.

Describe the action of both weight and thread. Explain.

3. Lay a card over the mouth of a bottle (Fig. 18), and place a small coin on the card above the opening. Suddenly drive the card off by striking it with the finger.

What becomes of the coin? Explain its behaviour.

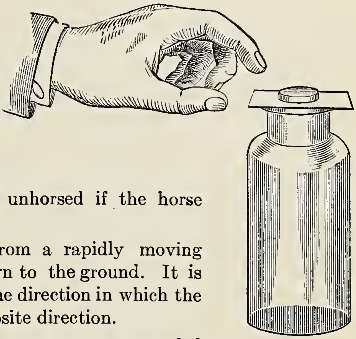


Fig. 18

4. Explain each of the following:

(*a*) A rider is liable to be unhorsed if the horse shies or stops suddenly.

(*b*) A person who steps from a rapidly moving car is in danger of being thrown to the ground. It is less dangerous to step out in the direction in which the car is moving than in the opposite direction.

(*c*) A circus rider can pass over a rope extended across the ring and regain his footing on his horse by leaping straight up when he comes to the rope.

(*d*) The outside bank is worn away when a river takes a sharp turn.

(*e*) "So suddenly did the motor-car stop that one of the occupants of the front seat was pitched through the windshield and those in the rear seat were propelled over into the front seat."—(From a newspaper).

(*f*) A well-loaded automobile, or a steamship with a full cargo, rides more smoothly than if it is without load.

(*g*) When a locomotive runs off the track, or in a collision between two trains, great damage is done by the cars telescoping one another.

5. Why does a base-ball player let his hands move backwards as he catches the ball?

**23. Mass.** What is there in a body which gives to it this characteristic property known as inertia? We are accustomed to say that it is its **mass**, though it is impossible to define or explain what mass is. Frequently the mass of a body is said to be the **quantity of matter** in it, but that does not really *explain* it, as we do not know what matter is. Such a definition would not supply a method of measuring the mass of a body.

**The mass of a body is proportional to its inertia.** Let us try the following experiment. *A*, *B*, *C*, *D* are cubes of the same size, made of cork, cement, iron and lead, respectively, and all painted precisely alike (Fig. 19). From their outward appearance one cannot judge their relative masses, but let us take a light ruler and strike them in succession with equal blows. The cork cube starts off with considerable speed and probably goes off the table. Its resistance to being moved is slight, or its inertia is small and so is its mass. The cement block may move a couple of feet, the iron possibly a foot but the lead one only a few inches. In this way we can arrange the masses in order and, indeed, get some idea of one in terms of the others. It is actually by comparing the effect of a force on various masses that we can compare them. As we shall see later, the force we usually employ is the attraction of the earth.

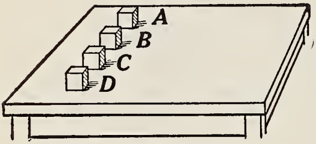


FIG. 19.—Comparison of masses.

**24. Units of Mass.** There are two units of mass in common use. In the metric system the fundamental unit is the kilogram. The world's standard kilogram is a cylinder of platinum-iridium alloy almost exactly $1\frac{1}{2}$ inches in diameter and in height (Fig. 20). It is preserved at Sèvres, France. A large number of equal standard masses have been made and distributed to different nations.

The kilogram is divided decimally as follows:

$$\frac{1}{1000}\text{ kilogram} = 1 \text{ gram.}$$
$$= 10 \text{ decigrams.}$$
$$= 100 \text{ centigrams.}$$
$$= 1000 \text{ milligrams.}$$

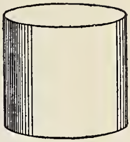


FIG. 20.—Standard kilogram, made of an alloy of platinum and iridium. Height and diameter each 1.5 inches.

The original kilogram was intended to represent the mass of 1000 c.c. (1 litre) of water when at its maximum density (at 4° C.)

**Hence 1 c.c. water = 1 gram-mass.**

In the English system the pound is the fundamental unit. The standard pound is a certain piece of platinum, which is preserved in the Standards Office in London, England. Its form is shown in Fig. 21.

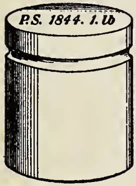



FIG. 21–Imperial Standard Pound Avoirdupois. Made of platinum. Height 1.35 inches; diameter 1.15 inches. "P.S." stands for *parliamentary standard.*

Unfortunately the pound is not divided decimally, and the calculations which involve the pound are more complicated than those in the metric system.

In the English system

$$1 \text{ grain} = \frac{1}{7000} \text{ pound (avoirdupois).}$$

$$1 \text{ ounce} = \frac{1}{16} \text{ pound} = 437.5 \text{ grains.}$$

Originally a grain of wheat was taken from the middle of the ear, and, after being well dried, was used as a standard *grain.**

The relation of the pound to the kilogram is officially stated by the British Government to be

*In addition we have two other sets of weights. *Troy* weight is used in weighing gold, silver, and precious stones. 24 grains = 1 pennyweight (dwt.), 20 dwt. = 1 ounce (oz.). 12 oz. = 1 lb. Thus 1 lb. troy = 5760 grains.

*Apothecaries'* weight is used in mixing medicines. 20 grains = 1 scruple (sc.), 3 sc. = 1 dram (dr.), 8 drs. = 1 oz., 12 oz. = 1 lb. Apothecaries' pound = Troy pound.

| | |
|---|---|
| 1 kilogram | = 2.2046223 pounds avoirdupois. |
| 1 gram | = 15.4323564 grains. |
| 1 ounce avoir. | = 28.349527 grams. |
| Approximately, 1 kg. | = $2\frac{1}{5}$ lb. |
| 1 oz. | = $28\frac{1}{3}$ grams. |

## EXERCISES

1. Find how many times the area of a circle contains the area of the square on the radius by the following method:

Cut out from the same sheet of paper as accurately as possible a square of side 6 cm. and a circle of radius 6 cm. Weigh each carefully. Divide the weight of the circle by the weight of the square. Taking $\pi = 3.1416$, find your percentage of error.

2. By weighing find the metric equivalent of an ounce weight.

3. Measure off 50 cm. of iron stove-pipe wire, weigh it and calculate the weight in grams per centimetre in length. Why use 50 cm., not 5?

4. Take another piece of the same wire, of unknown length, weigh it, and from the weight per centimetre determined in the last experiment, calculate the length of the wire. Verify your result by measuring its length with a metre scale.

What is your percentage of error?

5. Counterpoise a beaker on a balance and run into it from a burette 100 c.c. of water. Weigh the water.

What is the mass of the water?

What is the mass of one cubic centimetre of it?

6. Find the diameter of a capillary tube by the following method:

Weigh the tube empty, run into it a thread of mercury about 10 cm. long and weigh again. From the weight of the mercury calculate its volume (1 c.c. mercury = 13.59 gm.), then find the area of cross-section of the mercury column and finally the diameter.

**25. Gravitation Units of Force.** It was noted in Section 19 that, whenever the motion of a body is being changed, a force is acting on it. But force may be developed without motion being produced. For example, if we exert a muscular effort and pull one end of a spring rigidly fastened at the other end, we stretch it, but there is no motion of the spring as a whole. If we pull with a greater force, we stretch it still more. The stretch is proportional to the force.

Now take a standard pound-mass and hang it from the end of the spring. The spring is stretched a certain amount, and we know therefore that a force must be developed in the spring. This force is due to the pull or attraction of the earth on the pound-mass, and is called the weight of the mass. Next, suspend two standard pound-masses; the spring is stretched twice as much, the pull of the earth on them, (that is, their weight) being twice as great. If, further, we have a pointer attached to the spring which moves past a fixed scale (Fig. 22) by adding a succession of masses, we can calibrate the scale, and, in this way, construct an instrument for measuring the magnitude of forces similar to the familiar spring-balance. **The unit force we make use of is, therefore, the pull of the earth on a mass of 1 pound, or, briefly stated, 1 pd.-wt. This is called the gravitational unit of force. In the metric system the corresponding unit of force is, of course, 1 gram-force or 1 gram-wt., which is defined to be equal to the attraction of the earth on a mass of 1 gram, or, briefly, it is equal to the weight of 1 gram.**

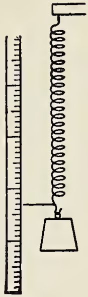


FIG. 22. — The weight of the body stretches the spring.

**26. Mass and Weight.** Suppose we take the spring with the standard pound-mass on the end of it (Fig. 22) to various places on the surface of the earth,—for instance, to Halifax, which is at sea level, to Lake Superior which is 602 feet above the sea, or to the summit of Mount Robson which is 13,000 feet above sea level; or let it be taken up several miles in a balloon. Will the spring in every case show the same amount of stretch, that is, be of the same length? With a sufficiently sensitive spring it will be found that the amount by which it is stretched is altered with its position on the earth's surface and with the height of the balloon. The attractive force exerted by the earth is the same as if the entire mass of the earth were concentrated at its centre, and the farther we get from the

centre the less is the pull of the earth on a given mass (see Chapter VII).

Now the mass of the standard pound obviously does not change,—it is the same body wherever it may be. Hence, since the stretch of the spring varies with the position of the body, the pull on the lower end of the spring, in other words the **weight** of the standard mass, must change. A clear distinction must therefore be made between mass and weight.

**A gram-mass is a certain quantity of matter which remains the same wherever it is taken; while a gram-weight or a gram-force varies with the position of the mass on the earth's surface, continually diminishing as it is taken up above sea level.**

**27. Absolute Unit of Force.** Although the variation in the weight of a gram or of a pound at various places on the surface of the earth is not great, it would never do to choose as an absolute standard a force which has not the same value at all places. Our absolute standard unit of force is defined in terms of the ability of a force to change the motion of a body. Before discussing this further we must consider certain other matters, which form the subjects of the next two chapters.

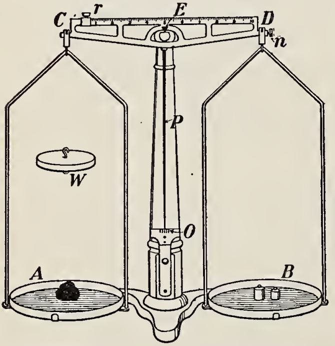



FIG. 23.—A simple and convenient balance. When in equilibrium the pointer $P$ stands at zero on the scale $O$. The nut $n$ is for adjusting the balance and the small weights, fractions of a gram, are obtained by sliding the rider $r$ along the beam which is graduated. The weight $W$, if substituted for the pan $A$, will balance the pan $B$.

**28. Comparison of Masses.** Masses may be compared:

(*a*) By the spring-balance, at the same place. If two bodies produce the same stretch in the spring, their masses are equal as explained in Sec. 25 above.

(*b*) By the common balance.

A good balance should have arms precisely equal in length, and the pans should also be precisely the same.

Upon pan *A* (Fig. 23) place a standard kilogram. That pan at once descends. This is due to the fact that the earth exerts an attractive force on the kilogram, and this force is proportional to its mass.

By carefully filing a piece of metal one can make another body which when put on *B* will just balance the body on *A*, and the masses of the two bodies will be equal.

With care one can produce two bodies which will have equal masses and each equal to one-half the original kilogram. Continuing this process, we can make a set of "weights" having any fractional masses we desire.

Next put any body on *A*, and by adding to *B* weights from one set we can balance *A* and thus determine its mass.

## QUESTIONS

1. State Newton's First Law of Motion. Define force, inertia, mass.

2. What is meant by a pound weight? A kilogram weight?

3. What causes the variation in a pound weight as one moves over the surface of the earth. Where should it be greatest and where least?

4. Distinguish clearly between weight and mass.

5. A spring balance calibrated in Toronto is taken to the equator and a piece of lead is put on the hook. The indicator points to 1 kilogram. Is the true mass of the lead greater or less than 1 kilogram? Where would the indicator point if the same piece of lead were placed on the hook at the north pole?

6. How would you use a spring balance to duplicate a 200 gm. "weight"?

7. Would an equal-arm balance in New Orleans give accurate results if the "weights" were made in England? Give reasons for your answer.

8. How would you duplicate a "weight," (1) if the arms of the balance are not of equal length, (2) without using a balance of the above type (Fig. 23) at all?

# CHAPTER IV

## Acceleration

**29. Accelerated Motion.** A person is not afraid to jump from a verandah to the ground, but hesitates to do so from the top of a high fence, and he would simply refuse to leap from an upstairs window unless it were done to save his life. The reason is obvious enough. The greater the distance a body falls through the air, the faster it moves, and in falling only a few feet a person may acquire a velocity great enough to injure him when he strikes the ground.

On going down a grade, even though the engineer shuts off the steam, the train continually gains in speed and the brakes may have to be set in order to observe the instruction "safety first." If a stone is thrown upwards its velocity gradually diminishes until the stone stops and it then comes downwards with continually increasing velocity.

When the velocity of a body is changing, the motion is said to be **accelerated.** If the velocity is diminishing we more often say that the motion is retarded, but a retardation may be considered a negative acceleration.

**30. How Acceleration is Expressed.** We are all familiar with the term acceleration as used in connection with the velocity of an automobile.

Let us suppose that at a given instant the speedometer of a car (Fig. 24) reads 10 miles per hour and that by pressing the accelerator we succeed in making the speedometer read 25 miles per hour at the end of 5 seconds. Then the gain in velocity has been 15 miles per hour in 5 seconds and if the

acceleration was uniform we can express it otherwise as 3 miles per hour per second. But 3 miles per hour is equivalent to 4.4 ft. per sec. Hence the acceleration can also be stated as 4.4 ft. per sec. per sec. Beginners sometimes have difficulty with the two time "labels" but the difficulty clears up when it is realized that in expressing an acceleration we must state how much the velocity changes in unit time.

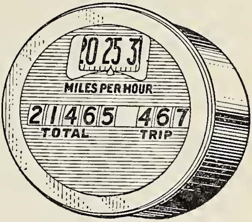



Fig. 24.—Speedometer.

**Acceleration is rate of change of velocity.**

## PROBLEMS

1. A railway train changes its velocity uniformly in 2 minutes from 20 kilometres an hour to 30 kilometres an hour. Find the acceleration in centimetres per second per second.

2. A stone sliding on the ice at the rate of 200 yards per minute is gradually brought to rest in 2 minutes. Find the acceleration in feet and seconds.

3. A point is moving with a uniform acceleration of 10 feet per second per second. (1) What is the total change in velocity in a minute? (2) What is the measure of the acceleration in feet per second per minute?

4. What velocity will a body acquire in half-an-hour if the acceleration is (1) 10 centimetres per second per minute, (2) 10 centimetres per second per second?

5. A point is travelling with an acceleration of 12 feet per second per hour. Find (1) what will be its change in velocity in a minute, (2) the measure of its acceleration in feet per second per second.

6. A train acquires a velocity of 30 feet per second in one hour. If its velocity is uniformly accelerated, find (1) the velocity which it will acquire in one minute, (2) the measure of its acceleration in feet per second per second.

7. A point is travelling with an acceleration of 12 feet per second per hour. How long will it take to acquire a velocity of 2 feet per second?

8. A train, moving with a uniform acceleration, acquires a velocity of 75 feet per second in a quarter of a minute. How long will it take to acquire a velocity of 100 yards per minute?

9. A point, moving with a uniform acceleration, acquires a velocity of 60 feet per second in 10 minutes. What is the measure of its acceleration in (1) feet per second per minute, (2) yards per second per minute, (3) feet per second per second, (4) yards per second per second?

10. A point travelling with a uniform acceleration, has its velocity increased 50 metres per second each minute. What is the measure of the acceleration in (1) metres per second per minute, (2) centimetres per second per minute, (3) metres per second per second, (4) centimetres per second per second?

11. A train moving with a uniform acceleration, acquires an additional velocity of 60 feet per second each minute. Find (1) the measure of its acceleration in feet per second per second, (2) the measure of the velocity it acquires each minute in feet per minute, (3) the measure of the acceleration in feet per minute per minute, (4) the measure of the acceleration in feet per minute per second.

12. A point is moving with a uniform acceleration and acquires an additional velocity of 20 cm. per second each second. Find the measure of the acceleration in (1) centimetres per second per minute, (2) centimetres per minute per minute, (3) metres per minute per minute, (4) metres per minute per second, (5) metres per second per second.

13. What is the measure of an acceleration of 30 feet per second per second when the units of displacement and of time are respectively (1) the foot and the second, (2) the foot and the minute?

**31. Measuring the Velocity of a Body.** In Fig. 25 is shown a small car or trolley mounted on light wheels which

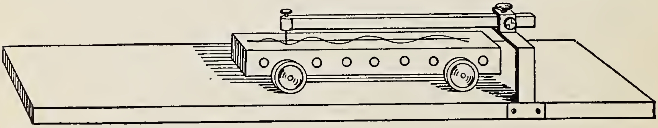


Fig. 25.—Measuring the velocity of the car.

turn with very little friction. It is about 2 feet long and $2\frac{1}{2}$ inches wide. When given a smart push on a level roadway the car runs for a considerable distance with almost uniform

speed. A metal bridge is fastened to the board and a flat steel spring is attached by one end to the bridge. The other end of the spring carries a soft brush which can be filled with ink. A long strip of paper is tacked on the top of the car and upon this the brush traces a record of the motion of the car.

First push the car along under the brush when the spring is at rest. The tracing on the paper is a long straight line. Next start the brush vibrating and give the car a quick push. It moves along approximately uniformly and the tracing on the paper has a wavy form like that shown in Fig. 26.

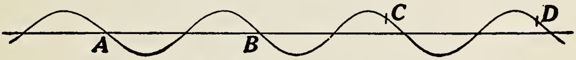


FIG. 26.—Uniform velocity.

(Before pushing the car, raise one end of the board slightly so that the car will not stop if started but will not start of itself. In this way allowance is made for the friction unavoidably present).

It is evident that while the car moved through a distance $AB$ or $CD$ the spring made a complete vibration. If, then, we know the period of the spring, that is, the time required for a complete vibration, we can determine the speed of the car. For example, if the period is $\frac{1}{5}$ sec. and $AB$ is 8.4 cm., the velocity (supposed uniform) is 42 cm. per second.

**32. Study of Acceleration.** Next raise one end of the board and allow the car to run down. Sometimes a trigger arrangement allows one to start the car moving and the brush vibrating at the same time, but in the experiment from which the following results were obtained the brush and car were started simply by hand.

In Fig. 27 is shown a trace obtained when the inclination of the board to the horizon was about 3°. The period of the spring was $\frac{1}{5}$ sec., and it is clear that the car moved through

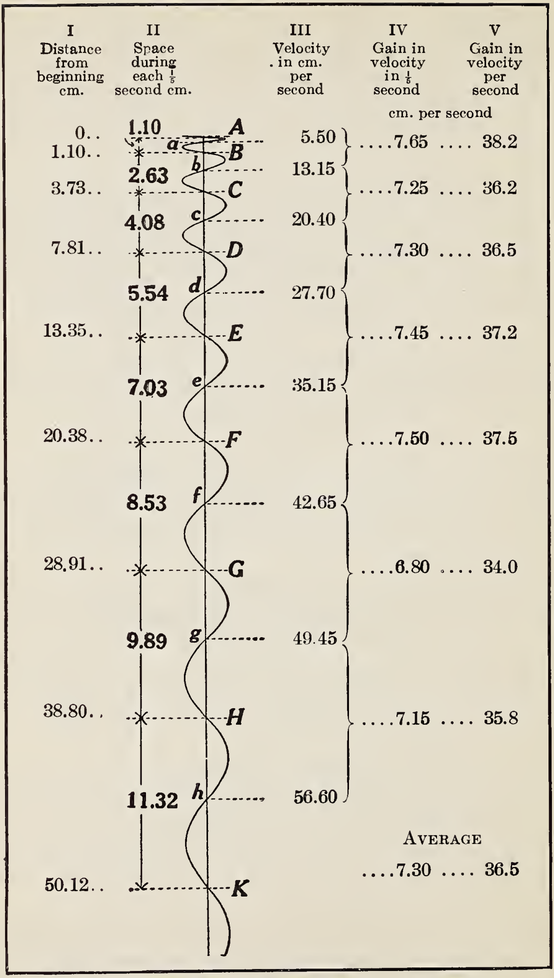



Fig. 27.—Trace of a body moving with uniform acceleration.

the distances $AB$, $BC$, $CD$ . . . . . . during successive vibrations. By means of a metre stick it was found that the

| distances | $AB$ | $AC$ | $AD$ | $AE$ | $AF$ | $AG$ | $AH$ | $AK$ |
|---|---|---|---|---|---|---|---|---|
| were | 1.10 | 3.73 | 7.81 | 13.35 | 20.38 | 28.91 | 38.80 | 50.12 cm. |

These are shown in column I. From these, by subtraction, we obtain

| the lengths | $AB$ | $BC$ | $CD$ | $DE$ | $EF$ | $FG$ | $GH$ | $HK$ | |
|---|---|---|---|---|---|---|---|---|---|
| to be | 1.10 | 2.63 | 4.08 | 5.54 | 7.03 | 8.53 | 9.89 | 11.32 | cm. |

These are given in column II.

Now during the first vibration, while the car moves from $A$ to $B$ its velocity is increasing, and since it passes over 1.10 cm. in $\frac{1}{5}$ sec. its *average* velocity $= 1.10 \times 5 = 5.50$ cm. per sec. If the velocity of the car is increasing uniformly this average velocity will be precisely the velocity at $a$, the mid-point of the vibration or $\frac{1}{10}$ sec. from the beginning. If the increase in the velocity is not perfectly uniform the velocity at $a$ will be approximately 5.50 cm. per sec.

During the second vibration the car travels from $B$ to $C$, a distance of 2.63 cm., and the average velocity is $2.63 \times 5 = 13.15$ cm. per sec. This may be taken as the velocity at $b$, $\frac{3}{10}$ sec. from the beginning.

Continuing this process for all the vibrations we find the

| velocities at | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ |
|---|---|---|---|---|---|---|---|---|
| are | 5.50, | 13.15, | 20.40, | 27.70, | 35.15, | 42.65, | 49.45, | 56.60 cm. per sec. |

These are given in column III.

We find, then, that at $a$ the velocity of the car is 5.50 cm. per sec., and at $b$, $\frac{1}{5}$ sec. later, the velocity is 13.15 cm. per sec. During this fifth of a sec. the increase in the velocity $= 13.15 - 5.50 = 7.65$ cm. per sec. At $c$ the velocity is 20.40 cm. per sec. and the increase in the previous fifth of a second $= 20.40 - 13.15 = 7.25$ cm. per sec. Proceeding in the same way, we obtain the increase in the velocity during the successive fifths of seconds to be

7.65, 7.25, 7.30, 7.45, 7.50, 6.80, 7.15 cm. per sec.

These are given in column IV.

The measure of the acceleration is the *rate of change of the velocity*, or the *change of velocity per unit of time.* While the car was passing from $a$ to $b$ (Fig. 27) the change in the velocity was 7.65 cm. per sec. The time taken to gain this was $\frac{1}{5}$ sec., and hence the acceleration was

7.65 cm. per sec. per $\frac{1}{5}$ sec., which is

the same as 38.25 cm. per sec. per sec.

The acceleration as determined from the next $\frac{1}{5}$ sec. was

7.25 cm. per sec. per $\frac{1}{5}$ sec.

or, 36.25 cm. per sec. per sec.

So on for the rest of the values. They are given in columns IV and V.

**33. Was the Acceleration Uniform?** On examining these values we see that they are approximately equal. Now make a graph as shown in Fig. 28, in which horizontal distances (abscissas) represent time, and vertical distances (ordinates) represent velocity. The dots $a'$, $b'$, $c'$ . . . . represent the velocities at $a$, $b$, $c$ . . . . (Fig. 27). Looking along them it is seen that they lie very nearly on a straight line. There are two reasons why they are not *exactly* on a straight line:—First, the acceleration may not have been perfectly uniform; second, there are unavoidable errors in all physical measurements. However, the velocity-time graph for a body moving with absolutely uniform acceleration is a perfectly straight line.

If we average the values in column IV we obtain 7.30.

We conclude, therefore, that the acceleration was very nearly uniform and that its value was approximately

7.30 cm. per sec. per $\frac{1}{5}$ sec.,

or, 36.5 cm. per sec. per sec.

On producing the line in Fig. 28 backward past $a'$, it is found to cut the "time line" about $\frac{1}{20}$ sec. to the left of 0, and

this indicates that the car began to move about $\frac{1}{20}$ sec. before the point $A$ on the curve was reached.

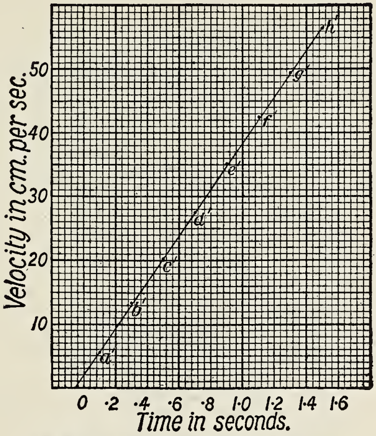



Fig. 28.—Graph showing uniform acceleration.

Moreover we see that the velocity of the car at the point $A$ was about 2 cm. per sec.

## PROBLEMS

1. On putting blocks under one end of the plane (Fig. 25) until its inclination to the horizon was about $4\frac{1}{2}°$ a tracing was obtained on which the following measurements were made:

$AB = 2.29$, $AC = 7.12$, $AD = 14.43$, $AE = 24.29$, $AF = 36.55$, $AG = 51.34$ cm.

From these deduce the approximate velocities at $a$, $b$, $c$ . . . . (Fig. 27), and then the mean acceleration in cm. per sec. per sec. Then from a graph as in Fig. 28 find how long before the instant represented by $A$ on the curve the car started to move.

2. With an inclination of about $6\frac{1}{4}°$ the following values were obtained:

$AB = 2.92$, $AC = 9.45$, $AD = 19.58$, $AE = 33.22$, $AF = 50.55$ cm.

Make similar use of these values.

3. In a laboratory experiment a body moved along a horizontal plane and the distances travelled over in successive intervals each of one-tenth of a second, were:

5.0, 8.2, 11.2, 14.4, 17.5, 20.7 cm.

Find the average acceleration in cm. per sec. per sec.

**34. Space, Acceleration, Velocity, Time.** Let a body move with a uniform acceleration of $a$ cm. per sec. per sec., and suppose that its velocity at a given instant is $u$ cm. per sec.

| | | | |
|---|---|---|---|
| At the beginning, velocity | $v = u$ | cm. per sec. | |
| At the end of 1 second, velocity | $v = u + a$ | " | " |
| " " 2 seconds, " | $v = u + 2a$ | " | " |
| " " 3 " " | $v = u + 3a$ | " | " |
| and " " $t$ " " | $v = u + ta$ | " | " |

Here the gain in velocity in 1 second is $a$ cm. per second; the gain in $t$ seconds is $at$ cm. per second; and the velocity at the end of the $t$ seconds is the original velocity + the gain, *i.e.*,

$$v = u + at \qquad (1)$$

If the initial velocity is zero, we have $u = 0$, and

$$v = at.$$

Next let us find the space traversed. It is evident that

Space $s$ = Average velocity × time,

or $s = \frac{1}{2}$ (Initial + Final velocity) × time,

$$\text{or} \qquad s = \left(\frac{u + v}{2}\right) t \qquad (2)$$

If now we desire an equation involving $s$, $u$, $a$ and $t$ we can obtain it by eliminating $v$ from (1) and (2).

$$\text{Thus} \qquad s = \left(\frac{u + u + at}{2}\right) t,$$

$$\text{or} \qquad s = ut + \tfrac{1}{2} at^2 \qquad (3)$$

By eliminating $t$ from (1) and (2) we obtain

$$s = \left(\frac{u + v}{2}\right) \left(\frac{v - u}{a}\right),$$

$$\text{or} \qquad 2as = v^2 - u^2,$$

$$\text{or} \qquad v^2 = u^2 + 2as \qquad (4)$$

**35. Examples.** In the last section there were developed four equations relating to the motion, in a straight line, of a body which is travelling with uniform acceleration. By using them it is easy to solve problems involving $u$, $v$, $a$, $s$ and $t$. It will be noted that each equation deals with four of these quantities, and consequently if three out of the four are given the fourth is readily found.

**Example 1.**—A body moving with uniform acceleration changes its velocity from 10 cm. per sec. to 100 cm. per sec. in 5 sec. Find the acceleration.

$$\begin{aligned}
\text{Here} \qquad u &= 10 \text{ cm./sec.} \\
v &= 100 \text{ cm./sec.} \\
t &= 5 \text{ sec.} \\
a &= ? \\
\text{But} \qquad v &= u + at. \\
\therefore 100 &= 10 + a \times 5, \\
\text{Whence} \qquad a &= 18 \text{ cm./sec./sec.}
\end{aligned}$$

**Example 2.**—A body passes a point with a velocity of 30 cm. per sec. and is subject to an acceleration in the opposite direction of 10 cm. per sec. per sec. When will it be 40 cm. from the point?

$$\begin{aligned}
\text{Here} \qquad u &= 30 \text{ cm./ sec.} \\
a &= -10 \text{ cm./sec./sec.} \\
s &= 40 \text{ cm.} \\
t &= ? \\
\text{But} \qquad s &= ut + \tfrac{1}{2} at^2. \\
\therefore 40 &= 30\,t - 5\,t^2, \\
5\,t^2 - 30\,t + 40 &= 0. \\
t^2 - 6\,t + 8 &= 0. \\
\text{Factoring, } (t-2)\,(t-4) &= 0. \\
\text{Whence} \qquad t &= 2 \text{ or } 4 \text{ sec.}
\end{aligned}$$

Let us consider the meaning of these two values. The conditions are represented in Fig. 29. The body will be at $B$, 40 cm. from the point $A$ in 2 seconds. At the end of 3 seconds it will be at rest at $C$. It will then start back and will be at $B$ once more at the end of 4 seconds.

The time when it reaches $C$ is obtained as follows:

$$v = u + at,$$

or $$0 = 30 - 10\,t;$$

Whence $$t = 3 \text{ seconds.}$$

The space $AC$ is given by $$s = ut + \tfrac{1}{2}\,at^2,$$

or $$s = 30 \times 3 - \tfrac{1}{2} \times 10 \times 9,$$
$$= 45 \text{ cm.}$$

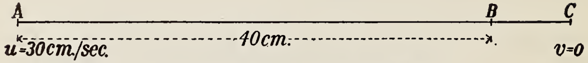



FIG. 29.—Motion of a body in the line $AC$.

For the velocity at $B$ on the "out" journey,

$$v = u + at,$$

or $$v = 30 - 10 \times 2 = 10 \text{ cm./sec.}$$

For the velocity at $B$ on the "in" journey,

$$v = u + at,$$

or $$v = 30 - 10 \times 4 = -10 \text{ cm./sec.}$$

These two values are equal in magnitude but are opposite in direction.

## PROBLEMS

(In solving these problems, it is advisable to write down the known and unknown quantities as in Sec. 35. A glance will then show which equation should be used. A diagram is often helpful.)

1. What is the initial velocity of a point, which, moving with a uniform acceleration of 10 centimetres per second per second, acquires in 10 seconds a velocity of 200 centimetres per second?

2. A body, moving at a certain instant with a velocity of 30 miles per hour, is subject to a uniform acceleration in the opposite direction, and comes to rest in 11 seconds. What was the measure of its velocity, in feet per second, 5 seconds before it stopped?

3. Find the initial velocity of a point which moves with a uniform acceleration of 20 centimetres per second per second, and acquires a velocity of 15 centimetres per second in 10 seconds. Interpret the result.

4. The velocity of a point increases uniformly in 20 seconds from 100 centimetres per second to 200 centimetres per second. Find (1) the

measure of the acceleration in centimetres per second per second, (2) the velocity 3 seconds after it was 150 centimetres per second, (3) when the body was at rest.

5. A point, which has an acceleration of 32 feet per second per second, is moving with a velocity of 10 feet per second. At the same place and at the same time another point, which has an acceleration of 16 feet per second per second, is moving in the same direction with a velocity of 170 feet per second. Find (1) when the two points will have equal velocities, (2) when the velocity of the second will be double that of the first.

6. A body, moving, with a velocity of 5 centimetres per second, has a constant acceleration of 10 centimetres per second per second, in the direction of its motion. Find (1) how far it will go in 10 seconds, (2) how long it will take to go 10 centimetres.

7. A body starts with a velocity of 15 centimetres per second, and has a constant acceleration of 10 centimetres per second per second in the opposite direction. When and where will it come to rest?

8. A body, starting from rest, moves with a uniform acceleration of 20 feet per second per second. Find (1) how far the body goes in 4 seconds, (2) how far it goes in 5 seconds, (3) how far it goes in the 5th second.

9. A body starts with a velocity of 6 feet per second and has a uniform acceleration of 3 feet per second per second in the direction of its motion. At the end of 4 seconds the acceleration ceases. How far does the body move in 10 seconds from the beginning of its motion?

10. With what uniform acceleration does a point, starting from rest, describe 640 feet in 8 seconds?

11. A point, starting from rest and moving with a uniform acceleration, has a displacement of 66 feet in the 6th second. What is the measure of the acceleration in feet per second per second, and what is its displacement in the 7th second ?

12. A train, having a velocity of 20 feet per second, attains a velocity of 30 miles per hour in passing over 128 feet. If the train is moving with a uniform acceleration, what is its acceleration ?

13. A trolley car, moving at the rate of 24 feet per second, is stopped with a uniformly decreasing motion in a space of 9 feet. What is the acceleration of the car ?

14. A particle starts with a velocity of 23 feet per second, and its velocity is uniformly decreased at the rate of 8 feet per second per second. Find how long it will take to describe a distance of 30 feet, and how much longer to come to rest.

**36. Further Study of Motion on an Inclined Plane.** The motion of a ball rolling down an inclined plane may be studied by means of the apparatus shown in Fig 30. It consists of a board 5 or 6 feet long in which is a circular groove 4 inches wide and having a radius of 4 inches. The surface is painted black and is made very smooth. Along the middle of the groove is scratched or painted a straight line; and near one end of the board is fastened a strip of brass, accurately at right angles to the length of the groove and extending to the middle of it. The board must be accurately made to give satisfactory results.

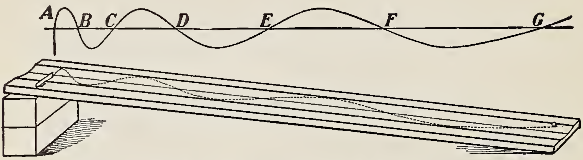



FIG. 30.—Apparatus to illustrate motion with uniform acceleration.

Lay the board flat on the floor, and place a sphere (a steel ball 1 in. to $1\frac{1}{2}$ in. in diameter), at one side of the groove and let it go. It will run back and forth across the hollow, performing oscillations in approximately equal times. By counting a large number of these and taking the average, we can obtain the time of a single one.

Next let one end of the board be raised and over the groove dust (through 4 or 5 thicknesses of muslin) lycopodium powder. Put the ball alongside the brass strip at one side of the groove and let it go. It oscillates across the groove and at the same time rolls down it, and the brass strip insures that it starts downwards without any initial velocity. By blowing the lycopodium powder away a distinct curve is shown like that in the upper part of Fig. 30.

It is evident that while the ball rolls down a distance $AB$ it rolls from the centre line out to the side of the groove and back again; while it rolls from $B$ to $C$, it rolls from the centre line to the other side of the groove and back again. These times are equal and each is about $\frac{1}{3}$ sec. In the same way $CD$, $DE$, $EF$ and $FG$ are each traversed in the same interval. This inverval is one-half of a complete oscillation and sometimes it is better to take the spaces traversed during complete oscillations. Such spaces are $AC$, $CE$, $EG$. By laying a metre scale along the middle of the groove the distances $AB$, $AC$, $AD$, . . . . . . may be measured.

The following are measurements obtained with 1 inch and $1\frac{1}{4}$ inch balls, rolling down a board 6 feet long. In the third, fifth and seventh

columns are shown the ratios of $AB$, $AC$, $AD$, $AE$, $AF$, and $AG$ to $AB$, which are referred to below.

| | 1 inch ball End raised 20 cm. | | $1\frac{1}{4}$ inch ball. End raised 22 cm. | | $1\frac{1}{4}$ inch ball. End raised $22\frac{1}{2}$ cm. | |
|---|---|---|---|---|---|---|
| | cm. | Ratio. | cm. | Ratio. | cm. | Ratio. |
| $AB$.......... | 4.55 | 1.0 | 4.40 | 1.0 | 4.45 | 1.0 |
| $AC$.......... | 18.80 | 4.1 | 18.35 | 4.2 | 18.65 | 4.2 |
| $AD$.......... | 40.40 | 8.9 | 39.50 | 9.0 | 40.25 | 9.0 |
| $AE$.......... | 70.28 | 15.4 | 70.90 | 16.1 | 72.95 | 16.4 |
| $AF$.......... | 111.90 | 24.6 | 108.45 | 24.6 | 111.00 | 24.9 |
| $AG$.......... | 161.30 | 35.4 | 157.10 | 35.7 | 161.00 | 36.2 |

The curves recording the motion in this experiment are quite similar to those obtained with the trolley, and if we know the period of oscillation of the ball we can calculate the acceleration.

## EXERCISES

1. Taking the time of a complete oscillation as $\frac{2}{3}$ sec., find the average acceleration in each of the above cases, as in Sec. 32.

2. Compare the value of $a$ obtained above with that calculated by using the equation $s = ut + \frac{1}{2} at^2$. Take $AG$ as $s$.

**37. Space Traversed.** Applying the formula $s = ut + \frac{1}{2}at^2$, and designating the time of a half oscillation by $x$, we obtain the following theoretical values:

$$\begin{aligned}
AB &= \tfrac{1}{2} ax^2, & &= 1 \times AB,\\
AC &= \tfrac{1}{2} a\,(2\,x)^2 = & 4 \times \tfrac{1}{2}\,ax^2 &= 4 \times AB,\\
AD &= \tfrac{1}{2} a\,(3\,x)^2 = & 9 \times \tfrac{1}{2}\,ax^2 &= 9 \times AB,\\
AE &= \tfrac{1}{2} a\,(4\,x)^2 = & 16 \times \tfrac{1}{2}\,ax^2 &= 16 \times AB, \text{ etc.,}
\end{aligned}$$

*i.e.*, the spaces $AB$, $AC$, $AD$, $AE$, etc., are proportional to 1, 4, 9, 16, etc., or the distance is proportional to the square of the time.

The actual measurements of the spaces are given in the above table, and also the ratios obtained on dividing each space by the first one. These ratios are very close to the theoretical values 1, 4, 9, 16, 25, etc., the discrepancies being due to imperfections in the board and small errors in measurement.

## CHAPTER V

### Acceleration due to Gravity

**38. Bodies attracted by the earth.** "Whatever goes up must come down" is a truth learned in early childhood. Whether the body be set free when high up in the air or when near the earth's surface, it begins to fall at once and does not cease until it reaches some obstacle which blocks the descent. We recognize also that at whatever place we may be the body descends in the vertical direction, that is, along a line perpendicular to the earth's surface at that place. Now the earth is (very approximately) a sphere and a vertical line at the equator makes a right angle with the vertical at the pole. It is clear that we can describe the motion of a falling body as being along the radius of the sphere.*

We 'explain' this phenomenon of falling bodies by saying that it is due to the attraction of the earth, which apparently tries to draw all bodies to its centre.

It was Galileo (1564-1642) who, as a result of experimental investigation, stated accurately the laws followed by bodies moving under the attraction of the earth. He showed that the acceleration of a falling body is uniform and is independent of the nature or the quantity of matter in it. Since his time it has been shown experimentally that the acceleration of gravitation, while constant at any one place, varies with the position of the place on the earth's surface.

**39. All Bodies falling freely have the same Acceleration.** Galileo asserted that all bodies, if unimpeded, fall at the same rate. Now, common observation shows that a stone or a

*On account of the earth being in rotation and also since it is not a perfect sphere this statement is not strictly accurate, but it is very approximately so.

piece of iron, for instance, falls much faster than a piece of paper or a feather. This is explained by the fact that the paper or the feather is more impeded by the resistance of the air.

From the top of the Leaning Tower of Pisa (see Sec. 153), Galileo allowed balls made of various materials to fall, and he showed that they fell in practically the same time. Sixty years later, when the air-pump had been invented, the statement regarding the resistance of the air was verified in the following way. A coin and a feather were placed in a tube (Fig. 31) four or five feet long and the air was exhausted. Then, on inverting the tube, it was found that the two fell to the other end together. The more completely the air is removed from the tube, the closer together do they fall.

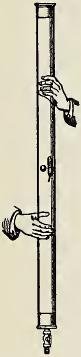


FIG. 31.—Tube to show that a coin and a feather fall in a vacuum with the same acceleration.

If a "guinea and feather" tube (Fig. 31) is not available the following simple experiment may be performed:—

Cut a paper disc slightly smaller than a quarter of a dollar, place it on the quarter and hold the coin between the thumb and forefinger, with its flat face horizontal. On releasing it the coin and the paper disc fall to the ground at the same rate. Here the falling coin prevents the air resistance from acting on the paper, and the true effect due to gravity is obtained.

**40. Determination of Acceleration due to Gravity.** The method which naturally suggests itself for determining the magnitude of the acceleration due to gravity is to time the fall of a body over a measured distance. In doing this, however, the velocity is gained so rapidly that it is difficult to measure the time with sufficient accuracy.

For example, we might stand on a bridge over a deep ravine and use a stop-watch to find how long it takes a stone to drop to the river below. The distance could then be determined by lowering another stone tied to the end of a ball of string, after which the string could be measured with a metre stick.

**Example**.—Let the distance be 50 metres and the time $3\frac{1}{5}$ sec. Then using the equation,

$$s = ut + \tfrac{1}{2}\,at^2,$$
$$5000 = \tfrac{1}{2}\,a \times \frac{256}{25}$$
$$a = 976.5 \text{ cm. per sec. per sec.}$$

An error of one-fifth of a second in determining the time would in this case result in an error of about 13 per cent. in the deduced value of $a$.

**41. Measurement of "g" in the Laboratory.**. A simple form of experiment is illustrated in Fig. 32. $P$ is a straight wooden rod about 4 feet long with a hole near one end and it swings on a pin $p$ in a block $B$ on the wall. A metal ball $b$ is attached to a silk thread which passes over a round disc $C$ mounted eccentrically.

First, let the rod hang vertically, and turn $C$ until the ball hangs just clear of the rod near the top. Then lower the ball and test if it hangs just free of the rod near the bottom. By turning a small weight $d$ this latter adjustment can be made, since a motion in $d$ slightly changes the position of the rod.

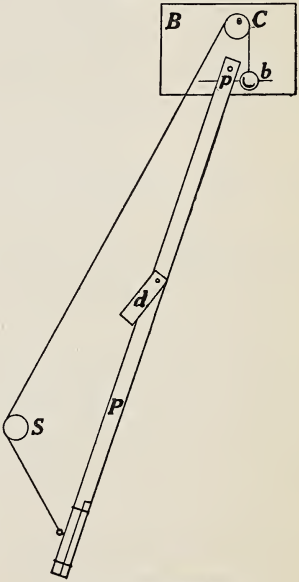


FIG. 32.—Determination of 'g'.

Cover about 20 cm. of the right-hand face of the rod with white paper, and by means of rubber bands fasten carbon paper over this with the ink-face inwards. Then pull the rod aside as shown in Fig. 32. By having the thread pass over a movable pin $S$ the centre of the ball may be made to be on the same level as a mark on the block $B$.

Then burn the thread near $C$, thus releasing $b$ and $P$ at the same time. At the moment when the pendulum is rapidly passing through its middle position it will strike the ball and a mark will be made on the white paper. By measuring from the centre of this mark to the mark on $B$ the distance the ball has fallen can be determined.

Next, pull the pendulum aside and count the number of swings in 30 sec., or the time required for any number of swings. Repeat both parts of the experiment and take the average of the values of the distance and the time of swing.

**Example**:—From a number of counts, 31 complete oscillations of the pendulum = 57 sec., or $\frac{1}{4}$ oscillation = 0.46 sec.

Average of measured distances = 104.3 cm.

Here the ball falls 104.3 cm. in 0.46 sec.

$$\text{Average velocity} = \frac{104.3}{0.46} = 226.7 \text{ cm. per sec.}$$

This is the velocity at half-time, or 0.23 sec. after the start.

In 0.23 sec. velocity gained = 226.7 cm. per sec.

" 1 " " " = 986 cm. per sec.

This is the measure of the acceleration.

This result might have been obtained somewhat more briefly by using the relation $s = \frac{1}{2}gt^2$, in which $g$ is the acceleration of gravity.

Here, $s = 104.3$ cm.

$t = 0.46$ sec.

and $g = \dfrac{2 \times 104.3}{(0.46)^2} = 986$ cm. per sec. per sec.

**42. Pendulum Method for Finding "g".** When it is desired to determine with accuracy the value of $g$ at any station a special form of pendulum is used. The period $t$ of a complete oscillation of a simple pendulum depends upon its length $l$ and the value of $g$. If the amplitude is small the relation connecting these quantities is

$$t = 2\pi\sqrt{\frac{l}{g}}. \quad *$$

If we can measure $t$ and $l$, we can deduce the value of $g$.

*The determination of this formula involves mathematics too advanced for this text.

Let us fasten a lead ball of about one inch diameter to a piece of thin fishing-line and set up a simple pendulum (Fig. 33) by clamping the other end of the line so that the distance from the point of support to the centre of the ball is about 100 cm. By pulling the bob aside through a short distance and taking the time for at least twenty complete oscillations, the time for one complete oscillation may be determined. If $l = 100$ cm., $t$ will be found to be nearly 2 sec.

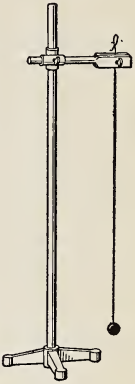


FIG. 33.—A simple pendulum.

Then, from the formula,

$$g = \frac{4\pi^2 l}{t^2} = \frac{4 \times 9.87 \times 100}{4}$$
$$= 987 \text{ cm. per sec. per sec.}$$

This experiment should be repeated using different lengths of string.

In a number of countries measurements of $g$ have been made at many stations, since in this way the form of the earth can be determined. This work is called a gravimetric survey.

At the equator, $g = 978.1$; at the pole, 983.1; at Washington, 980.1; at Toronto, 980.6.

For middle latitudes the value may be taken as 981; using feet and seconds as units, $g = 32.2$.

## PROBLEMS

(Unless otherwise stated, take as the measure of the acceleration of gravity, with centimetres and seconds, 980; with feet and seconds, 32.)

1. A body falls freely for 6 seconds. Find the velocity at the end of that time, and the space passed over.

2. The velocity of a body at a certain instant is 40 cm. per sec., and its acceleration is 5 cm. per sec. per sec. What will be its velocity half-a-minute later?

3. What initial speed upwards must be given to a body that it may rise for 4 seconds?

4. The Eiffel Tower is 300 metres high, and the tower of the City Hall, Toronto, is 305 ft. high. How long will a body take to fall from the top of each tower to the earth?

5. On the moon the acceleration of gravity is approximately one-sixth that on earth. If on the moon a body were thrown vertically upwards with a velocity of 90 feet per second, how high would it rise, and how long would it take to return to its point of projection?

6. A body moving with uniform acceleration has a velocity of 10 feet per second. A minute later its velocity is 40 feet per second. What is the acceleration?

7. A body is projected vertically upward with a velocity of 39.2 metres per second. Find

(1) how long it will continue to rise;

(2) how long it will take to rise 34.3 metres;

(3) how high it will rise.

8. A stone is dropped down a deep mine, and one second later another stone is dropped from the same point. How far apart will the two stones be after the first one has been falling 5 seconds?

9. A balloon ascends with a uniform acceleration of 4 feet per second per second. At the end of half-a-minute a body is released from it. How long will it take to reach the ground?

10. A train is moving at the rate of 60 miles an hour. On rounding a curve the engineer sees another train $\frac{1}{4}$ mile away on the track at rest. By putting on all brakes a retardation of 3 feet per second per second is given the train. Will it stop in time to avoid a collision?

11. A body drops vertically from rest. What velocity will it have (1) at the end of 5 seconds, (2) when it has fallen 1600 feet?

12. A body is thrown vertically downward with an initial velocity of 100 feet per second. Find what velocity the body will have (1) at the end of 10 seconds, (2) when it has fallen 900 feet.

13. A body is thrown vertically upward with an initial velocity of 4900 centimetres per second. Find its velocity (1) at the end of 3 seconds, (2) when it has risen 117.6 metres.

14. A body falls from rest for 4 seconds. Find the distance fallen (1) in the four seconds, (2) in the fourth second, (3) when it has a velocity of 100 feet per second.

15. A body is thrown vertically downward with an initial velocity of 1470 centimetres per second. Find the distance traversed in the fourth second.

16. A body is thrown vertically upward with an initial velocity of 100 feet per second. Find the height to which it will rise.

17. A body is projected vertically upward with an initial velocity of 160 feet per second. Find the distance traversed (1) in 5 seconds, (2) in the 5th second.

18. A body is thrown vertically upward with an initial velocity of 50 feet per second. What is its height when its velocity is 30 feet per second?

19. A stone is thrown vertically into the shaft of a mine with a velocity of 5.4 metres per second, and reaches the bottom in 4 seconds. What is the depth of the mine?

20. A particle is projected vertically upward with a velocity of 96 feet per second. In what time (1) will its velocity be 48 feet per second, (2) will its displacement be 144 feet?

21. A body drops vertically from rest. Find (1) when its velocity is 2450 centimetres per second, (2) when the body is 99.225 metres from the point from which it dropped.

22. A stone is projected vertically downward with a velocity of 100 feet per second. Find (1) when its velocity is 292 feet per second, (2) when it is 900 feet from the point of projection.

23. With what velocity must a body be thrown vertically upward (1) that it may rise for 3 seconds, (2) that it may have a velocity of 30 feet per second at the end of the 3rd second, (3) that it may rise 100 feet?

24. With what velocity must a body be thrown vertically downward (1) that it may have a velocity of 100 feet per second at the end of the 2nd second, (2) that it may describe 204 feet in 3 seconds?

25. A body, thrown vertically upward, passes a point 173 feet from the point of projection with a velocity of 50 feet per second. How much further will it go, and what was the velocity with which it was projected?

**43. Path of a Projectile.** Suppose a person to be at the top of a tower 64 ft. high. If he drops a stone it will fall vertically downwards and will reach the ground in 2 sec. If now it is thrown outwards in a horizontal direction, will it reach the ground as quickly?

In this case a force acting in a horizontal direction gives the body a velocity in that direction, and the question is, will that force in any way change the action of the force of gravity? Can the horizontal velocity alter in any way the vertical velocity?

The best way to answer this question is to test it by experiment. Many pieces of apparatus have been devised for this purpose, one of the simplest being shown in Fig. 34.

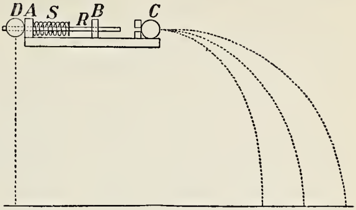



FIG. 34.—The ball $C$, following a curved path, reaches the floor at the same time as $D$ which falls vertically.

$A$ and $B$ are two upright supports through which a rod $R$ can slide. $S$ is a spring so arranged that when $R$ is pulled back and let go it flies to the right. $D$ is a metal sphere through which a hole is bored to allow it to slip over the end of $R$. $C$ is another sphere, at the same height above the floor as $D$.

The rod $R$ is just so long that at the moment it strikes $C$, the sphere $D$ is set free. Thus $C$ is projected horizontally outwards while $D$ drops directly down.

By pulling $R$ back to different distances, different velocities can be given to $C$, and thus different paths described, as shown in the figure.

It will be found that, no matter which of the curved paths $C$ takes, it will reach the floor in the same time as $D$ takes.

It is evident that the horizontal distance travelled by $C$ can easily be found if we know its initial height above the ground and the velocity with which it was projected horizontally. For example, with the measurements given in the first paragraph of this section, if the horizontal velocity were 100 ft. per sec., the horizontal distance travelled would be 200 ft.

The curve traced out by the body $C$ is a parabola.

**44. Effect of Air Resistance.** In actual cases, especially when the velocity is high, the resistance of the air causes the path to deviate considerably from a true parabola.

In the United States during the Great War experiments were made on the path of a bomb when dropped from an aeroplane. Each bomb was fitted with a strong electric light, and when dropped at night looked like a bright shooting star. The path of the light was photographed by two cameras placed 2630 ft. apart. By an electrical arrangement equally spaced moments of time were simultaneously marked on the plates in the two cameras, and it thus was possible to determine, with an error of less than 2 ft., the position of the bomb in the air at the end of successive seconds.

In one experiment a 50-lb. bomb was dropped from an aeroplane when it was 5539 ft. above the ground and travelling horizontally at the rate of 98 ft. per sec. The distances in feet which the bomb had travelled forward ($x$) and downward ($y$) at the end of successive seconds of time are given in the accompanying table and from them the path of the bomb (Fig. 35) was plotted.

| $t$ sec. | $x$ ft. | $y$ ft. | $t$ sec. | $x$ ft. | $y$ ft. |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 11 | 1065 | 1912 |
| 1 | 98 | 16 | 12 | 1158 | 2268 |
| 2 | 196 | 64 | 13 | 1251 | 2652 |
| 3 | 294 | 144 | 14 | 1343 | 3064 |
| 4 | 392 | 256 | 15 | 1433 | 3502 |
| 5 | 490 | 400 | 16 | 1523 | 3965 |
| 6 | 587 | 576 | 17 | 1611 | 4453 |
| 7 | 684 | 782 | 18 | 1698 | 4965 |
| 8 | 780 | 1019 | 19 | 1783 | 5499 |
| 9 | 875 | 1287 | 19.075 | 1790 | 5539 |
| 10 | 970 | 1585 | | | |

## EXERCISES

(*a*) From the values of $x$ and $y$ plot on graph paper the path of the bomb. Plot also the path if there had been no air resistance. Compare.

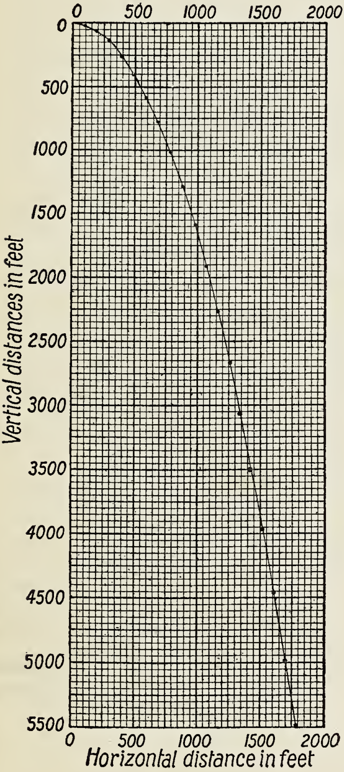



FIG. 35.—Path of a bomb

(*b*) From the values of $x$ find the distance the bomb went forward (horizontally) during each successive second and hence deduce the average velocity forward during each second. Then, using time for abscissas and velocity for ordinates, draw a graph.

(*c*) From the values of $y$ find the distance the bomb went downwards (vertically) during each successive second, and deduce the average velocity downwards during each second. Then, using time for abscissas and velocities for ordinates, draw a graph.

(*d*) By comparing the average velocities each second downwards deduce the change of velocity from one second to the next. This is the acceleration. Was it uniform?

## PROBLEMS

(Take $g$ = 32 ft. or 980 cm. per sec. per sec.)

1. From a window 16 ft. above the ground a ball is thrown in a horizontal direction with a velocity of 50 ft. per sec. Where will it strike the ground?

2. A cannon is discharged in a horizontal direction over a lake from the top of a cliff 19.6 m. above the water, and the ball strikes the water 2500 m. from the shore.

Find the velocity of the ball outwards, supposing it to be uniform over the entire range.

3. An aeronaut when 2112 ft. above the earth's surface and rising vertically at the rate of 16 ft. per sec., throws an object in a horizontal direction with a velocity of 40 ft. per sec. How long will it take to reach the earth and where will it strike?

4. How far would the bomb in Fig. 35 have dropped and how far would it have travelled horizontally in 19 seconds if there had been no air resistance?

5. How long would it have taken the bomb to drop 5539 feet if there had been no air resistance? How far would it have moved horizontally in this time?

6. Neglecting air resistance, how far in advance of his target must a pilot, flying with a ground speed of 90 miles per hour at a height of 576 feet, release a bomb in order that it may strike the target?

7. Plot on a piece of squared paper the path of the projectile in problem 3.

8. Plot the path of the projectile in problem 6.

# CHAPTER VI

## Force

**45. Mass and Velocity.** A lead bullet has small mass (about half-an-ounce) and if thrown against a wooden wall it will do little or no damage; but if fired from a modern rifle, with a speed of 2,000 feet per second, it will pass through several feet of wood and can do great damage. A small mass, when combined with a great velocity, can produce a great result.

Sometimes a large vessel on entering a lock of a canal fails to stop, and though its speed may be quite small (no greater than a walk) it breaks through the strong gates of the lock. A great mass, even though moving with a small velocity, can produce a great effect.

When a person wishes to drive a nail he does not choose a light wooden stick for the purpose but takes a hammer with massive steel head, and if the nail is a large one he gives to the hammer a great velocity.

In ancient times the walls of fortifications were broken down by means of the battering-ram. This was a long heavy log suspended in a horizontal position by ropes and made to swing back and forth in a lengthwise direction. When vigorously operated by a large number of men even the strongest wall could not stand against the continued blows of the end of the log.

In each of the above illustrations it was the combination of mass with velocity which produced the result named, and of course the greatest effect is obtained when a great mass is moving with a great velocity.

Now the word **momentum** is used to express the combination of mass with velocity. Thus, by definition,

$$\textbf{Momentum} = \textbf{Mass} \times \textbf{Velocity},$$

or

$$\mathbf{M} = \mathbf{mv}.$$

No definite name has been given to unit of momentum; we simply speak of a body having so many units of momentum.

**46. Force, Time and Momentum.** It is pleasant to spend some weeks during the summer beside the water. Often some friends will come to call upon you, and you go down to the landing to bid them good bye as they are leaving in a row-boat. Suppose the boat and its occupants to weigh 480 pounds. Taking hold of the end of the boat you push with a force of 25 pounds (that is, the force required to lift a 25-lb. mass) for 3 seconds and give the boat a velocity of 5 ft. per sec. Had you pushed for only 1 sec. the velocity given, of course, would have been $\frac{1}{3}$ of $5 = 1\frac{2}{3}$ ft. per sec. This is the gain of velocity in unit of time, or the acceleration.

On another day you push on a motor-boat, the mass of which is 2400 lb. (5 times as great as the row-boat). You apply the same force but the boat moves much more slowly. How long will it require to give it the same velocity? You find that it takes 15 sec., just 5 times as long. Observe, however, that in the same time the momentum acquired by the motor-boat is equal to that acquired by the row-boat.

Similarly, for the same applied force, a heavily loaded automobile gains speed more slowly than the same car when lightly loaded. Moreover if we wish to stop a moving automobile quickly a great force is needed, while a smaller force will produce the same change in momentum if applied for a longer time.

**It is evident then that the change in momentum is proportional to the magnitude of the force applied and to the length of time it is applied.**

**47. How Force is Measured.** The magnitude of a force is measured by the rate at which it changes momentum.

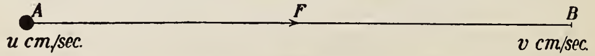



Fig. 36.—Diagram to illustrate change in momentum.

Let a body of mass $m$ grams have a velocity at $A$ (Fig. 36) in the direction $AB$, of $u$ cm. per sec., and let a force $F$ act on it in the direction $AB$, with the result that after a time $t$ sec., it has a velocity at $B$ of $v$ cm. per sec.

Then, by our definition of force,

$F$ is proportional to $\dfrac{mv - mu}{t}$,

or $F = k\dfrac{m(v-u)}{t}$, where $k$ is some constant.

But $v = u + at.$

Hence $F = k\dfrac{m\,at}{t} = k\,ma.$

If, now, we agree to define unit force as that force which gives unit mass unit acceleration

$$1 = k \times 1 \times 1, \text{ or } k = 1.$$

Therefore, under this definition of unit force

$$\mathbf{F} = \mathbf{ma},$$

or, alternatively,

$$\mathbf{F} = \frac{\mathbf{mv} - \mathbf{mu}}{\mathbf{t}}.$$

**48. Newton's Second Law.** The method of measuring force described in the preceding article was first defined by Newton in his Second Law of Motion which may be stated thus:

"Change of motion is proportional to the impressed motive force and takes place along the straight line in which that force is impressed".

In the definitions at the beginning of his "Principia," Newton states that by "motion" he means the product of mass and velocity or what we now call *momentum*. Also he makes it clear that force is to be measured by the change of motion (momentum) produced *in a given time.*

We may, then, recast the **Second Law** as follows:

**Rate of change of momentum is proportional to the impressed force and takes place in the direction in which the force acts.**

Stating this algebraically

$$F = k\,\frac{mv - mu}{t};$$

or if we define unit force as that force which in unit time produces unit change of momentum,

$$F = \frac{mv - mu}{t},$$

which reduces to

$$F = ma.$$

It should be noted also that, under our definition of unit force,

$$\mathbf{Ft = mv - mu.}$$

This form of the equation is useful in cases where the force acts only for a short time, as in the case of a bat hitting a ball. In such cases each body is said to be acted on by an **impulse** which is equal to the change in its momentum. We have then

**Impulse = force × time = change in momentum.**

**49. Verification of Newton's Second Law.** Arrange the trolley as in Fig. 37. Before attaching the light cord, however, raise the left-hand end of the board slightly so that the car

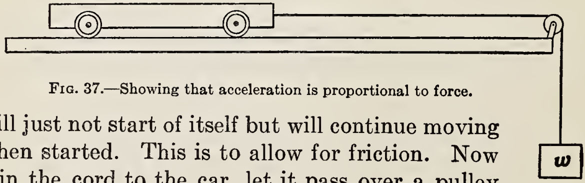



FIG. 37.—Showing that acceleration is proportional to force.

will just not start of itself but will continue moving when started. This is to allow for friction. Now join the cord to the car, let it pass over a pulley and attach a small weight $w$ to it. This weight will keep a constant tension in the cord during the motion of the car which should move with uniform acceleration. Use different

weights for $w$ and obtain tracings with each. The experiments must be made with great care. The masses of the wheels and the pulley should be as small as possible, and it is difficult to get rid entirely of friction.

First, let us keep constant the mass which is accelerated and find how the acceleration varies with the applied force.

Let $w$ consist of two 50-gram weights and let the mass of the car be 1300 gms.

Then the force producing acceleration is the pull of the earth on the 100-gm. mass and the total mass accelerated is 1400 gm. From the tracing shown in Fig. 38*a*, the acceleration is found to be 70 cm. per sec. per sec.

Next, remove one of the 50-gm. weights from $w$ and place it in a slot in the car. The force now producing motion is the pull of the earth on a 50-gm. mass and the total mass accelerated is still 1400 gm. (1350 gm. + 50 gm.). By measuring the tracing in Fig. 38*b*, the acceleration is found to be 35 cm. per sec. per sec.

Evidently, then, twice the applied force produces twice the acceleration, if the mass accelerated is kept constant; and a series of similar experiments would always show that

$$a \propto F, \text{ if } m \text{ is constant.}$$

Next let us investigate the effect of varying the mass accelerated while the applied force is kept constant. By placing masses in the slots provided in the trolley for this purpose, the mass of the car is increased to 2700 grams, while $w$ is again made 100 gm. The mass accelerated is now $2700 + 100 = 2800$ gm , and the applied force is the pull of the earth on a 100-gm. mass.

It is found that we again obtain the curve in Fig. 38*b* and that the acceleration is again 35 cm. per sec. per sec.

We conclude, then, that if the applied force is kept constant, doubling the mass accelerated results in the acceleration being halved, or, more generally,

$$a \propto \frac{1}{m}, \text{ if } F \text{ is constant.}$$

Combining the two results

$$a \propto \frac{F}{m} \text{ or } ma \propto F$$

or $$F = k\, ma.$$

This is Newton's Second Law stated algebraically.

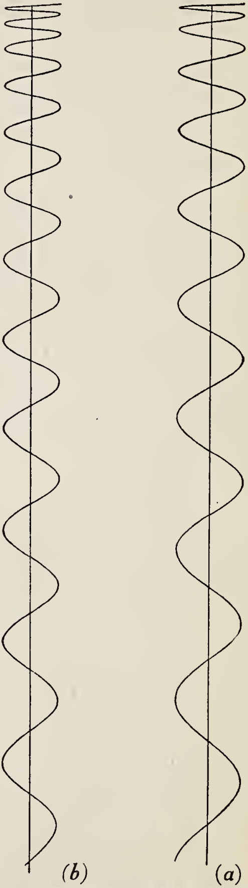


FIG. 38.—Tracings obtained with a force, (*a*) of 100 grams-wt., (*b*) of 50 grams-wt.

**50. Measurement of Force.** We are now in a position to discuss more fully the units used in measuring a force. We have already stated two exactly equivalent definitions of unit force:

**(1) Unit force is that force which in unit time produces unit change in momentum.**

**(2) Unit force is that force which gives unit mass unit acceleration.**

**51. Absolute Units of Force.** In the metric (or C.G.S.) system, the units of length, mass and time are 1 cm., 1 gm., and 1 sec., respectively. The unit of force is called a **dyne.**

By definition,

**1 dyne force is that force which in 1 second produces 1 (metric) unit change in momentum;**

whence

$$F \text{ (in dynes)} = \frac{mv - mu}{t},$$

where $m$ is measured in grams, $v$ and $u$ in cm. per sec. and $t$ in seconds.

Or alternatively,

**1 dyne force is that force which gives 1 gram mass an acceleration of 1 centimetre per second per second;**
whence

$F$ (in dynes) $= m$ (in grams) $\times a$ (in cm. per sec. per sec.).

In the British (or F.P.S.) system the units of length, mass and time are 1 foot, 1 pound and 1 sec., respectively. The unit of force is called a **poundal.**

By definition,

**1 poundal force is that force which in 1 second produces 1 (British) unit change in momentum;**
whence

$$F \text{ (in poundals)} = \frac{mv - mu}{t},$$

where $m$ is measured in pounds, $v$ and $u$ in ft. per sec. and $t$ in seconds.

Or, alternatively,

**1 poundal force is that force which gives 1 pound mass an acceleration of 1 foot per second per second;**
whence

$F$ (in poundals) $= m$ (in pounds) $\times a$ (in ft. per sec. per sec.)

These units are called "absolute" because they do not depend on any particular place on the earth, or indeed, in the universe. Should one be transported to the moon he could use dynes and poundals as units of force just as we do on the earth.

## PROBLEMS

1. What force expressed in poundals will give a mass of 25 pounds an acceleration of 25 feet per second per second?

2. A force of 225 poundals gives a certain mass an acceleration of 15 feet per second per second. Find the measure of the mass.

3. Find the magnitude of the force expressed in dynes in each of the following cases:

(1) The force which will produce in a mass of 20 grams an acceleration of 10 cm. per sec. per sec.

(2) The force which will produce in a mass of 5 kg. an acceleration of 5 cm. per sec. per sec.

(3) The force which will produce in a mass of 30 grams an acceleration of 10 metres per sec. per sec.

(4) The force which will produce in a mass of 10 kg. an acceleration of 20 cm. per min. per min.

(5) The force which, acting on a mass of 3 grams for 12 seconds, will impart to it a velocity of 120 cm. per sec.

(6) The force which, acting on a mass of $m$ grams for $t$ seconds, will impart to it a velocity of $v$ cm. per sec.

4. Find the acceleration expressed in cm. per sec. per sec. in each of the following cases:

(1) A force of 10 dynes acts on a mass of 10 grams.

(2) A force of 15 dynes acts on a mass of 5 kg.

(3) A force of 9800 dynes acts on a mass of 5 grams.

5. Find the mass of the body acted upon by the force in each of the following cases:

(1) A force of 5 dynes produces in a body an acceleration of 10 cm. per sec. per sec.

(2) A force of 10 dynes acting for 5 seconds imparts to a body a velocity of 20 cm. per second.

(3) A force of 30 dynes produces in a body an acceleration of 5 metres per min. per min.

(4) A force of 1,960,000 dynes acting for 2 minutes imparts to a body a velocity of 60 cm. per sec.

6. A mass of 400 grams is acted on by a force of 2000 dynes. Find the acceleration. If it starts from rest, find, at the end of 5 sec., (1) the velocity generated, (2) the momentum.

7. A force of 10 dynes acts on a body for 1 min., and produces a velocity of 120 cm. per sec. Find the mass, and the acceleration.

8. Find the force which in 5 sec. will change the velocity of a mass of 20 grams from 30 cm. per sec. to 80 cm. per sec.

9. A force of 50 poundals acts on a mass of 10 lb. for 15 sec. Find the velocity produced, the acceleration and the momentum.

10. Two masses, $3m$ and $5m$, are acted on by forces which produce in their motions accelerations of 7 and 9 respectively. Compare the magnitude of the forces.

11. A force acts on a mass of $m$ grams. Compare the acceleration with that produced by the same force acting on a mass of (1) $nm$ grams, (2) $\frac{m}{n}$ grams.

12. A force is capable of producing in a certain mass an acceleration of $b$ cm. per sec. per sec. and in another mass an acceleration of $bc$ cm. per sec. per sec. Compare the masses.

**52. Gravitation Units of Force.** However, we live on the earth and the force with which we are best acquainted is the force of gravity. It is quite natural, therefore, that there should have arisen a unit of force which depends upon gravity. In the metric system, as was stated in Sec. 25, the unit is the **gram-force,** and it is defined thus:

1 gram-force = the attraction of the earth on 1 gram-mass;

or, it is the *weight* of 1 gram-mass.

Thus a gram-force and a gram-mass are quantities of entirely different kinds. A gram-mass is a certain quantity of matter, which will remain the same wherever it may be taken; while a gram-force varies with one's position on the earth's surface and would change entirely if one should go off into space.

We can compare a dyne with a gram-force in the following way:

Allow a gram-mass to fall freely. Its acceleration is $g$ cm. per sec. per sec. Thus,

1 gm.-force acting on 1 gm.-mass gives it an acceleration of $g$ cm. per sec. per sec.
But 1 dyne " " 1 " " " " 1 " " "

**Hence, 1 gm.-force = $g$ dynes.**

$g = 980$ (approx.), and hence 1 dyne $= \frac{1}{980}$ (approx.) of the weight of 1 gram of matter, or a little more than the weight of 1 milligram mass.

In the English system the gravitation unit of force is the earth's attraction on 1 pound-mass, or the *weight* of 1 pound-

mass. Thus 1 pound-mass differs in nature from 1 pound-force and it is convenient to write 1 pound as "1 lb." when referring to mass, and "1 pd." when referring to force or weight.

If 1 lb. mass is allowed to fall freely its acceleration is $g$ ft. per sec. per sec.

We can then say,

1 pd.-force acting on 1 lb. mass gives it an acceleration of $g$ ft. per sec. per sec.
But 1 pdl. " " 1 " " " " 1 " " "

**Hence, 1 pd.-force = $g$ poundals.**

Here, $g = 32$ (approx.), and 1 pdl. $= \frac{1}{32}$ of wt. of 1 lb. mass (approx.)
$=$ wt. of $\frac{1}{2}$ oz. (approx.)

**53. Examples.** 1. A trolley (Fig. 39), weighing 1300 gm., is placed on a horizontal track and is put in motion by a mass of 100 gm. hanging from the end of a light horizontal string attached to the trolley and passing over a light pulley at the end of the track. Neglecting friction and the mass of the string and pulley, find

(*a*) the acceleration with which the trolley moves;

(*b*) the tension in the string.

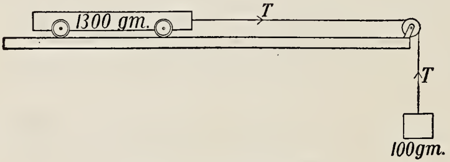



FIG. 39.—A trolley problem.

(*a*) The force producing motion is the pull of the earth on the 100 gm. mass = 100 × 980 dynes. The total mass accelerated = 1300 + 100 = 1400 gm.

$$\text{But} \qquad F = ma;$$

$$\therefore 100 \times 980 = 1400a,$$

$$\text{or} \qquad a = 70 \text{ cm. per sec. per sec.}$$

(Question: Why do we not take into account the pull of the earth on the trolley?)

(*b*) (1) Considering the motion of the trolley only, we can think of it moving with an acceleration of 70 cm. per sec. per sec. because of a tension of $T$ dynes in the string.

But $F = ma,$

$$\therefore T = 1300 \times 70 = 91{,}000 \text{ dynes.}$$

(2) Considering the motion of the 100 gm. mass only, we find it subjected to the following forces:

Downward.....................100 × 980 = 98,000 dynes.
Upward......................................$T$ dynes.

Hence resultant downward force causing it to move with an acceleration of 70 cm. per sec. per sec. = (98,000 − $T$) dynes.

But $F = ma;$

$$\therefore 98{,}000 - T = 100 \times 70,$$

or $T = 91{,}000$ dynes, as before.

2. A light string has masses of 230 gm. and 260 gm. attached to its ends. The string is placed over a light "frictionless" pulley (Fig. 40). Find

(*a*) the acceleration with which the system moves;

(*b*) the tension in the string.

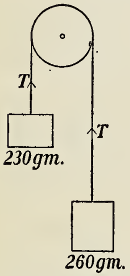



FIG. 40.—Problem with weights on a cord.

Note: The tension of the string acts vertically upwards in each part of the string. It causes the 230 gm. mass to rise in spite of gravity and retards the free fall of the 260 gm. mass.

(*a*) The force producing motion is the difference between the weights of the two masses, i.e., 30 gm., or 30 × 980 dynes. The total mass accelerated = 490 gm.

Now $F = ma.$

Hence, $30 \times 980 = 490\,a,$

or $a = 60$ cm. per sec. per sec.

(*b*) (1) Considering the 260 gm. mass only, we find it acted on by the following forces:

Downward..........260 × 980 dynes,
Upward.....................$T$ dynes.

The resultant *downward* force causing it to move with an acceleration of 60 cm. per sec. per sec. = 260 × 980 − $T$ dynes.

But $F = ma.$

$$\therefore 260 \times 980 - T = 260 \times 60,$$

whence $T = 239{,}200$ dynes.

(2) Considering the 230 gm. mass only, we have the following forces:

Downward............................$230 \times 980$ dynes,

Upward....................................$T$ dynes.

The resultant *upward* force causing it to move with an acceleration of 60 cm. per sec. per sec. $= T - 230 \times 980$ dynes.

But $\qquad F = ma.$

$$\therefore T - 230 \times 980 = 230 \times 60,$$

or $\qquad T = 239{,}200$ dynes, as before.

## PROBLEMS

1. Express:

   (1) A force of 10 kg. in dynes.

   (2) A force of 10 dynes in grams force.

   (3) A force of 12 pounds in poundals.

   (4) A force of 320 poundals in pounds.

2. A mass of 25 pounds lies on a table. Find the force it exerts on the table in (*a*) pounds, (*b*) poundals.

3. A mass of 5 kg. is acted on by a force which imparts to it an acceleration of 100 cm. per sec. per sec. Find the force in (*a*) dynes, (*b*) grams.

4. A certain force acts on a mass of 150 grams for 10 seconds, and produces in it a velocity of 50 metres per second. Compare the force with the weight of a gram.

5. A certain force acts on a mass of $m$ gm. and generates in it an acceleration $a$ cm. per. sec. per sec. Find the mass which the force would support.

6. How long must a force of 5 units act upon a body in order to give it a momentum of 3000 units?

7. What force acting for one minute upon a body whose mass is 50 grams will give it a momentum of 2250 units?

8. A force of 980 dynes acts vertically upward upon a mass of 5 grams, at a place where $g = 981$ cm. per sec. per sec. Find the acceleration of the body.

9. A mass of 10 kg. is acted upon for one minute by a force which can support a mass of 125 grams. Find the momentum which it will acquire.

10. A falling weight of 160 grams is connected by a string to a mass of 1800 grams lying on a smooth flat table. Find the acceleration and the tension of the string.

11. A mass of 3 kg. is drawn along a smooth horizontal table by a mass of 4 kg. hanging vertically. Find the displacement in 3 seconds from rest and the tension of the string.

12. A body of mass 9 grams is placed on a smooth table at a distance of 16 cm. from its edge, and is connected by a string passing over a pulley at the edge with a body of mass 1 gram. Find (1) the time that elapses before the body reaches the edge of the table, (2) its velocity on leaving the table.

13. A mass of 10 grams hanging freely draws a mass of 60 grams along a smooth table. Find (1) the displacement in 5 seconds, (2) the displacement in the 8th second, and (3) the velocity acquired between the 7th and the 12th seconds.

14. Two masses of 100 and 120 grams are attached to the extremities of a string passing over a smooth pulley. If the value of $g$ is 979 cm. per sec. per second, find the velocity after 8 seconds and the tension of the string.

15. A mass of 52 grams is drawn along a table by a mass of 4 grams hanging vertically. If at the end of 4 seconds the string breaks, find the space described by each body in 4 seconds more.

16. Masses of 800 and 180 grams are connected by a string over a smooth pulley. Find the space described in (1) 5 seconds, (2) the 5th second.

17. To the ends of a light string passing over a small smooth pulley are attached masses of 977 grams and $x$ grams. Find $x$ so that the former mass may rise through 200 cm. in 10 seconds. ($g = 981$).

18. If bodies whose masses are $m_1$ and $m_2$ are connected by a string over a smooth pulley, find the ratio of $m_1$ to $m_2$ if the acceleration is $\frac{1}{4}\,g$.

**54. Force Produced by a Fluid in Motion.** A wind is simply a portion of the atmosphere in motion and when it strikes a surface a force is exerted upon it,—sometimes sufficient to do great damage, as shown by the destruction caused by a tornado. The force from a current of air can cause a windmill to rotate and thus pump water or grind grain, while the force from a current of water can put in motion great turbine water-wheels, sometimes with the power of several thousand horses. A stream of water from a fire-hose can break a window, tear up shingles or do other things.

Whole hills have been removed by directing streams of water against them, thus loosening the earth and then carrying it away.

Consider a cube of matter 1 cm. to the edge and having mass 1 gram. If a force of 1 dyne act at right angles to one face of this for 1 sec. it will be given a velocity of 1 cm. per sec., and it will possess 1 unit of momentum.

If this matter were water the dyne force would give it unit of momentum as before.

Next imagine 1 c.c. of water (1 gm. mass) to be projected perpendicularly against a surface with such a speed that its forward momentum is destroyed in 1 sec. Then it will exert upon the surface a force of 1 dyne.

Suppose that a stream of water with area of cross-section 1 sq. cm., moving with a speed of 3 metres per sec., falls at right angles upon a wall, and suppose further that its forward momentum is completely destroyed by the impact.

In this case a cylinder of water (Fig. 41) 1 sq. cm. in section and 300 cm. long strikes a surface and has its momentum destroyed in 1 sec. The volume of this cylinder = 300 c.c. and its mass = 300 grams. As its velocity = 300 cm. per sec., its

$$\text{Momentum} = 300 \times 300 = 90{,}000 \text{ C.G.S. units,}$$

and this is destroyed in 1 sec.

The force against the surface must therefore

$$= 90{,}000 \text{ dynes} = 92 \text{ gms.-wt. (nearly).}$$

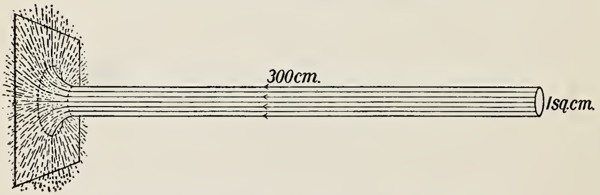



Fig. 41.—Force produced on a surface by a stream of water.

At first sight it would appear that we should be able to say that the *pressure* is 90,000 dynes per sq. cm. Experiment, however, has shown that this is incorrect, on account of the spreading out of the stream when it hits the wall. Actually the pressure at the centre of the stream is 45,000 dynes per sq. cm. and the pressure decreases as the distance from the centre increases.

**Examples.**—(1) A stream of water 1 sq. cm. in cross-section, moving with a speed of 20 metres per sec., strikes a wall at right angles. Find the force exerted against the wall.

Vol of water striking wall in 1 sec. = 2000 c.c. = 2000 grams.

Its velocity = 2000 cm. per sec.

Hence, its momentum = 2000 × 2000 = 4,000,000 units, and this is destroyed in 1 sec.

But the force = rate of change of momemtum

Hence force = 4,000,000 dynes,

= 4077 gm.-wt. (approx.).

(2) A hose delivers 600 gallons per minute with a speed of 60 ft. per sec. against a wall which it strikes at right angles. Find the force against the wall.

600 gal. per min. = 10 gal. per sec.

= 100 lb. " "

Mass of water striking wall in 1 sec. = 100 lb.

Its velocity = 60 ft. per sec.

Momentum destroyed in 1 sec. = 100 × 60

= 6000 F.P.S. units.

Hence force = 6000 poundals

= 187.5 pd.

**55. General Formula.** It will be useful for us to obtain the general formula for the force produced by a jet striking a surface at right angles.

Let the velocity of the jet = $v$ cm. per sec. and let its area of cross-section = 1 sq. cm. Then a cylinder of fluid $v$ cm. long and 1 sq. cm. in cross-section will fall upon 1 sq. cm. of the surface in 1 sec. Let $\rho$ = density of the fluid.

Mass of fluid = $\rho v$ grams,

Velocity = $v$ cm. per sec.

Hence, momentum = $\rho v^2$ units, which is destroyed in 1 sec.

Consequently $Ft = \rho v^2$

and as $t = 1$, $F = \rho v^2$ dynes.

In the F.P.S. system, $F = \rho v^2$ pdl.

It is to be observed that the force is not proportional to the velocity, but to its square.

In the case of a jet of water of area of cross section $A$ striking a surface at right angles the total force $= A\rho v^2$ dynes (or pdl.) provided the area of the surface is at least four times the area of the jet. It has been found from actual experiment that if the area is less, not all of the forward momentum of the fluid is destroyed and the formula no longer holds.

**56. Force Produced by an Air Current.** Similarly forces are produced by currents of air, but as the density of air is much less than that of water the forces are ordinarily much smaller.

**Example:**—Consider a column of air 1 sq. ft. in cross-section and travelling 30 miles per hr., striking a *large* surface at right angles.

Now 30 m.p.h. = 44 ft. per sec.

The air in a cylinder 44 ft. long and 1 sq. ft. in cross-section will have its momentum destroyed in 1 second.

| | |
|---|---|
| Its volume | = 44 cu. ft. |
| and its mass | $= 44 \times 0.08$ lb. |
| The momentum | $= 44 \times 0.08 \times 44$ (F.P.S.) units, |
| | = 154.9 units. |
| Hence, the force | = 154.9 pdl. |
| | = 4.8 pd. (approx.). |

If, however, we consider a *wind* striking the obstacle, not all of the forward momentum is destroyed by the impact and we cannot obtain the total thrust by simply multiplying the area of the surface in square feet by the force produced by the unit air column just considered.

For flat plates, it has been found experimentally that

$$F = k\,A V^2 \text{ pd.},$$

where $k = 0.00328$ for plates 5 ft. or more in diameter.

$A$ = area of plate in sq. ft.

and $V$ = velocity in miles per hour.

For a wind blowing with a velocity of 30 m.p.h. the force on a plate whose area = 100 sq. ft. would be

$$F = 0.00328 \times 100 \times 900 \text{ pd.},$$
$$= 295.2 \text{ pd.}$$

whereas, if the theoretical formula held, the force would be $100 \times 4.8 = 480$ pd.

## PROBLEMS

(For wind problems, use $F = 0.00328\,AV^2$ pd., where $A$ = area in sq. ft., and $V$ = velocity in m.p.h.)

1. A jet of water, 5 sq. cm. in section and moving with a velocity of 20 m. per sec., strikes a board at right angles. Find the force against the board.

2. A fire-hose delivers 400 gal. of water per minute at a speed of 20 ft. per sec. The water strikes a fence at right angles. Find the force exerted on the fence.

3. A wind with a velocity of 40 mi. per hr. strikes at right angles a sign 30 ft. long and 6 ft. high. Find the total force against the sign.

4. The wind-shield of an automobile has an area of 3 sq. ft. and is set vertically. Find the total force exerted by the air against the wind-shield when the car has a velocity of 30 miles per hour.

5. Find the force exerted on the same wind-shield when the car is being driven at 35 miles per hour against a wind blowing 15 miles per hour.

# CHAPTER VII

## Gravitation

**57. Why does a body fall to the Earth?** We have seen in Chapter V that the earth attracts bodies on it, so that they move with an acceleration of about 978 cm. per sec. per sec. at the equator and about 983 cm. per sec. per sec. at the poles. No one has yet found out why bodies tend to fall toward the earth, but we now know that this phenomenon is only a particular example of the action of a universal law. The same force which makes a stone fall to the earth, determines the path of the moon in its journey round the earth and also the orbits of the planets as they revolve round the sun.

**58. The Motion of the Planets.** For many centuries it was commonly believed that the earth was immovable and was at the centre of the uni-

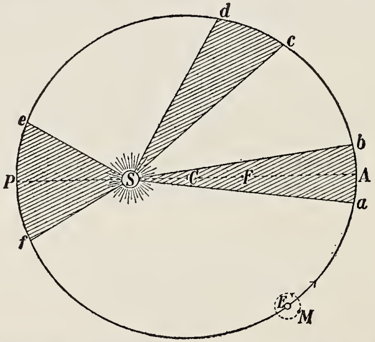



Fig. 42.—Diagram to illustrate Kepler's Laws. $S$ is the sun at one focus of the ellipse, $F$ is the other focus, $C$ is the centre, $P$ is perihelion, $A$ is aphelion. $CP$ or $CA$, that is one half of the axis-major, is the mean distance. $E$ is a planet moving in the elliptical orbit and $M$ is a moon revolving about it. Suppose the planet moves from $a$ to $b$, or $c$ to $d$, or $e$ to $f$ in the same length of time, say 30 days, then the shaded areas are equal. (The diagram is not drawn to scale; the orbits of the planets are more nearly circular).

verse, and that the sun and the other heavenly bodies revolved about it. This view is known as the Ptolemaic theory or hypothesis, since it was

proposed and supported by Claudius Ptolemy in a great work which he composed about 150 A.D. At last, however, it was overthrown, being superseded by the theory put forward by Nicolaus Copernicus (1473-1543), and hence known as the Copernican theory. According to it, the sun is the central body of our system, and the planets, of which the earth is one, revolve about it in circles. Following Copernicus came a famous Danish astronomer named Tycho Brahe (1564-1601). He was an enthusiastic observer, and, although he lived before the telescope was invented, he made many accurate measurements of the positions of the planets in the sky, especially of the planet Mars. These observations were placed in the hands of his pupil, John Kepler (1571-1630), who spent many years studying them and at last deduced from them three simple laws of planetary motion, as follows:

**(1) The orbit of each planet is an ellipse (not a circle as Copernicus thought) with the sun in one of its foci (Fig. 42).**

**(2) The radius vector (i.e., the line joining the sun and the planet) describes equal areas in equal times.**

**(3) The square of the periodic time of a planet is proportional to the cube of its mean distance.** (The mean distance is the semi-axis major of the elliptical orbit).

These laws form the basis of mathematical astronomy.

Kepler did not assign any reason why the planets should move in accordance with these laws, but he simply found that they must move thus in order to satisfy Tycho's observations.

**59. Newton's Law of Gravitation.** Contemporary with Kepler was Galileo Galilei (1564-1642), a distinguished Italian astronomer and physicist. He invented one form of telescope in 1609 and with it made many discoveries which strongly favoured the Copernican theory. In addition, he made many experiments in mechanics and stated some of its fundamental principles, thus effectually preparing the way for his successors.

It was felt by scientific men that there must be some physical principle which would supply the reason for Kepler's laws, and for fifty years the matter was a favourite subject for conjecture and discussion. At last Isaac Newton (1642-1727) proved that if we start with the simple hypothesis that the sun attracts each planet with a force which is inversely proportional to the square of its distance, Kepler's laws must necessarily follow.

On further consideration Newton was led to the view that each body attracts every other body in the same way that the sun attracts the planets. This is known as the **Principle of Universal Gravitation** and may be stated as follows:

**The attraction between any two bodies varies directly as the product of their masses and inversely as the square of the distance between them.**

Let $m_1$, $m_2$ be the masses of the two bodies, $r$ the distance between them.

The force of attraction is proportional to $\frac{m_1 m_2}{r^2}$,

or
$$F = k\,\frac{m_1 m_2}{r^2},$$
where $k$ is a numerical constant.

**60. Application to the Earth.** Consider a mass $m$ at $A$ on the earth's surface (Fig. 43), and let the mass of the earth be $M$.

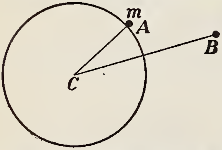


Fig. 43.—Attraction of the earth on a mass on its surface and also twice as far away from the centre.

Then, according to Newton's Law, the force of attraction between the two bodies is proportional to $M \times m$. But what is the distance between them? They are actually in contact.

Now it can be shown by mathematical calculation that a homogeneous sphere* attracts as though all the matter in it were concentrated at its centre. This point is its **centre of mass.** Indeed every body has a centre of mass and the distance between two bodies is to be understood as the distance between their centres of mass. The centre of mass of a body coincides with its centre of gravity (see Chapter XV).

If $m$ is a pound-mass on the earth's surface, the attraction of the earth on it is 1 pound-force or 1 pound-weight.

* A sphere is homogeneous when it is similar in all directions from the centre. It may differ at different distances from the centre but it is the same at the same distance.

If $m$ is a gram-mass, the attraction is 1 gram-force or 1 gram-weight.

If the mass of $m$ is 100 grams, the attraction is 100 grams-force.

If the mass of the earth could be doubled without altering its radius, the attraction would be doubled, since the force is proportional to the product of the masses.

Again, suppose the pound-mass to be at $B$, 2 radii or 8000 miles from $C$. The attraction is now not $\frac{1}{2}$, but $\frac{1}{2^2}$ or $\frac{1}{4}$ of a pound-weight or pound-force.

If it were 6000 miles or $\frac{3}{2}$ of the radius, the force $= \frac{1}{(\frac{3}{2})^2} = \frac{4}{9}$ of a pound-weight.

Read Section 26 again.

**61. Example of Newton's Law—Attraction on the Moon.** Let us calculate the weight of a pound-mass on the surface of the moon.

The moon's diameter is 2163 miles and the earth's is 7918 miles, but for ease in calculation we shall take these numbers as 2000 and 8000 respectively (Fig. 44).

Assuming, then, the radius of the moon to be $\frac{1}{4}$ that of the earth, its volume is $\frac{1}{64}$ that of the earth, and if the two bodies were equally dense the moon's mass would also be $\frac{1}{64}$ of the earth's mass. But the density of the moon is only $\frac{6}{10}$ that of the earth and consequently the mass of the moon is $\frac{6}{640}$ that of the earth.

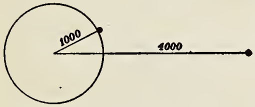



FIG. 44.—Attraction on the moon is one-sixth that on the earth.

Therefore the attraction on a pound-mass at a distance of 4000 miles from the moon's centre would be $\frac{6}{640}$ of a pound-force.

But the distance is 1000 miles, or $\frac{1}{4}$ of this, and the attraction on this account would be $4^2$ or 16 times as great.

Hence, attraction $= 16 \times \frac{6}{640} = \frac{1}{6}$ (approx.) pd.*

Hence, if we could visit the moon, retaining our muscular strength, we would lift 600 pounds with the same ease that we lift 100 on the earth.

* A more accurate calculation is

$$\left(\frac{2163}{7918}\right)^3 \times \left(\frac{7918}{2163}\right)^2 \times \frac{6}{10} = \frac{6489}{39590} = \frac{1}{6.101}.$$

If you can throw a base-ball 100 yards here, you could throw it 600 there.

On the surface of the sun, so immense is that body, the weight of a pound-mass is 27 pounds-force.

## QUESTIONS AND PROBLEMS

1. If the earth's mass were doubled without any change in its dimensions, how would the weight of a pound-mass vary?

Could one use ordinary balances and the same weights as we use now?

2. The attraction of the earth on a mass at one of its poles is $\frac{1}{568}$ greater than at the equator. Why is this?

3. A spring-balance would have to be used to compare the weight of a body on the sun or the moon with that on the earth. Explain why.

4. Find the weight of a body of mass 100 kilograms at 6000, 8000, 10,000 miles from the earth's centre.

5. The diameter of the planet Mars is 4230 miles and its density is $\frac{7}{10}$ that of the earth. Find the weight of a pound-mass on the surface of Mars.

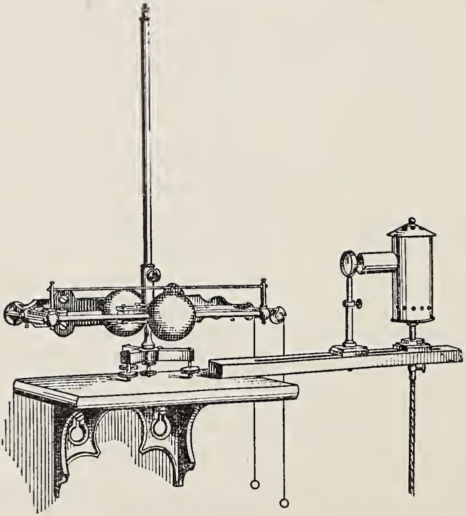


Fig. 45.—A modern laboratory form of Cavendish's apparatus. The horizontal glass case which contains the silver spheres is seen between the large lead balls; the long vertical tube encloses the quartz suspension and the mirror. Light from the lamp at the right enters the right-hand aperture, strikes the mirror and is reflected to a scale through the front aperture.

**62. The Cavendish Experiment.** The mass of the earth is so great that its attraction upon a mass at its surface is easily detected and measured, but between ordinary bodies the attraction is extremely small and to measure it is a task of great difficulty.

The first successful attempt to determine experimentally the attraction between two known masses was made in 1798 by Henry Cavendish,* an eccentric but very able English physicist and chemist. Fig. 45 shows the general appearance of a modern laboratory form of the apparatus which he used, while Figs. 46 and 47 show more of the details.

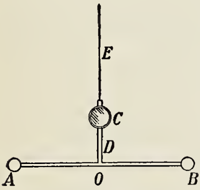


Fig. 46.—Elevation of Cavendish's Apparatus.

$AB$ (Fig. 46) is a light wire, about 5 cm. long, on the ends of which are silver spheres, each of mass approximately 1 gm. It is supported by a rod $D$ with a small mirror $C$ on it, and the whole is suspended by a fine fibre of quartz $E$ about 60 cm. long. Such an apparatus is called a torsion pendulum. When it is turned through an angle about the vertical and let go it is brought back by the torsion of the fibre and continues to oscillate for some time. It is very sensitive to air currents and is protected by a glass case. If we know the dimensions and mass of each part of the pendulum and observe the time required for an oscillation we can calculate the force required to turn it through any given angle. The angle through which the pendulum turns can be determined from the motion of a beam of light reflected from the mirror $C$ to a graduated scale.

The pendulum is so hung that the arm $AB$ is between two rods $L$, $M$ (Fig. 47), on which slide two large lead spheres $F$, $G$, about 8 cm. in diameter and weighing about 3 kg.

First, suppose the lead spheres are in the positions $F, G$. It is evident that the pull of $F$ on $A$ will be greater than the

* Cavendish was the first to recognize hydrogen as a distinct substance.

pull of $G$ on $A$ (indeed the latter may be neglected) and the pull of $G$ on $B$ will be greater than that of $F$ on $B$. Consequently the arm $AB$ will turn through a small angle in the clockwise direction. Next, let $F$ and $G$ be moved into the positions $H$ and $K$. It is clear that now $AB$ will turn in the contra-clockwise direction.

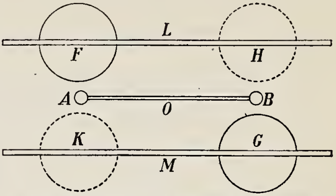


FIG. 47.—Plan of Cavendish's Apparatus.

By measuring the deflection of the spot of light on the scale when the large spheres are shifted, the angle of rotation of the pendulum is found. The force which produces this is the attraction of the lead spheres on the silver spheres. If the mass of a lead sphere is $M$ grams, that of a silver sphere is $m$ grams and the distance between their centres is $r$ cm., the force of attraction,

$$F = k\frac{Mm}{r^2} \text{ dynes.}$$

We know $F$ from the angle through which the pendulum is turned, and we know $M$, $m$ and $r$, and hence $k$ can be determined.

**63. Results of Experiments.** The experiment has been repeated frequently, with many variations in the apparatus. A very careful series of experiments was conducted by C. V. Boys, 1890 to 1893, who obtained the value 0.000,000,064,8 for $k$ when $F$ is measured in dynes. In other words, Boys found that two small spheres, each containing 1 gram of matter,* when placed with their centres 1 cm. apart, attract each other with a force of 0.000,000,064,8 dyne = $\frac{1}{15,000,000}$ dyne (approximately).

**64. "Weighing the Earth."** Having determined the gravitation constant $k$, it is possible to calculate what must be the

* A lead sphere 5.5 mm. in diameter (the size of a large pea), contains 1 gram of matter.

mass of the earth in order that it may exert the attraction upon a body at its surface which we have observed.

The earth acts for purposes of attraction as if all its mass were concentrated at its centre, and we know that it attracts a mass of 1 gram at its surface with a force of approximately 980 dynes.

From the equation $F = k\,\frac{m_1 m_2}{r^2}$ we have, then,

$$980 = \frac{1}{15{,}000{,}000} \cdot \frac{M \times 1}{R^2},$$

where $M$ is the mass of the earth in grams and $R$ its radius in cm. Taking the radius of the earth as approximately 6370 km., we obtain

$$980 = \frac{M}{15 \times 10^6 \times (6370 \times 10^5)^2},$$

whence $M = 6 \times 10^{27}$ grams (approx.),

$= 6.6 \times 10^{21}$ tons (approx.).

**65. The Density of the Earth.** Knowing the mass of the earth and its volume we can easily find its density. Cavendish found it to be 5.45 gm. per c.c.; Boys found 5.53 gm. per c.c.

This is about twice the density of substances in the crust of the earth; consequently (as we might expect), its density is greater as we descend below the surface.

**66. Von Jolly's Experiment.** The simplest method of finding the mass of the earth is that devised by Von Jolly at Munich in 1881.

The apparatus (Fig. 48) consists of a very accurate balance equipped with two sets of scale pans, the lower pans being suspended from the upper by long wires. Two equal spherical masses, $m_1$ and $m_2$, will, of course, balance one another if both are placed in the upper pans or in the lower pans. If, however, they are placed as in the figure, the attraction of the earth on $m_1$ will be greater than on $m_2$ because $m_1$ is nearer the centre of the earth.

The masses are placed as shown and equilibrium is restored by placing an additional small mass $a$ on the upper pan. A large mass $M$ is then brought underneath the lower pan and quite close to it. This large mass is so far from $m_2$ that the attraction between $M$ and $m_2$ is negligible and

equilibrium is disturbed because of the attraction between $M$ and $m_1$. Equilibrium is again restored by the addition of the small mass $b$ to the upper pan.

It is evident now that the pull of $M$ on $m_1$ is just equal to the pull of the earth on $b$.

Hence

$$k\frac{E\,b}{R^2} = k\frac{M\,m_1}{d^2},$$

where $E$ is the mass of the earth, $R$ is its radius, and $d$ is the distance between the centres of $m_1$ and $M$.

Therefore $$E = \frac{M\,m_1\,R^2}{b\,d^2},$$

and $E$ follows since all of the quantities on the right hand side of the equation are known.

In Von Jolly's experiments

$$m_1 = 5.00 \text{ kg.}$$
$$M = 5775.2 \text{ kg.}$$
$$b = 0.589 \text{ mg.}$$
$$d = 56.86 \text{ cm.}$$
$$R = 6366 \text{ km.}$$

whence $$E = 6.15 \times 10^{27} \text{ gm.}$$

This is about 2 per cent. too great but the difficulty in determining $b$ with greater accuracy readily accounts for the error.

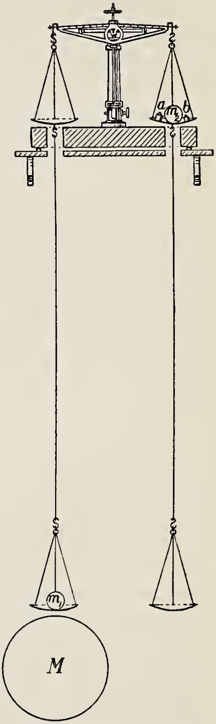



Fig. 48.—Von Jolly's apparatus for determining the mass of the earth.

## PROBLEMS

1. Find the gravitational attraction between two 11,000-ton ocean liners which are 100 metres apart. (Take 1 kg. = 2.2 lb.).

2. Calculate the attraction between two lead spheres, each having a mass of 10 kg., placed with their centres 20 cm. apart.

# CHAPTER VIII

## Action and Reaction

**67. Newton's Third Law.** On pressing together the thumb and a finger the force exerted by the thumb upon the finger is obviously equal to that exerted by the finger upon the thumb; or the *action* of the thumb upon the finger is equal to the *reaction* of the finger upon the thumb.

Again, consider a small vessel which is experiencing some difficulty in coming up to a dock. Someone in the vessel throws a rope to a person standing on the dock. He pulls steadily upon it and slowly the boat is brought in to the dock. In this case by exerting muscular effort a tension is produced in the rope, and it is evident that this pulls the boat in one direction and with an equal force pulls the man in the opposite direction. The action of the man upon the boat is equal and opposite to the reaction of the boat upon the man.

Someone may ask, if such is the case, why does not the man move towards the boat just as the boat moves towards the man? The answer to this becomes clear if we consider the man and the boat separately.

The forces acting upon the boat are:

(i) the pull of the rope in one direction, and

(ii) the friction of the water in the opposite direction.

The former is greater than the latter and so the boat moves forward.

The forces acting upon the man are:

(i) the pull of the rope in one direction, and

(ii) the friction of the dock on which he stands in the opposite direction.

The pull of the rope is not sufficient to overcome the friction of the dock and so he remains where he is.

But if two boats are floating on still water and a line is thrown from one to a person in the other, when he pulls on it each boat will move towards the other. In this case the friction of the water is not sufficient to balance the pull of the rope in the opposite direction and both bodies move.

A magnet attracts a piece of iron; does the iron attract the magnet? Lay the magnet on one piece of wood and the iron on another and float them on the surface of water. There is no doubt about what happens, each moves towards the other.

We are thus led to conclude that,

**Reaction is always equal and opposite to action;**

or in other words,

**The actions of two bodies upon each other are always equal and in opposite directions.**

This is Newton's **Third Law of Motion.**

**68. Impact of Two Bodies.** Suspend two exactly similar ivory or steel balls, *A* and *B*, side by side, as in Fig. 49. Draw *A* aside to *C* and let it go. Its velocity continually increases until it strikes *B*, when it suddenly comes to rest while *B* starts off.

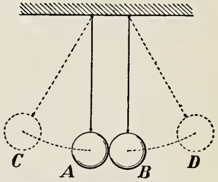


Fig. 49.—The action of *A* on *B* is equal to the reaction of *B* on *A*.

Repeat the experiment and observe closely the distance through which *B* swings. It will be found to move to *D*, approximately as far from *B* as *C* is from *A*. From this we conclude that *B* starts off with approximately the velocity which *A* has when it strikes *B*. The momentum possessed by *A* is thus transferred to *B*.

The action of *A* consists in exerting a force upon *B* which gives to *B* a certain velocity, that is, produces a certain momentum. The reaction of *B* consists in exerting on *A* an equal force in the opposite direction. As the balls are exactly

similar this force gives to $A$ an equal backward velocity which brings it to rest.

In this particular case the momentum of $A$ is handed on, practically without loss, to $B$; so that in the impact of $A$ and $B$ there is no change in the amount of momentum possessed by the two bodies in the horizontal direction from left to right.

If the balls are of wood the transfer of momentum is not complete. $A$ loses some but not all of its momentum, and $B$ gains the amount that $A$ loses. If made of wax or putty they may stick together, but in every case what momentum $A$ loses $B$ gains.

**69. Law of Conservation of Momentum.** Next, try an experiment with two trolleys (Fig. 50). First raise one end

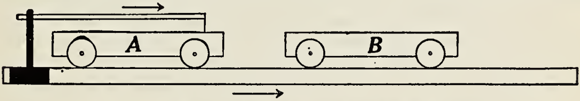



FIG. 50.—Experiment with two cars and a single tracing.

of the track until the cars will just run down with uniform velocity when they are started. This is to allow for friction.

Place $B$ at rest and arrange a vibrating brush so that it will write a tracing upon $A$ just before $A$ strikes $B$ and for some time afterwards. Also arrange that when $A$ strikes $B$ they will be automatically coupled together and so must move off with the same velocity.

Find the masses of $A$ and $B$ by weighing them and from the tracing find the velocity of $A$ before impact and of the two combined after impact.

Let $m_1$ = mass of $A$,

$m_2$ = mass of $B$,

$u$ = velocity of $A$ before impact,

$v$ = velocity of $A$ and $B$ after impact.

Then momentum before impact $= m_1 u$,
and " after " $= (m_1 + m_2)v$.

Now the *action* of $A$ on $B$ is to give to $B$ a momentum in the horizontal direction from left to right, and the *reaction* of $B$ on $A$ reduces the momentum of $A$ by the same amount, and so the entire momentum in the direction named should be the same after and before impact, or

$$m_1 u = (m_1 + m_2)v.$$

**Example.**—In Fig. 51 is shown a tracing obtained with two cars, each having a mass of 1.12 kg. The long waves of the tracing give

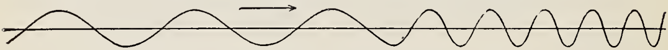


Fig. 51.—Tracing giving velocity before and after impact.

the velocity before impact. The wave-length comes out 10.0 cm., which is the distance travelled in $\frac{1}{5}$ sec., or the velocity was 50.0 cm. per sec.

The short waves give a velocity after impact of $4.9 \times 5 = 24.5$ cm. per sec., and the mass moving at this velocity = 2.24 kg.

Hence, before impact momentum $= 1.12 \times 50.0 = 56.0$ units,
after " " $= 2.24 \times 24.5 = 54.9$ "

These are approximately equal.

Again, try this experiment but do not use the automatic coupling. A brush for each car will be required as in Fig. 52.

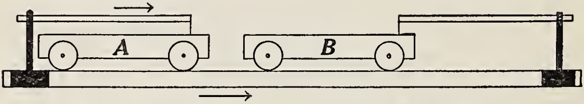



Fig. 52—Experiment with two cars and two tracings.

Have $B$ at rest, start $A$ with a smart push, and at the same time start both brushes vibrating. There will be a tracing on $A$ showing its velocity before and after impact, and one on $B$ showing its velocity after impact.

Let $m_1$ = mass of $A$,
$u_1$ and $u_2$ = its velocities before and after impact,

$$m_2 = \text{mass of } B,$$
$$v = \text{its velocity after impact.}$$

Then by the impact $A$ loses $m_1(u_1 - u_2)$ units of momentum and $B$ gains $m_2 v$ units.

According to the Third Law these quantities should be equal.

**Example.**—The following are the results of an experiment:

| | | | |
|---|---|---|---|
| Two cars, each | | = 1.12 kg. | |
| Velocity of $A$ before impact | = | $10.7 \times 5 = 53.5$ | cm. per sec. |
| " " $A$ after " | = | $2.8 \times 5 = 14.0$ | " " " |
| " " $B$ " " | = | $8.1 \times 5 = 40.5$ | " " " |
| Momentum lost by $A$ | = | $1.12\ (53.5 - 14.0) = 44.2$ | units |
| " gained " $B$ | = | $1.12 \times 40.5 = 45.4$ | " |

These quantities are approximately equal.

The experiments with the trolleys can be varied in many ways, by loading with different masses and giving different velocities.

Thus we arrive at the principle of **Conservation of Momentum:**

**In all cases of impact between two bodies, the momentum lost by one body is equal to that gained by the other, or the total amount of momentum after impact is equal to the total amount before impact.**

## 70. Examples.

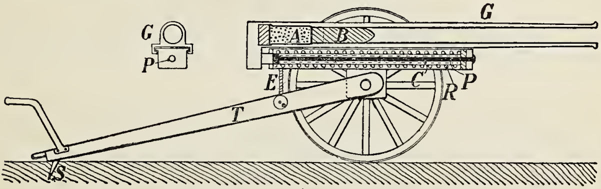



FIG. 53.—Mechanism of a field gun. $A$, charge; $B$, shell; $G$, gun supported on cradle on which it slides; $C$, inner cylinder containing oil, which recoils with gun; $P$, stationary piston with holes through which oil passes slowly to check recoil; $R$, running-up spring coiled around $C$; $E$, elevating screw; $T$, trail; $S$, spade.

The field gun shown in Fig. 53 provides an interesting example of action and reaction.

The force of the explosion, resulting from the ignition of the charge $A$, drives the shell $B$ forwards and the gun $G$ backwards at the same time. This force acts on shell and gun until the shell leaves the muzzle. The spade $S$, at the end of the trail $T$, keeps the carriage from running backwards, while the recoil of the gun is reduced to zero in about four feet by the action of the hydraulic buffer shown beneath the gun. The coil spring $R$, which is compressed during recoil, then returns the gun to its firing position.

Since the same force acts on gun and shell for the same time, the forward momentum of the shell as it leaves the gun will be equal to the backward momentum of the gun.

**Numerical Problems.**

(1) If the shell weighs 18 lb. and has a muzzle velocity of 2000 ft. per sec., find the velocity of recoil of the gun which weighs 1200 lb.

Momentum of shell $= 18 \times 2000$ units,

" " gun $= 1200 \times v$ "

Hence $1200\,v = 36{,}000$, or $v = 30$ ft. per sec.

(2) If the force of the explosion acts on the shell for $\frac{1}{20}$ sec., find the average force in tons.

From Newton's Second Law:

$$Ft = mv - mu,$$

Hence $$F \times \tfrac{1}{20} = 36{,}000,$$

or $$F = 720{,}000 \text{ pdl.}$$

$$= \frac{720{,}000}{32 \times 2000} = 11\tfrac{1}{4} \text{ tons.}$$

## PROBLEMS AND QUESTIONS

1. On stepping from a row-boat to the shore the boat moves backward, but on stepping from a steamboat no backward motion is noticeable. Why is this?

2. The bow of a row-boat is just touching a pier and a boy in the stern walks towards the bow with the intention of stepping onto the pier. What happens as he is moving forward? If he stops before he reaches the bow what happens?

3. If the sphere $B$ (Fig. 49) has a mass twice as great as $A$, what will happen (1) when $A$ and $B$ are of ivory? (2) when they are of sticky putty?

4. A hollow iron sphere is filled with gunpowder and exploded. It bursts into two parts, one part being one quarter of the whole. Find the relative velocities of the fragments.

5. A rifle weighs 8 lb. and a bullet weighing 1 oz. leaves it with a velocity of 1500 ft. per sec. Find the velocity with which the rifle recoils.

6. A gun weighing 6 tons fires an 18-lb. shell with a muzzle velocity of 1500 ft. per sec. Find the velocity of the recoil.

7. A shell of mass 12 lb. is discharged into a box of sand suspended by a rope and weighing 900 lb., and the combined mass begins to swing with a velocity of 25 ft. per sec. Calculate the velocity of the shell.

8. A railway train of mass 200 tons and moving at 6 ft. per sec. strikes a freight car of mass 40 tons standing still, and is automatically coupled to it. Find the speed with which the entire train begins to move.

9. A base-ball weighing 5 oz. and travelling forward at the rate of 40 ft. per sec. is struck and driven directly backward at the rate of 60 ft. per sec. What is the change in momentum? If the bat was in contact with the ball for $\frac{1}{25}$ sec. find the average value of the force exerted by the bat.

10. A 16-lb. shell leaves the muzzle of a gun with a velocity of 2000 ft. per sec. If the force of the explosion acts on the shell for $\frac{1}{20}$ sec., find the average force in tons. If the mass of the gun is 1000 lb., find the velocity of recoil.

**71. Centrifugal and Centripetal Force.** Tie a metal ball securely to the end of a string and whirl it about in a circle. You feel a distinct pull on the hand, and the faster the ball moves the stronger is the pull. We recognize that there is a tension in the string. Owing to this tension there is a pull upon the hand and an equal pull on the ball. Considering the hand and the ball as two bodies acting upon each other through the string connecting them, we may say that the action of the hand upon the ball is equal to the reaction of the ball upon the hand and in the opposite direction.

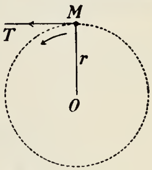


FIG. 54.—Motion in a circle.

Suppose that at some moment the ball is at $M$ (Fig. 54). The direction of its motion at this instant is along the line $MT$ which is the tangent to the curve at $M$; and if the string were to break the ball would move off in the line $MT$. But the string constrains

the ball to move out of this straight line path and to go along the curved path. Consequently it exerts a force on the ball in accordance with Newton's First Law of Motion. This force is always directed (along the string) towards the centre *O* and since it appears to cause the body to seek the centre it is known as the **centripetal force.** The force acting upon the hand is directed from the centre and is called the **centrifugal force.**

The pull on the hand leads most persons to think of a whirled body as endeavouring to fly off radially from the centre, but such is really not the case. The body, according to Newton's First Law, simply tries to continue in the straight line in which it at any moment may be considered as moving. It is clear also that the greater the mass of the whirled body, the greater is its inertia and consequently the greater is the centripetal force required to make it move in its curved course.

The grandest examples of bodies moving under centripetal forces are to be found in the solar system. If the attraction of gravitation should cease, the planets and their satellites would move off in straight lines.

**The centripetal force is the force required to overcome the inertia of a body when it is being deflected from a rectilinear into a circular path.**

**The centrifugal force is the reaction opposing the centripetal force; it is the resistance which the inertia of a body in motion opposes to whatever deflects it from its rectilinear path.**

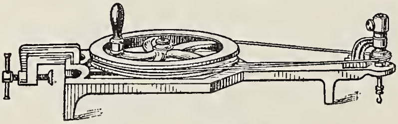


Fig. 55.—A whirling table.

**72. Experiments with Rotating Bodies.** The apparatus shown in Fig. 55 is called a whirling table. By means of it various bodies can be rotated rapidly.

The factors on which the magnitude of the centrifugal force depends may be investigated by the apparatus illustrated in Fig. 56. *A* is a mass of about 500 gm. which can move vertically on the support *B*. Two pieces of silk cord are attached to *A* and pass over the pulleys *P*. Masses of 10 gm. and 20 gm. slide on the horizontal rods *R* and may be attached to the cords by clamping screws.

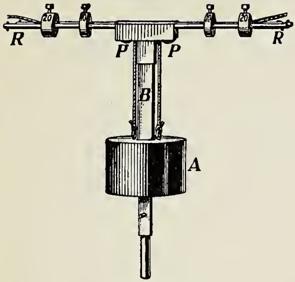


FIG. 56.

(1) Attach the 10-gram masses to the cords at about the middle of the horizontal rods and rotate the apparatus first slowly and then more rapidly. At a certain rate of speed the centrifugal force is sufficiently great to raise the mass *A*. Next clamp the 20-gram masses to the strings at the same points. It will be found that *A* rises when the apparatus is rotating at a slower speed.

(2) Attach the masses to the cords at points nearer the ends of the horizontal rods. It will be found that a slower rate of rotation is sufficient to cause *A* to rise.

It is evident, then, that the magnitude of the centrifugal force depends on the mass rotated, on the angular velocity, and on the distance of the rotating mass from the centre of rotation.

Actually

$$F = k\, m\, \omega^2\, r$$

where $F$ is the centrifugal force, $k$ is a constant, $m$ is the mass rotated, $\omega$ is the angular velocity* and $r$ is the radius of rotation.

**73. The Shape of the Planets.** The apparatus shown in Fig. 57 consists of two thin steel circles attached to collars at *A* and *B*. The collar at *A* is fixed in position on the vertical rod *AB* while that at *B* slides freely on the rod. When rotated rapidly the rings bulge out at the equatorial region and flattening occurs along the axis of rotation. The centrifugal force exerted by a mass at *C* is greater than that exerted by an equal mass at *D* rotating with the same angular velocity.

* The angular velocity is measured in radians per second. $\pi$ radians = 180 degrees.

Now the earth is somewhat flattened at the poles. It is believed that at one time the earth was in a plastic condition and that the flattening is due to its rotation upon its axis. The equatorial diameter is 7926.6 miles and the polar diameter 7899.6 miles. The difference is 27 miles which is about $\frac{1}{300}$ part of the diameter. This is so slight that if a person could observe the earth from a point far out in space the eye could not detect the flattening.

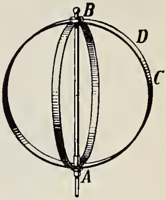


FIG. 57

But the flattening is much greater in the case of some of the other planets. Jupiter's equatorial diameter is 88,200 miles, its polar, 83,000 miles, or $\frac{1}{17}$ part less. The flattened form of the planet is easily observed in the telescope. Jupiter rotates on its axis in 9h. 55m., from which we deduce that a point on its equator moves with a velocity 29,437 miles per hour! We should not be surprised at the flattening.

The planet next in order from the sun and second in size in the system is Saturn. Its equatorial diameter is 75,000 miles, and polar diameter 68,000 miles, or $\frac{1}{11}$ part less. Thus its flattening is greater than that of Jupiter, but it is not usually

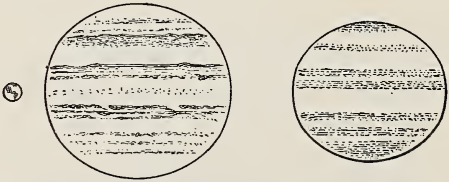


FIG. 58.—Shapes and relative sizes of the earth, Jupiter and Ball of Saturn.

so easy to detect, chiefly on account of the wonderful rings which surround the ball of the planet. Every fifteen years, however, the rings are turned edgewise to us and then the flattening is very evident. In Fig. 58 the shapes and the relative sizes of the earth, Jupiter and Saturn are shown.

## PROBLEMS

1. Calculate the linear velocity of a point on the earth's equator, taking the diameter as 7926.6 miles and the period of rotation as 23 h. 56 m.

2. Do the same for Saturn, taking the diameter as 75,000 miles and the period as 10 h. 14 m.

**74. The Centrifuge.** Place upon the spindle of a rotator a glass globe (Fig. 59*a*), containing some coloured water and a little mercury, and start it rotating. Both liquids creep up and form a band at the equator, with the heavier substance next the glass (Fig. 59*b*).

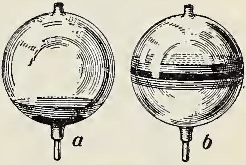


Fig. 59.—Apparatus to illustrate the action of a centrifuge.

If we consider equal sized particles of water and mercury at the same distance from the axis of rotation, then, since the mass of the mercury is much greater than that of the water, the centripetal force required to keep the mercury in this circle is greater than that needed for the water. The centripetal force is provided by the reaction of the walls of the globe transmitted through the liquid. If, then, there is a force just sufficient to hold the water in the circle it will not be great enough to retain the mercury, which consequently will move farther away from the axis of rotation.

The very useful instrument shown in Fig. 60 is called a centrifuge. Near the ends of its two arms are suspended two metal tubes closed at the lower end, and within these are placed glass tubes containing the substance to be investigated. The metal tubes can swing back and forth about horizontal pivots near the open end.

Inside the body of the instrument are multiplying gears, and when the handle is turned the tubes can be made to revolve very rapidly, often more than 2,000 times per minute. This causes the tubes to point directly outwards from the axis of rotation and the heavier portion of the substance is

driven to the bottom of the tubes. This apparatus is useful in testing blood and some other liquids.

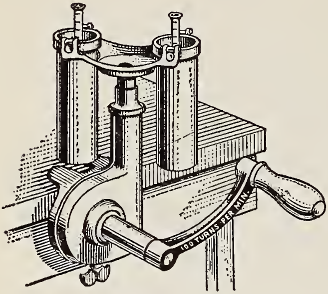


Fig. 60.—A centrifuge.

In the Babcock milk tester, used for determining the amount of butter-fat in milk, the fat is separated by a centrifugal machine similar to the above.

**75. The Cream Separator.** Milk consists of a liquid with small globules of fat distributed through its mass. Not very many years ago the ordinary practice was to place the milk in pans and allow the globules, which are lighter than the liquid, slowly to rise and collect at the surface as cream. This was then skimmed off and afterwards churned into butter. It has been found, however, that the cream can be taken from the milk much more completely and in a small fraction of the time by means of the now familiar cream separator.

The essential portion of the machine is a steel bowl which is rotated very rapidly. That used in a well-known type of separator is illustrated in Figs. 61, 62, 63, 64. The outside of the bowl, together with the gears for rotating it, is shown in Fig. 61. The lower part (see Fig. 62) is hollowed out underneath and is heavier around the rim, so that the bowl rests upright when placed on the end of a vertical axis, which is shown beneath. The conical shell of the bowl fits snugly into the lower part and rests on a rubber washer. By screwing down the nut at the top the bowl is made milk-tight. In Fig. 62, is shown the arrangement within the bowl. The central shaft is a projection upwards from the lower part of the bowl. It is hollow, thus forming a tube,

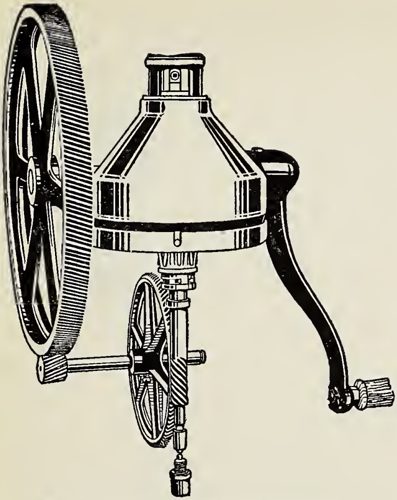


Fig. 61.—Bowl and gears of the separator.

and in the wall of the tube are slots opening into three enclosed gutters which lead into the bowl and end at some distance from the shaft. (See 2, 2, in Fig. 62). A series of conical 'discs' pressed from thin sheet-metal fit over the shaft. In these, holes are punched, being arranged to be just above the ends of the gutters.

The operation of the machine is somewhat as follows:—The bowl is put into very rapid rotation and the milk is admitted at 1. It passes down and comes out of the openings 2, 2, and is thus delivered between the discs some distance from the axis of the bowl. The centrifugal action causes the heavier liquid portion of the milk to go outwards along the under surface of the discs, and collect at the outer wall of the bowl.

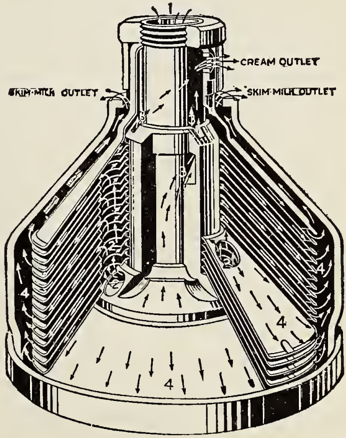



Fig. 62.—Inner view of separator bowl.

The cream finds its way inwards along the upper surface of the discs (Fig. 63). In this way the cream gathers at the

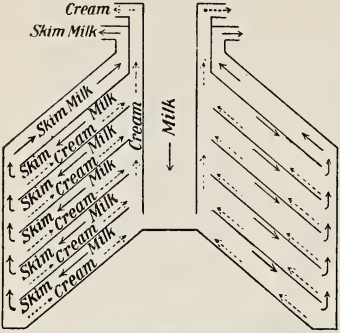



FIG. 63.—Showing passage of milk outwards and cream inwards.

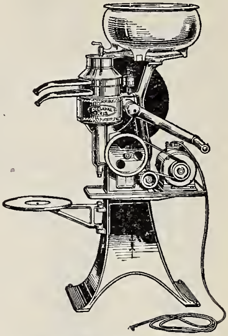


FIG. 64.—Separator, with electric motor.

centre and rises and, passing through 3, 3, comes out at the square outlet in the cap at the top. The skim-milk rises at the outer wall of the bowl and finds its way out of a rectangular opening beneath that for the cream. The cream and the skim-milk spurt out into vessels placed over the top of the bowl (see Fig. 64) and pass off by way of spouts leading from these vessels. The entire operation requires only a few minutes.

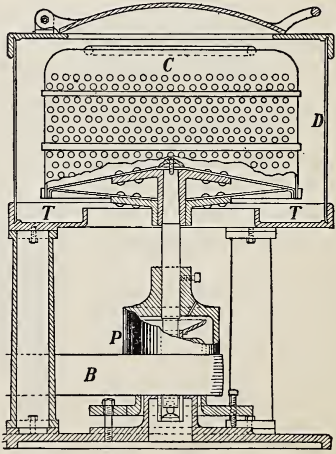



FIG. 65.—An extractor used in drying clothes.

The speed is very high, ranging (in the type of machine shown here) from 6000 revolutions per

minute for the larger sizes to over 8000 for the smaller ones, and on account of this high speed the machine must be well made and must be kept well oiled and in good order.

**76. The Extractor.** Centrifugal action is used extensively in drying clothes in laundries, in drying sugar and other crystals and in extracting honey from the comb.

Fig. 65 illustrates an extractor used in drying clothes. The wet clothes are placed in the perforated cylinder *C* which is made to rotate at a high speed by the belt *B* which drives the pulley *P*. The water is thrown through the holes in *C*, strikes the walls of the outer cylinder *D* and runs off through a waste pipe attached to the trough *T*. The other extractors mentioned are similar in construction.

### EXERCISES AND PROBLEMS

1. In a separator (Fig. 61) the crank revolved 60 times per minute, the large gear-wheel on the crank-shaft had 177 teeth, the small pinion into which it meshed had 19 teeth, the large worm wheel on its shaft had 103, and the worm screw on the axis supporting the bowl had 7 teeth. Calculate the revolutions per minute of the bowl.

2. In another machine the crank revolved 48 times per minute and the teeth on the gears were 247, 19, 93, 8, respectively. Find the revolutions per minute of the bowl.

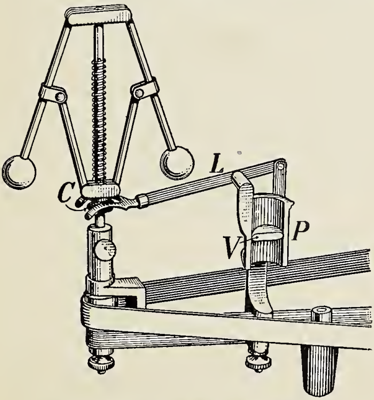



Fig. 66.—Model of steam engine governor.

3. Explain the action of the steam engine governor, a model of which is shown in Fig. 66. *P* is the steam pipe leading from the boiler to the cylinder and *V* is a valve actuated by the lever *L* which connects with the sliding collar *C*.

4. Why does a cyclist lean as he goes round a corner?

5. Explain the action of a "sling" used to throw a stone.

6. Why does a car skid in rounding a corner if the street is slippery?

7. Draw a diagram to show how the mud flies off a rotating car wheel.

8. Explain the action in the circus performance known as "looping the loop".

9. A man stands on a platform balance and swings a pail of water in a vertical circle. Will the reading of the balance vary? Explain.

10. Does the rotation of the earth affect the weight of a body? If so, where will the effect be greatest? Will it be shown on an ordinary balance or a spring balance?

11. Why are railway tracks and highways banked at the curves?

12. Explain the tendency of belts on high-speed pulleys to slip.

# CHAPTER IX

## Work, Energy, Power

**77. Meaning of 'Work' in Mechanics.** When water is drawn from a well by means of a bucket on the end of a rope, or when bricks are hoisted during the erection of a building, or when land is ploughed, or when a blacksmith files a piece of iron, or when a carpenter planes a board, it is recognized that *work is done.*

Let us analyse these operations. In the case of drawing water from the well, the water is attracted towards the earth, and the person pulling on the rope must exert upon the bucket a force just sufficient to overcome this attraction, that is, the weight of the bucket. The bucket is then displaced through a certain distance in the direction in which the force acts. **When a force acts upon a body and causes it to move in the direction of the force** we say that the **force does work,** though it would be more accurate to say that the agent exerting the force does work. The force in this case which resists the motion and which is overcome is gravity and it is customary to say that the work is done *against gravity.* We might also describe the production of work by stating that when motion takes place against resistance work is done.

In the raising of the bricks the circumstances are precisely similar to those just described. A force acts upon the bricks and displaces them in the direction of the force and the *force does work.*

In the other three cases a force has to be exerted sufficient to overcome the resistance opposed to it, a resistance similar to friction. The force is applied to the object (plough, file, plane) which moves in the direction of the force.

We thus see that the work done depends upon two factors,

(i) the force acting on the body;

(ii) the distance through which the body moves in the direction of the force.

As a force acts *at a point* in a body it is rather more accurate to speak of the motion of the point of application of the force than of the motion of the body as a whole.

It is evident that if a force twice as great is exerted, twice the work will be done.

Also, if the displacement of the body is doubled the work will be doubled.

Hence, if $F$ = force, and $s$ = space moved through, the work done, $W = Fs$.

This is a very important formula,

**Work done = force exerted × displacement.**

It must be clearly understood that unless motion takes place no work is performed from the standpoint of mechanics. The iron pillars supporting a building may be exerting great force but they are not doing any work. Atlas may hold up the world on his shoulders but he does not perform any work in doing so.

Work is done when a mass is moved with acceleration because force must be exerted to produce this acceleration. No work is necessary to keep a body moving with uniform velocity unless gravity or friction or resistance of some other sort has to be overcome. Anyone who has turned the crank of a cream separator or who has ridden a bicycle up a hill or against a wind will recognize the truth of these statements.

**78. Units of Work.** By choosing various units of force and of length we obtain different units of work.

| | | |
|---|---|---|
| If | unit of force | = 1 pound-force or pd.-wt., |
| and | unit of length | = 1 foot. |
| then | unit of work | = **foot-pound (ft.-pd.).** |

**Thus, 1 foot-pound is the work done when a force of 1 pound is exerted through a distance of 1 foot.**

This gravitation unit, the ft.-pd., is in general use by British and American engineers.

The corresponding metric engineering unit is the kilogram-metre, and as 1 kg. = 2.205 pd., and 1 m. = 3.28 ft.,

$$1 \text{ kg.-m.} = 2.205 \times 3.28 = 7.23 \text{ ft.-pd.}$$

In more purely scientific work the absolute units of force are used.

In the British (F.P.S.) system the absolute

| | | |
|---|---|---|
| | unit of force | = 1 poundal, |
| | unit of length | = 1 foot, |
| and hence | unit of work | = 1 **foot-poundal (ft.-pdl.)**. |
| Now, | 1 pd.-force | = $g$ pdl. ($g$ = 32), |
| and consequently | 1 ft.-pd. | = $g$ ft.-pdl. |

In the C.G.S. system the absolute

| | | |
|---|---|---|
| | unit of force | = 1 dyne, |
| | unit of length | = 1 cm., |
| and hence, | unit of work | = 1 **dyne-cm.** |

To this unit has been given the special name **erg.**

**Thus, 1 erg of work is done when 1 dyne force is exerted through a space of 1 centimetre.**

| | | |
|---|---|---|
| Now, | 1 gm.-force | = $g$ dynes ($g$ = 980), |
| and hence, | 1 gm.-cm. of work | = $g$ ergs, |
| and | 1 kg.-m. of work | = 98,000,000 ergs. |

An erg is a very small quantity and another unit, introduced through its convenience in electrical calculations, is often used, namely the **joule,** which = 10,000,000 or $10^7$ ergs.

**79. How to Calculate Work.** A bag of flour, 98 pounds, has to be carried from the foot to the top of a cliff, which has a vertical face and is 100 feet high.

There are three paths from the base to the summit of the cliff. The first is by way of a vertical ladder fastened to the face of the cliff. The second is a zig-zag path 300 feet long, and the third is also a zig-zag route, 700 feet long.

To perform this task a person, if he were strong enough, might strap on his back the mass to be carried and climb vertically up the ladder, or he might take either of the other two routes. The distances passed through are 100 feet, 300 feet, 700 feet, respectively, but the result is the same in the end, the mass is raised through 100 feet against gravity.

The force required to lift the mass is 98 pounds-force, and it acts in the vertical direction. The distance *in this direction* through which the body is moved is 100 feet, and therefore the

$$\text{Work} = 98 \times 100 = 9800 \text{ foot-pounds.}$$

Along the zig-zag paths the effort required to move the mass is not so great but the length of path is greater and the total work is the same in the end.

Again, let a loaded sleigh be drawn on a level road a distance 5 ft. by a force $F$ pd. acting in a direction making an angle $\theta$ with the horizontal (Fig. 67).

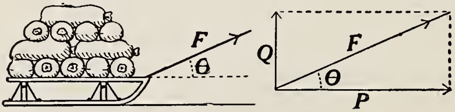



Fig. 67.—Calculation of work done in drawing a sleigh.

Here the displacement of the body is 5 ft., but it is not in the direction of the force.

The force $F$ may be resolved into components $P$ and $Q$ (Fig. 67), where $P = F \cos \theta$, $Q = F \sin \theta$. (See Sec. 111).

The force $Q$ is perpendicular to the direction of motion, and hence does no work.

The force $P$ is in the direction of motion, and the work done by it $= P \times s = (F \cos \theta) \times s$.

**Examples.** (1) Let $F = 25$ pd., $\theta = 20°$, $s = 1$ mile $= 5280$ ft.

Work done $= 25 \times \cos 20° \times 5280 = 12{,}403$ ft.-pd.

(2) A canal horse tows a boat by means of a rope which is inclined 30° to the direction of motion. The tension of the rope is 100 pd. Find the work done in going 2 miles.

Work $= 100 \times \cos 30° \times 5280 \times 2 = 91{,}435$ ft.-pd.

(3) A force is applied to a mass of 10 kg. and gives it a uniform acceleration of 60 cm. per sec. per sec. It moves through a distance of 5 metres. Find the work done by the force.

We must first determine the magnitude of the force, and it will be best to use C.G.S. units throughout.

| | | |
|---|---|---|
| Mass | $m = 10{,}000$ grams. | |
| Acceleration | $a = 60$ cm. per sec. per sec. | |
| Hence force | $F = ma = 600{,}000$ dynes | (Sec. 48) |
| and work done | $= 600{,}000 \times 500$ ergs, | |
| | $= 300{,}000{,}000$ ergs $= 30$ joules. | |

**80. The Inclined Plane.** Let us investigate experimentally the work done in moving a body up an inclined plane.

In Fig. 68, $C$ is a well-oiled car connected to a weight $P$ by a string which passes over a light "frictionless" pulley. The string between the car and the pulley is parallel to the plane.

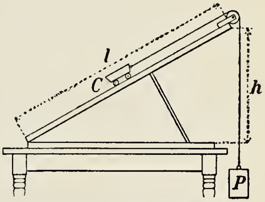


FIG. 68.—Show that $Pl = Wh$.

Set the inclined plane at an angle of about 30°. Place a weight of (say) 200 gm. in $C$ and adjust $P$ until $C$ just moves up the plane without acceleration. This can be done by attaching a pail to the cord and using sand, water or shot to increase the weight. Let $P_1$ be the weight in this case. Then lighten $P$ until $C$ just moves down the plane. Let $P_2$ be the weight now. Take $\frac{1}{2}(P_1 + P_2) = P$ as the weight required to balance $C$ if there were no friction.

Let $W$ be the total weight of the car and its load.

It is evident that when $C$ passes from one end of the plane to the other it rises through a distance $h$, the height of the plane, and hence the work done against gravity is $Wh$; while $P$ passes through a distance $l$, the length of the plane, and so does work $Pl$.

The distances $h$ and $l$ may be measured with a metre rod.

In the following table are some values obtained by experiment:

| $P_1$ | $P_2$ | $P = \frac{1}{2}(P_1+P_2)$ | $l$ | $Pl$ | $W$ | $h$ | $Wh$ |
|---|---|---|---|---|---|---|---|
| 181.5 gm. | 159.5 gm. | 170.5 gm. | 76 cm. | 12958 | 468.9 gm. | 27.7 cm. | 12988 |
| 261.9 | 226.5 | 244.2 | 76 | 18559 | 669.4 | 27.7 | 18542 |

In every case $Pl = Wh$ within the limits of experimental error.

We conclude, then, that the work done on $W$ is equal to the work done by $P$ (neglecting friction).

Neglecting friction, the work done by the engine of an automobile in driving it up a hill at uniform speed is equal to the work which would be done in lifting the car vertically through a distance equal to the height of the hill.

**Question.**—In view of the last statement, why is so much labour spent to lessen the grade of hills on highways?

## PROBLEMS

1. A force of 10 pounds acts through a space of 10 feet. Find the work done in (*a*) foot-pounds, (*b*) foot-poundals.

2. A force of 20 pounds acts through a space of 32 feet. Find the work done in (*a*) foot-pounds, (*b*) foot-poundals.

3. Find the work done in exerting a force of 1000 dynes through a space of 1 metre.

4. A block of stone rests on a horizontal pavement. A spring-balance, inserted in a rope attached to it, shows that to drag the stone requires a force of 90 pounds. If it is dragged through 20 feet, what is the work done?

5. The weight of a pile-driver, of 2500 pounds mass, was raised through 20 feet. How much work was required?

6. A coil-spring, naturally 30 centimetres long, is compressed until it is 10 centimetres long, the average force exerted being 20,000 dynes. Find the work done. Find its value in kilogram-metres. ($g = 980$).

7. Two men are cutting logs with a cross-cut saw. To move the saw requires a force of 50 pounds, and 50 strokes are made per minute, the length of each being 2 feet. Find the amount of work done by each man in one hour.

8. To push his cart a banana man must exert a force of 50 pounds. How much work does he do in travelling 2 miles?

9. Find the work done in raising 1000 litres of water from a well 10 metres deep.

10. Supposing that a man, whose weight is 100 kg., in walking raises his whole mass a distance of 10 cm. at every step, and that the length of the step is 50 cm., find how much work he does in walking 500 metres.

11. A ladder 10 metres long rests against a vertical wall, and is inclined at an angle of 60° to it. How much work is done in ascending it by a man weighing 80 kg.?

12. How much work is done in lifting 8 kg. to a height of 12 metres above the surface of the moon, where $g$ is 150 cm. per sec. per sec.?

13. A circular well 1.4 metres in diameter is 10 metres deep. Find the work expended in raising the material, supposing that a cubic metre of it weighs 2500 kg.

14. The cylinder of a steam engine (see Sec. 92) has a diameter of 14 cm. and the piston moves through a distance of 20 cm. Find the work done per stroke if the pressure of the steam in the cylinder be constant, and equal to 5 kg. per square centimetre.

**81. Definition of Energy.** In preparing the foundation for a bridge, a wharf or other structure, frequently piles are driven into the ground, and the method of doing this is well known. After sharpening one end of a log it is stood upright and then a heavy iron mass is raised to a considerable height and allowed to fall upon the upper end of the log. This is repeated time after time, and with each blow the log sinks farther into the earth, until at last it is down far enough. In Fig. 69 is shown a pile-driver.

Now to thrust the log into the earth requires a great force and therefore in driving it home considerable work is performed. The ability to do this work is possessed by the mass of iron moving with the velocity it has acquired in falling. *A mass in motion is able to do work.*

A hammer moving with some speed is able to drive in a nail against considerable resistance; and a rifle bullet of mass but half an ounce, moving with very great velocity, can penetrate almost anything in its path, and thus perform much work.

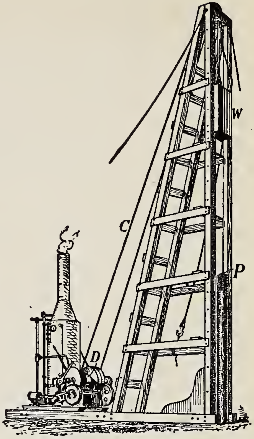


FIG. 69.—A pile-driver. The heavy mass $W$ is raised by the cable $C$ which passes around the drum $D$, driven by the engine. On falling, $W$ drives the pile $P$ into the ground.

**Ability to do work is called *Energy*.**

Again, the velocity possessed by the pile-driver weight arises from its having fallen from a height. By doing work on this body, against the force of gravity, it is given an advantageous position and we can look on it as possessing energy by virtue of its position. The source of the energy, however, does not reside in the body but rather in its separation from the earth. The body and the earth form a *system* and by changing the shape of the system energy is given to it.

By the performance of work we give the body energy of position, and as it falls its energy of position is changed into energy of motion, which is used up in doing work.

We see then that there are two kinds of energy:

(i) Energy of position, or **potential** energy (P.E.).

(ii) Energy of motion, or **kinetic** energy (K.E.).

As another example of a body possessing energy of position, a spring wound up may be mentioned. It is able to drive a clock, or a phonograph, or do other kinds of work.

**82. How to Measure Energy.** Since energy is the ability to do work it can be measured in the same units as are used in measuring work.

Suppose a mass $m$ grams to be lifted through a height $h$ cm. (Fig. 70).

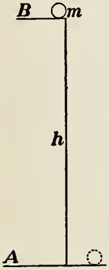



FIG. 70.—The potential energy at height $B$ is equal to the kinetic energy on reaching $A$.

Force exerted $= m$ gm.-force $= mg$ dynes.

Displacement of body $= h$ cm.

Hence work done $= mgh$ ergs.

Now allow the mass to fall. Upon reaching the former level $A$ it will have acquired a velocity $v$ such that

$$v^2 = 2gh, \text{ or } gh = \tfrac{1}{2} v^2 \qquad \text{(Sec. 34)}$$

The P.E. possessed by the body at $B$ is $mgh$ ergs (the work expended in putting it there), and if we assume for the present that this energy of position is completely changed into energy of motion on reaching $A$,

the K.E. at $A$ $= mgh$ ergs.

But $gh = \frac{1}{2} v^2$,

and therefore the K.E. $= \frac{1}{2} mv^2$ ergs.

In the F.P.S. system,

let mass $= m$ lb., and velocity $= v$ ft. per sec.

Then K.E. $= \frac{1}{2} mv^2$ ft.-pdl. $= \frac{1}{2} \frac{mv^2}{g}$ ft.-pd.

since 1 pd.-force $= g$ pdl. ($g = 32$) (Sec. 52)

**83. More General Solution.** In the last section the expression for the K.E. was obtained on the assumption that it was developed through the force of gravity, but as the result is very important it is desirable to show that we reach the same formula for any force.

Imagine that we have been transported far off in space, away from the earth or any other body.

Let a force $F$ dynes act for $t$ seconds on a mass $m$ grams initially at rest at $A$ (Fig. 71).

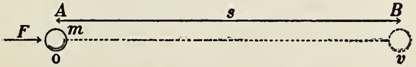



FIG. 71.—The calculation of kinetic energy.

Let it move from $A$ to $B$ and let the velocity at $B$ be $v$ cm. per sec., and the space traversed be $s$ cm.

The force $F$ dynes acts on the mass which moves through a space $s$ cm., and hence the force does $Fs$ ergs of work. Consequently at the end of the time the body possesses K.E. $= Fs$ ergs.

$$\begin{array}{lrl} \text{But} & F &= ma. \\ \text{Hence} & \text{K.E.} &= mas. \\ \text{Also,} & v^2 &= u^2 + 2as, \\ & &= 0 + 2as. \\ \text{Hence} & as &= \tfrac{1}{2}v^2, \\ \text{and} & \text{K.E.} &= \tfrac{1}{2}mv^2 \text{ ergs.} \end{array}$$

Whenever, then, a body of $m$ grams is moving with a velocity of $v$ cm. per sec. it possesses $\frac{1}{2}mv^2$ ergs of kinetic energy.

If the initial velocity of the body is $u$ cm. per sec. and the final velocity $v$,

$$\begin{array}{lrl} \text{Then work done} & &= Fs \text{ ergs, as before.} \\ \text{But} & Fs &= mas, \\ & &= m\left(\dfrac{v^2 - u^2}{2}\right), \\ & &= \dfrac{mv^2}{2} - \dfrac{mu^2}{2}. \end{array}$$

In this case the work done is equal to the difference between the kinetic energy at $B$ and at $A$.

**84. Examples.** (1) Find the K.E. of 1 kg. after falling 1 metre.

In this case force acting on mass $= 1000 \times 980$ dynes.

Distance fallen through $= 100$ cm.

K.E. acquired = work done $= 980 \times 1000 \times 100$,

$= 98{,}000{,}000$ ergs.

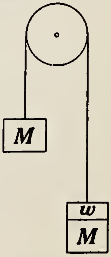


Fig. 72.

(2) A mass of 6 kg. is moving with a velocity of 60 cm. per sec. What is its K.E.? If brought to rest by a constant force in a distance 100 cm., what is the force?

Here, mass $m = 6000$ gm., velocity $v = 60$ cm. per sec.

$$\text{K.E.} = \tfrac{1}{2}\,mv^2 = 10{,}800{,}000 \text{ ergs.}$$

Also, if $F$ = opposing force in dynes,

$$Fs = \tfrac{1}{2}\,mv^2, \text{ and } F = 108{,}000 \text{ dynes.}$$

(3) In Fig. 72 let $M = 14$ lb., $w = 1$ lb. Find the velocity, and the acceleration, when the masses have moved through 2 ft.

Mass $M$ rises and gains P.E. $= 14 \times 2$ ft.-pd.
$= 14 \times 2 \times 32$ ft.-pdl.
Mass $M$ has also gained K.E. $= \frac{1}{2} \times 14 \times v^2$ "
Mass $M + w$ has lost P.E. $= 15 \times 2 \times 32$ "
" " gained K.E. $= \frac{1}{2} \times 15 \times v^2$ "

Considering the two masses as one "system," there has been no change in the total energy, or the loss = the gain.

Hence, $15 \times 2 \times 32 = 14 \times 2 \times 32 + \frac{1}{2} \times 14 \times v^2 + \frac{1}{2} \times 15 \times v^2$,
and $v^2 = 128/29$, or $v = 2.10$ ft. per sec.
Also $v^2 = 2as$, and $a = 1.10$ ft. per sec. per sec.

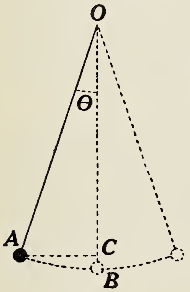


Fig. 73.—Finding velocity at lowest point $B$.

(4) A mass $m$ grams hangs at the end of a light cord $l$ cm. long. It is drawn aside through the angle $\theta$ from position $OB$ to $OA$ (Fig. 73), and allowed to swing. Find its velocity at its lowest point.

In position $A$ the energy of the mass is entirely potential.

P.E. $= m \times BC$ gm.-cm. $= mg \times BC$ ergs.

When in position $B$ its P.E. has been completely changed into K.E. and $= \frac{1}{2}\, mv^2$ ergs.

Hence, $\frac{1}{2}\, mv^2 = mg \times BC$ or $v^2 = 2g \times BC$.

Hence, the velocity at $B$ is the same as if the mass had simply fallen freely through a distance $CB$.

(5) If $OA = 100$ cm. and $\theta = 60°$, find the velocity at $B$.

$$\begin{aligned} \text{P.E. at } A = mgh &= m \times 980 \times BC, \\ &= m \times 980\,(100 - 100 \cos 60°), \\ &= m \times 980 \times 50 \text{ ergs.} \end{aligned}$$

Now K.E. at $B$ $= \frac{1}{2}\, mv^2$.

Hence $\frac{1}{2}\, mv^2 = m \times 980 \times 50$,

or $v^2 = 2 \times 980 \times 50$,

and $v = 140\sqrt{5}$ cm. per sec.

**85. Transformation and Transference of Energy.** Energy has been defined as ability to do work, and anything from which we can get work is a source of energy. We must therefore consider falling water, coal, an electric current, the sun, as among our sources of energy. We thus see that energy

appears in many forms. The various effects due to heat, light, sound and electricity are simply manifestations of it.

In dealing with the motion of bodies we were led to believe that there are two distinct forms of energy, namely, energy of position and energy of motion. Now it is difficult to determine accurately the nature of some of the forms of energy met with, but the farther the investigation proceeds the more firmly becomes the conviction that all energy can be considered to be either potential or kinetic. When sound is produced the particles of air or other substance are in vibration. Heat and light are believed to be due to vibrations of some material particles, and similarly electricity is conceived to possess energy in the potential or the kinetic form, or perhaps both at the same time.

**The utmost that a machine, whether a living body or an inanimate thing, can do is to transform energy from one form into another or transfer it from one body to another. It can never create it.** The energy of coal when burned in the furnace is changed into the energy of heat, and this is changed into the energy of steam. The steam drives the engine, which can pump water, saw wood or make a dynamo generate an electric current. The energy of the current may be conveyed to another place and there produce heat or light or chemical action or drive a motor.

**It has been established, or at least made extremely probable, by numerous careful experiments extending over many years, that there is no change in the total amount of the energy in our universe.** This is now looked upon as one of the grand laws of nature and is known as the law of the **Conservation of Energy.**

We start with a definite amount of energy in the coal, and if it were possible to keep a strict account of the different forms into which it is changed and the amount in each form and could add them all together, we would have at last precisely the same total that we had at the beginning.

It should be observed, however, that energy may be in existence without being available for use. Thus there is much heat energy in the ocean but we cannot at present make any commercial use of it. When a railway train is brought to rest, the energy of motion of the train is changed by the friction into heat energy, which is radiated or conducted away and is lost to us. In any 'system' the tendency is towards dissipation and degradation of its energy to a condition where it cannot be used.

The law of the **Conservation of Matter** has been accepted for many years and is the basis of analytical chemistry. **Matter can be changed into many forms but the sum total remains the same. It cannot be created or destroyed.**

Force, on the other hand, is of an entirely different nature. On pulling a string, tension is developed in it, which disappears when we let go. Matter and energy are bought and sold but force cannot be. We are concerned not with the force but with the results produced by the force, that is, with the work done.

**86. The Ballistic Pendulum.** We can apply the Law of Conservation of Energy to the problem of finding the velocity of a projectile by means of the ballistic pendulum.

In Fig. 74, $A$ is a spring gun which fires the ball $B$ into the hollow cylinder $C$ where it is retained by a spring catch. The cylinder is supported by a light rod $D$ pivoted freely at $F$. When the gun is fired the pendulum swings through an arc of about thirty degrees and remains at its extreme position by reason of a pawl attached to $C$ which engages with a ratchet $E$. (See upper smaller diagram).

By measuring the height of a mark on $C$ in its new position, above the original position of the mark, we can calculate the gain in potential energy; and this potential energy must be equal to the kinetic energy at the beginning of the swing.

Let the mass of $B$ be $m_1$ gm.
and " " " $C$ " $m_2$ gm.
Let the new position be $h$ cm. above the old.
Then the gain in P.E. $= (m_1 + m_2)\, gh$ ergs.

Let the velocity at the beginning of the outward swing be $v_2$ cm. per sec.

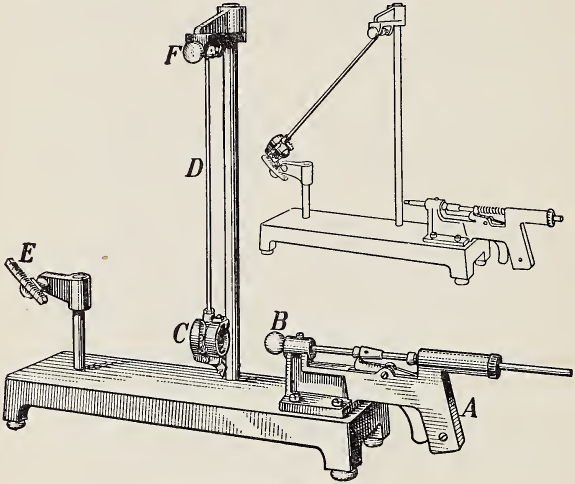



FIG. 74.—The ballistic pendulum.

Then the K.E. at the beginning of the swing

$$= \tfrac{1}{2}\,(m_1 + m_2)\, v_2^2 \text{ ergs.}$$

Hence $\tfrac{1}{2}\,(m_1 + m_2)\, v_2^2 = (m_1 + m_2)\, gh$,

and $v_2 = \sqrt{2gh}$ cm. per sec.

Let the velocity of $B$ be $v_1$ cm. per sec. before impact.

By the Law of Conservation of Momentum

$$m_1 v_1 = (m_1 + m_2)\, v_2,$$

$$\text{or } v_1 = \frac{(m_1 + m_2)\, v_2}{m_1} \text{ cm. per sec.;}$$

and since $m_1$, $m_2$ and $v_2$ are known, $v_1$ is easily calculated.

A form of ballistic pendulum is used for finding the velocity of a rifle bullet.

**Numerical Example.**

Let $h = 8.5$ cm.

$m_2 = 223.0$ gm.

$m_1 = 68.0$ gm.

Also $v_2 = \sqrt{2gh} = \sqrt{2 \times 980 \times 8.5} = 129.1$ cm. per sec.

Then $$v_1 = \frac{(m_1 + m_2)\,v_2}{m_1} = \left(\frac{68 + 223}{68}\right) \times \frac{129.1}{1}$$

$= 552.4$ cm. per sec.

## PROBLEMS

1. A mass of 64 pounds is moving with a velocity of 10 feet per second. Find its kinetic energy in (1) foot-poundals, (2) foot-pounds.

2. A mass of 10 grams is thrown vertically upward with a velocity of 980 cm. per second. Find its kinetic energy (1) at the instant of projection, (2) at the end of one-half second, (3) at the end of one second, (4) at the end of two seconds.

3. Find the kinetic energy of a cannon-ball whose mass is 10 lb. discharged with a velocity of 50 yards per second.

4. A stone of mass 6 kg. falls from rest. What will be its kinetic energy at the end of five seconds?

5. A 100-gram bullet strikes an iron target with a velocity of 400 metres per second and falls dead. How much kinetic energy has the bullet lost?

6. A stone whose mass is 100 lb. is carried to the top of a wall 40 feet high. What potential energy does the stone possess? If the stone is dropped, what kinetic energy will it have when it strikes the ground?

7. A hammer whose mass is one pound strikes a nail with a velocity of 20 feet per second. Find the kinetic energy possessed by the hammer when it is about to touch the nail. If it drives the nail a distance of $1\frac{1}{2}$ inches, find the average force in pounds exerted by the hammer upon the nail.

8. A cricket ball, whose mass is 100 grams, is given by a blow a velocity of 20 metres per second. What is the measure of the work done?

9. Calculate the kinetic energy possessed by a stone whose mass is 1 kg. after it has fallen from rest through a space of 1 metre.

10. Find the energy required to project a golf ball whose mass is 43 grams a distance of 100 metres vertically upwards.

11. A stone whose mass is 100 pounds falls freely from a point 400 feet above the ground. Find in foot-pounds (1) its kinetic energy, (2) its potential energy, at the end of the fourth second.

12. A mass of 20 pounds hanging at the end of a light cord 16 feet in length is drawn aside through an angle of 90° and then let go. Find (1) its kinetic energy in foot-poundals, (2) its velocity, when it reaches its lowest point.

13. A bullet weighing 2 oz. is fired into a bag of sand weighing 19 pd. 14 oz., suspended from a cord 10 ft. long, and remains imbedded in the sand. If the bag swings to one side until the cord makes an angle of 30° with the vertical, find the velocity with which the bullet struck the bag.

14. Compare the kinetic energy of a meteor 1 gm. in mass, travelling 25 miles per sec., with that of a 1-ton truck moving at 45 miles per hr. (1 kg. = 2.2 lb.).

**87. Power.** In stating the amount of work done the question of *time* does not enter at all. A man could dig a big cellar quite as well as a steam shovel can, if he were given time enough, but when he got through, the need for the building to be erected over it might be past. In ordinary life we must consider time, or the rate at which work is performed.

**The *Power* or *Activity* of an agent is its rate of doing work.**

**88. The Horse-power; the Watt.** The chief use of the steam engine at first was to pump water from the mines. Horses had been utilized for this as well as for many other purposes, and it was natural that James Watt,* after making the engine really efficient, should rate it in terms of the power of a horse. In order to do this he made experiments with strong dray-horses and finally he decided to call a horse-power the ability to perform 33,000 ft.-pd. of work in 1 minute, or 550 ft.-pd. in 1 second. As a matter of fact, this is much

*Watt died in 1819, more than one hundred years ago. The story of his life is very interesting.

greater than any ordinary horse can continuously perform, but it would seem that Watt was anxious that purchasers of his engines should be satisfied with their capabilities, and that they should be able to do more than their name would demand.

In the C.G.S. system the unit of power is the ability to do 1 erg per second, but this is an extremely small quantity and it is more convenient to choose a unit 10,000,000 times as great. This unit is called a **watt.**

$$\begin{aligned} 1 \text{ watt} &= 10{,}000{,}000 \text{ ergs per second,} \\ &= 1 \text{ joule per second.} \\ 1000 \text{ watts} &= 1 \text{ kilowatt (k.w.).} \end{aligned}$$

The horse-power can be expressed in watts as follows:

$$\begin{aligned} 1 \text{ ft.} &= 30.48 \text{ cm.,} \\ 1 \text{ pd.} &= 453.59 \text{ gms.-wt.} = 453.59 \times 981 \text{ dynes.} \end{aligned}$$

$$\begin{aligned} \text{Hence, } 550 \text{ ft.-pd.} &= 550 \times 30.48 \times 453.59 \times 981 \text{ ergs,} \\ &= 746 \times 10^7 \text{ ergs,} \\ &= 746 \text{ joules.} \end{aligned}$$

Hence, 1 h.p. $= 550$ ft.-pd. per sec. $= 746$ watts $= \frac{3}{4}$ k.w. (approx.),

and $$1 \text{ k.w.} = \frac{1000}{746} \text{ h.p.} = 1\tfrac{1}{3} \text{ h.p. (approx.).}$$

**A *Watt* is the power of an engine which can do 1 joule, or $10^7$ ergs, of work per second.**

**A *Horse-Power* is the power of an engine which can do 550 ft.-pd. of work per second or 33,000 ft.-pd. per minute.**

**89. Experimental Determination of Horse-power.** The usual method of determining the power of a motor or engine is to make it do work against the friction produced by a stationary belt which passes around a pulley driven by the motor. The work done can be measured and hence the power can be found. This is called the brake method of determining horse-power.

Suppose we wish to find the h.p. of a small water motor. The apparatus may be arranged as in Fig. 75. First, the bar at the top is pushed up until each balance reads about 300

gm. Then the water is turned on, and after the motor has attained full speed the reading on each balance is taken. The speed of the motor must also be found. This is done by means of a revolution counter (Fig. 76). The number of revolutions the motor makes in (say) 30 sec. is observed and the number per second calculated. Further, the circumference of the pulley must be measured.

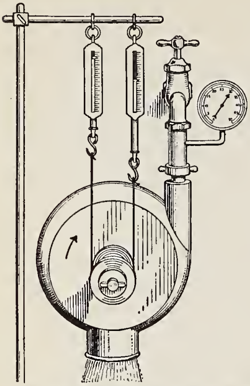


FIG. 75.—Testing the horse-power of a small water motor.

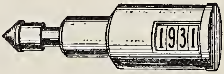



FIG. 76.—A revolution counter.

Let the reading on one balance be 550 gm.; that on the other, 50 gm.; the revolutions per sec. be 25; and the circumference of the pulley be 20 cm.

Then the friction = 550 − 50 = 500 gm.

In 1 revolution the motor does work against this force through a distance equal to the circumference of the pulley.

Hence, work done per sec.

$$= F \times s,$$

$$= \frac{500 \times 980 \times 20 \times 25}{10^7} = 24.5 \text{ joules},$$

and the power

$$= 24.5 \text{ watts},$$

$$= \frac{24.5}{746} = \frac{1}{30} \text{ h.p. (approx.)}$$

By varying the tension of the cord and along with it the friction produced we can find a certain speed at which the motor develops its maximum horse-power.

**90. Power of Heat Engines.** Both steam and gas engines convert heat energy into mechanical energy, the motion being

produced by the force exerted by an expanding gas against a piston moving in a cylinder. The steam engine is an *external* combustion engine, because the fuel is burned *outside* the cylinder; while the gas engine is an *internal* combustion engine since the fuel is burned *inside* the cylinder.

The mechanical efficiency of any machine

$$= \frac{\text{Work output}}{\text{Work input}} \times 100\%;$$

and the mechanical efficiency of a steam or a gas engine

$$= \frac{\text{Brake horse-power}}{\text{Indicated horse-power}} \times 100\%.$$

The brake horse-power is determined by a method similar to that described in the preceding section; while the indicated horse-power is found by considering the pressure in the cylinder, the area of the piston, the length of stroke and the number of working strokes per minute.

The mechanical efficiency of heat engines may be as high as about 85% but the *thermal* efficiency, or ratio of the mechanical work done to the heat energy contained in the fuel, is much lower, ranging from a maximum of 17% for steam engines to about 30% for oil engines.

**91. Power of a Gas Engine.** The method of determining the power and efficiency of a gas engine is shown diagrammatically in Fig. 77.

Let us assume that the engine is of the four-stroke cycle type. As the piston $A$ moves to the right, the explosive mixture is drawn into the cylinder $B$ from the carburetor through the intake valve $C$. On the return stroke of the piston the mixture is compressed in the cylinder. Then the mixture is fired by the spark at the spark-plug $D$, and the explosion pushes the piston to the right again. On the next return stroke of the piston the exhaust valve $E$ opens and the burned gases escape. The cycle,—intake stroke, compression stroke, power stroke and exhaust stroke,—then repeats. The motion of the piston is communicated to the fly-wheel $F$ by the connecting rod $G$ which is attached to the crank $H$.

The brake horse-power is determined as already described in Sec. 89, from the known values of the weight $W$, the reading of the balance $S$, the circumference of the fly-wheel, and the number of revolutions per second.

To determine the input we must know the force behind the piston and the distance the piston moves during each stroke. The latter is easily measured, while the former is determined by an instrument called an

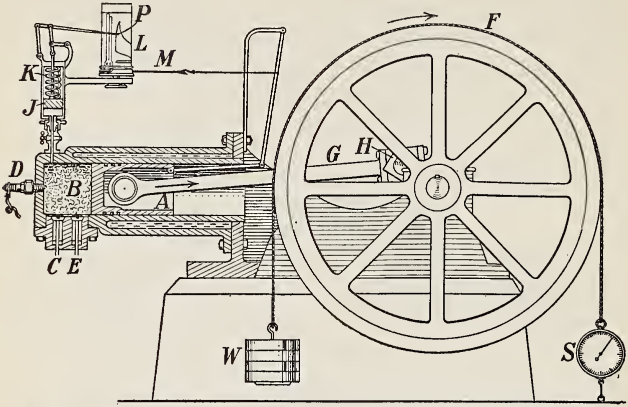



FIG. 77.—A gas engine equipped with indicator and brake for finding horse-power.

indicator. In the diagram is shown a Crosby pattern indicator attached to the cylinder.

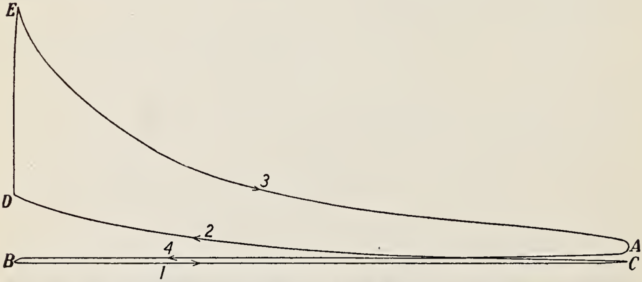



FIG. 78—Indicator diagram (actual size) for a gas engine. In this diagram a vertical distance of 1 inch represents a pressure of 200 pd. per sq. in.

The gas pressure in the cylinder acts against the piston $J$ and forces it upward against the back pressure of the spring $K$. The movement of $J$ is recorded by a pencil point $P$ on a sheet of paper wrapped around the

drum $L$, and the drum is made to rotate by a cord $M$ attached by links to the moving piston. The slack of the cord is taken up by a spring inside the drum.

It is evident that the pencil point will indicate the pressure inside the cylinder at every part of the stroke. A diagram for a complete cycle is shown in Fig. 78. Here $AB$ and $BC$ are lines made during the exhaust and intake strokes. These are nearly horizontal because the pressure is approximately atmospheric. $CD$ indicates what happens during the compression stroke. The explosion occurs at $D$, followed by the sharp rise of pressure shown between $D$ and $E$. Between $E$ and $A$ the pressure gradually decreases as the piston moves to the right.

The average pressure is obtained by dividing the area of the diagram by its length and multiplying the quotient by the spring factor.

Let $P$ = average pressure in pounds per square inch,
$A$ = area of engine piston in square inches,
$L$ = length of stroke in feet,
$N$ = number of explosions per minute.

Then work done per minute $= Fs$,
$= PALN$ ft.-pd.,

and the indicated horse-power $= \dfrac{PALN}{33{,}000}$.

**Example.**

*Input*:

Average pressure = 100 pd. per sq. in.,
Diameter of piston = 7 inches,
Explosions per min. = 90,
Length of stroke = 14 inches.

Indicated horse-power $= \dfrac{100 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{14}{12} \times 90}{33{,}000} = 12.25$ h.p.

*Output*:

Weight attached to belt, $W$ = 150 pd.,
Reading of balance $S$ = 10 pd.,
Diameter of fly-wheel = 4 ft.,
Revolutions per min. = 180.

Brake horse-power $= \dfrac{140 \times \frac{22}{7} \times 4 \times 180}{33{,}000} = 9.6$ h.p.

Mechanical efficiency $= \dfrac{9.6}{12.25} \times 100 = 78.4\%$.

**92. Power of a Steam Engine.** The essential working parts of a simple steam engine are shown in Fig. 79.

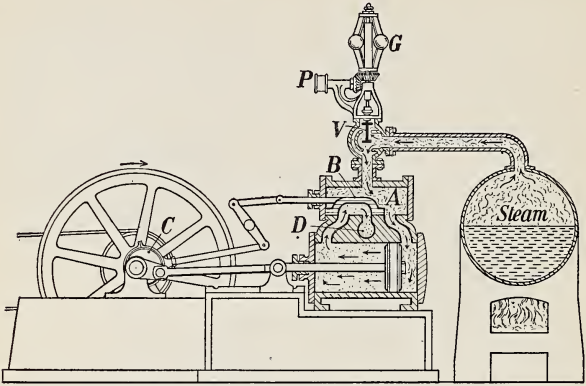



Fig. 79.—Essential working parts of a steam engine. (The governor $G$ has been turned through a right angle to show the pulley $P$ and the bevel gears).

One of the main differences between this engine and the gas engine described in the preceding section is that both ends of the steam engine cylinder are closed. The steam passes from the boiler into the steam-chest $A$ and thence, first into one end and then into the other end of the cylinder, according to the position of the slide-valve $B$. As the piston moves in one direction the slide-valve moves in the opposite direction, being actuated by the eccentric $C$. As the piston is driven forward by the steam from the boiler, the steam on the other side of the piston escapes into the air, or into a condenser, through the exhaust pipe $D$. The speed is controlled by the governor $G$, which actuates the valve $V$.

The brake horse-power is found by the same method as is used in the case of the gas engine.

In calculating the indicated horse-power indicator diagrams must be taken for both ends of the cylinder, and in applying the formula

$$\text{I.H.P.} = \frac{PALN}{33{,}000}$$

it is evident that $N$ will represent the number of single strokes of the piston, while in the gas engine it represented the number of explosions per minute.

## PROBLEMS

1. At what rate (measured in horse-power) is work being done when 1100 pounds of water are lifted every second from a well 100 feet deep?

2. Find the horse-power of an engine that will pump every hour 660 tons of water from a mine 600 feet deep.

3. Water flows into a mine 990 feet deep at the rate of 100 cubic feet per minute. What is the horse-power of the engine that will keep the mine dry? (A cubic foot of water weighs 62.5 pounds).

4. It is estimated that 700,000 tons of water pass over Niagara Falls every minute, and fall 160 feet to the lower level. If it were permissible to take one-tenth of the water for commercial purposes, what horse-power could be developed therefrom?

5. A motor is capable of hoisting 1320 tons of coal from the bottom of a mine 1200 feet deep per hour. Find its horse-power.

6. A force of 10 dynes acting on a mass moves it through 60 cm. in 10 seconds. What is the power?

7. A force of 30 dynes acting on a mass moves it through 2 metres in a minute. What is the power?

8. A mass of 20 grams is lifted vertically a distance of 1 metre in 196 seconds. What is the rate of working?

9. It is found that six million dynes are required to keep a street-car in motion, while it passes over 1 kilometre in 10 minutes. Determine the rate of working in watts.

10. A force of ten million dynes is required to draw a car along a track at the rate of 36 kilometres per hour. What is the rate of working in watts?

11. A man pumps 600 kilograms of water from a well 10 metres deep in 49 minutes. At what rate, measured in watts, is he working?

12. Calculate the horse-power of a steam engine which will raise 1,200 kilograms of water per minute from a well 149.2 metres deep.

13. A man whose mass is 60 kilograms walks up a hill 298.4 metres high in 14 minutes. What is the average power which he exerts compared with a horse-power?

14. A boy can carry 300 litres of water to the top of a hill 80 metres high in 1 hour. State in watts his rate of working.

15. If 596,800 litres of water flow per minute over a dam 6 metres high, what is the power of the fall?

16. A hoist used in the erection of a building raises in 2 hours 30,000 bricks, each weighing 5 pounds, and 2000 feet of lumber, weighing 3 pounds per foot, through a height of 50 feet. Calculate the work done.

Calculate also the horse-power developed by the engine running the hoist, supposing 20 per cent. of the energy developed to be lost in friction.

17. A street-car, of mass 18 tons, is propelled at 10 miles per hour up a hill rising 1 foot in 100 feet measured along the track. Neglecting friction, find the horse-power developed by the motors.

18. An engine is drawing a train whose mass is 360,000 kilograms up a smooth inclined plane of 1 in 30, at the rate of 22,380 metres per hour. What is the horse-power of the steam engine?

19. A man cycles up a hill, rising 1 in 14, at the rate of 6000 metres per hour. The mass of the man and the machine is 60 kilograms. At what rate is he working?

20. A train consists of 30 cars, and each car with its load weighs 14,920 kg., the resistance to motion on a level track is at the rate of 15 kg. per 1000 kg. of load. Find in horse-power at what rate an engine is working that hauls this train at the rate of 30 km. per hour.

21. What is the horse-power of an engine which keeps a train whose mass is 60,000 kg. moving on a horizontal track at a uniform rate of 44,760 metres per hour, the resistance due to friction, etc., being $\frac{1}{50}$ of the weight of the train?

22. An engine, whose horse-power is 1000, pumps water from a depth of 1000 feet. Find the number of tons raised per hour.

23. An engine of 98 horse-power, working 10 hours a day, supplies 3000 houses with water, which it raises to a mean level of 149.2 metres. Find the average supply to each house.

24. The piston of a steam engine is 10 inches in diameter and the stroke is 16 inches long. If the average pressure of steam on this piston throughout the full length of the stroke is 70 pd. per square inch, and if the engine makes three double strokes (backward and forward movements) per second, determine its horse-power.

## CHAPTER X

### SOME TRANSFORMATIONS OF ENERGY

**93. Heat a Mode of Motion.** Until almost the middle of the last century it was the generally accepted belief that heat was a subtle fluid called *caloric,* which was distributed amongst the molecules of a body. When a piece of iron was hammered the caloric was driven out from its hiding place and revealed itself in a rise in the temperature of the iron.

An interesting investigation into the nature of heat was made in 1798 by Count Rumford.* While engaged in boring cannon at the arsenal in Munich he was surprised at the great amount of heat generated in the operation, and in order to make a thorough inquiry into the matter he prepared a hollow bronze cylinder which he mounted so that it could be rotated by horse-power while a blunt steel tool was pressed against the bottom inside. In one experiment the cylinder was immersed in about 20 pounds of water. The temperature steadily rose and in $2\frac{1}{2}$ hours the water actually boiled. Rumford found that as long as he kept the machine going the heat continued to be produced and he concluded that as the supply was inexhaustible **heat could not be a material substance but must be a form of motion.**

**94. Relation between Heat and Mechanical Work.** Though heat could be obtained at the expense of mechanical work, the precise relation between these two was not determined until Joule published the results of experiments which he began in 1840. If the work is really all spent in producing heat, then with every form of experiment one should obtain

* Rumford's name was Benjamin Thompson. He was born in 1753 at Woburn, near Boston, Mass., went to England in 1775, and at the close of the Revolutionary War went to Bavaria in 1783. He was made Count by the Elector of Bavaria and chose his title from the name of a small town (now called Concord) in New Hampshire.

the same amount of heat for a given amount of work. **The quantity of work which is required to create one unit of heat is called its mechanical equivalent.**

The essential features of one method used by Joule to determine this mechanical equivalent is illustrated in Fig. 80; but it should be understood that the actual apparatus used was much more complicated, and the method of calculating the results included many corrections difficult to make. A paddle-wheel is made to revolve in a vessel *A*, filled with water, by the descent of a weight *C* on the end of a cord which is wound about *B*. A thermometer *T* measures the rise in temperature. The heat generated is calculated from the mass of the water and its rise in temperature, and the amount of work which is equivalent to it is measured by the weight *C* and the distance through which it falls.

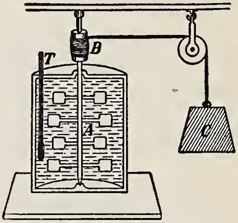



**Fig. 80.—Principle of Joule's apparatus for determining the mechanical equivalent of heat.**

As the result of many careful and tedious experiments Joule calculated that the mechanical equivalent of one British thermal unit (that is, the heat required to raise 1 pound of water through 1° F.) was 772 foot-pounds of work. Later investigations by Rowland and others give the value as

**778 foot-pounds for 1 B.T.U.,**

which is the same as

**427 gram-metres for 1 calorie,**

or, **$4.187 \times 10^7$ ergs for 1 calorie.**

One calorie is the amount of heat required to raise 1 gram of water through 1° C.

**95. Determination of the Mechanical Equivalent.** By means of the apparatus shown in Fig. 81 the mechanical equivalent may be determined rapidly and with considerable

accuracy. $C$ is a drum made of thin brass which can be rotated about the horizontal axis $B$. On the other end of this axis is the driving wheel $A$ which can be turned by hand or driven by a small electric motor $M$ through reduction gearing $G$. The number of revolutions made by the drum is automatically recorded by the counter $N$. Around the drum is wound a silk belt, making one and one-half complete turns. Unequal adjustable weights $E$ and $F$ are suspended from the ends of this belt and a light spring balance $D$ is added to secure stability of equilibrium.

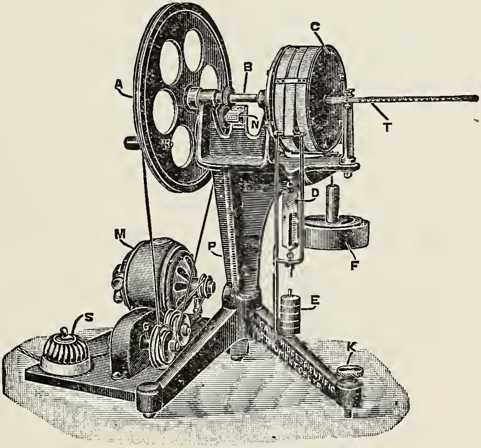



Fig. 81.—Callendar's apparatus for determining the mechanical equivalent of heat. The electric motor shown in the picture is not essential. It may be dispensed with and the machine driven by hand.

A measured quantity of water is poured into the drum $C$. A thermometer $T$ is inserted through a hole in the centre of the face of the drum. Its stem is bent almost at a right angle so that the bulb may be fully immersed in the water, and any rise in the temperature of the water is shown by the reading on the thermometer. As the drum revolves the friction of the

belt upon its circumference warms it and the heat is conducted to the water within.

If $W$ ergs be the work expended in rotating the drum and $H$ calories be the heat generated, then

Mechanical Equivalent of Heat = $W/H = J$ ergs per calorie.

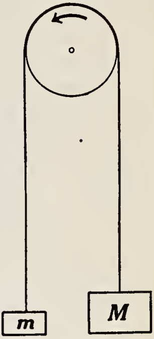



FIG. 82.—The drum rotates in the direction of the arrow and friction of the belt sustains the weight $M$.

First, let us calculate the work expended in rotating the drum. Consider a drum (Fig. 82) with a belt over it with weights $M$, $m$ grams on its ends, and suppose the drum to be rotated in the direction of the arrow, thus keeping the weights in equilibrium. Then it is evident that the weight $m$ added to the friction of the belt balances the weight $M$.

Hence the friction = $M - m$ grams.

Now in one rotation of the drum work has been done against this friction through a distance equal to the circumference of the drum.

Let circumference of drum = $c$ cm.
and the number of rotations = $n$.

Then the work done,

$$W = (M - m)\, nc \text{ gram-cm.}$$
$$= (M - m)\, ncg \text{ ergs.}$$

Next, calculate the heat produced.

| | |
|---|---|
| Let mass of drum | $= w_2$ grams, |
| and its specific heat | $= s$. |
| Then its water equivalent | $= w_2 s$ grams. |
| Let mass of water | $= w_1$ grams, |
| and rise of temperature | $= T°$ C. |

Then heat generated, $H = (w_1 + w_2 s)\, T$ calories.

$$\text{Hence, } J = \frac{W}{H} = \frac{(M - m)\, ncg}{(w_1 + w_2 s)\, T} \text{ ergs per calorie.}$$

**Example.**—From an actual experiment.

$$M - m = 1935 \text{ grams}$$
$$c = 47.5 \text{ cm.}$$
$$n = 410 \text{ revolutions}$$
$$g = 980$$
$$w_1 = 300 \text{ grams}$$
$$\left.\begin{aligned} w_2 &= 403.1 \text{ grams} \\ s &= .092 \end{aligned}\right\} w_2 s = 37 \text{ grams}$$
$$T = 2.6° \text{ C.}$$

From which $J = 4.23 \times 10^7$ ergs per calorie.

**Question.**—Why is a silk belt used rather than one of leather or of metal?

**96. Alternative Method of Determining J.** A different form of apparatus for the determination of the mechanical equivalent is shown in Figs. 83 and 84. The calorimeter consists of a conical brass vessel $A$ (Fig. 83),

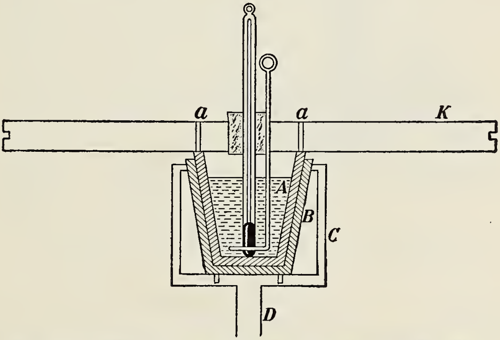



Fig. 83.—Diagram of double-cone form of mechanical equivalent apparatus.

carefully ground to fit a second conical brass vessel $B$. The latter is mounted in a bakelite insulating cup $C$, fastened to the upper end of the shaft $D$, which may be rotated by a belt passing around the pulley $E$ (Fig. 84). A revolution counter $F$ is geared to the shaft.

The inner vessel, which contains the water, stirrer and thermometer, is held stationary during the experiment by weights $G$, attached to a cord which passes over the light pulley $H$. The other end of the cord is fastened

to the circumference of the fibre disc $K$ which engages with two studs $a$, $a$ in the top of $A$. An annular weight $L$ provides pressure between the cups.

When the shaft is rotated heat is developed because of the friction between the outer rotating brass vessel and the inner stationary one. The rubbing surfaces are kept lubricated by a few drops of thin oil which reduces the friction to a suitable value.

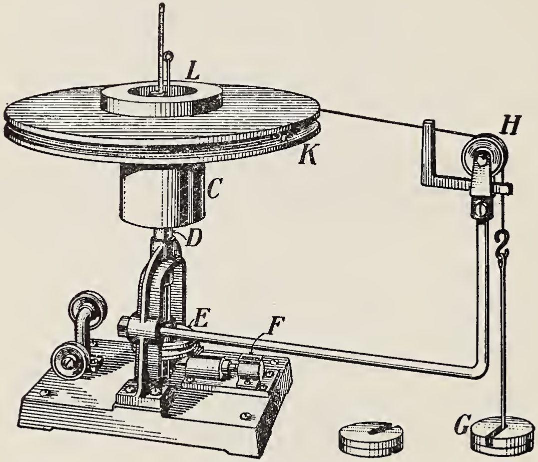



Fig. 84.—General appearance of double-cone form of mechanical equivalent apparatus.

The two brass vessels, with stirrer, are weighed and placed in position in the bakelite cup. $A$ is then nearly filled with water at a temperature about eight degrees below that of the room. The disc and weights are placed in position and the thermometer is inserted in the calorimeter.

After a few preliminary turns to ascertain the correct mass to place at $G$ so that it will stay *just suspended* when the shaft is rotating, the reading of the thermometer is taken very carefully and the reading of the revolution counter noted. The shaft is then rotated, usually by a hand-driven wheel, until the temperature rises as much above room temperature as it was below when the first reading was taken. This final temperature must also be taken very accurately. The two brass vessels, with water and stirrer, are again weighed to determine the weight of the water.

The mechanical equivalent is then determined as in the following example:—

**Heat Developed**

| | |
|---|---|
| Weight of vessels and stirrer | 260 gm. |
| Water equivalent of vessels and stirrer $(260 \times 0.090)$ | 23.4 gm. |
| Water equivalent of thermometer* | 0.5 gm. |
| Weight of water | 22.1 gm. |
| Total water equivalent | 46.0 gm. |
| Initial temperature | 15°C. |
| Final temperature | 25°C. |
| Heat developed $= 46.0 \times 10 =$ | 460 cal. |

**Work Done**

In calculating the work done it is evident that the same results could have been obtained if the outer vessel had been held stationary and the inner one had been made to rotate by letting $G$ descend such a distance as would cause the disc and inner cup to rotate the number of times indicated by the revolution counter.

| | |
|---|---|
| Number of revolutions | 1760 |
| Circumference of disc | 75 cm. |
| Weight of $G$ | 150 gm. |

$$
\begin{aligned}
W &= Fs, \\
&= 150 \times 980 \times 1760 \times 75, \\
&= 1940.4 \times 10^7 \text{ ergs.}
\end{aligned}
$$

$$\text{Hence } J = \frac{1940.4}{460} = 4.24 \text{ joules per calorie.}$$

## PROBLEMS

(Take $J = 778$ ft.-pd. per B.T.U. or $4.19 \times 10^7$ ergs per calorie)

1. A stone weighing 5 kg. drops from a height of 100 metres and strikes a pile of sand. Find the heat developed in calories.

2. The Falls of Niagara are 166 ft. high. Find the rise in temperature due to the impact of the water on the rocks below, assuming that all its mechanical energy is changed into heat energy. (Consider a mass of 1 lb.).

---

* In calculating the water equivalent of the thermometer only that part actually in the water need be considered.

3. A jet of water is driven with a velocity of 150 metres per sec. against a wall. If the mechanical energy is used up in heating the water, find its rise in temperature.

4. How much heat is generated when a train of 200 tons moving with a velocity of 60 miles per hr. is brought to rest by the brakes?

5. Find the heat developed when a motor car weighing 3000 lb. and moving at 30 miles per hr. is brought to rest by the action of the brakes.

6. In an experiment a 5 h.p. motor in 1 min. raised the temperature of 1 gal. of water through 22° F. Calculate the ft.-pd. expended per B.T.U. What per cent. is the result in error?

7. If 1 lb. of good coal can raise the temperature of 60 lb. of water from 0° to 100° C. find the energy in ft.-pd. in 1 lb. of coal. If the efficiency of a steam engine is 8 per cent., what must be the consumption of coal per hr. to produce 150 h.p.?

8. The heat energy available in a certain sample of coal is 14,500 B.T.U. per lb. If all of this could be converted to mechanical energy, how many horse-power hours would be produced by burning one ton of the coal?

9. If the atmosphere were removed and the sunlight fell perpendicularly upon the earth's surface, each square centimetre would receive in 1 min. approximately 2 calories of heat. Assuming that 40 per cent. of the sun's radiation is lost by absorption in the atmosphere, find in k.w. the energy falling upon 1 sq. metre of the deck of a steamer when the sun is directly overhead.

10. In an experiment with Callendar's apparatus the following observations were made:—Number of revolutions, 305; circumference of drum, 48.5 cm.; $M = 4000$ gm.; $m = 50$ gm.; mass of water, 250 gm.; mass of brass drum, 380.5 gm.; initial temperature, 16.4°C.; final temperature, 21.4°C. Taking specific heat of brass = .090, find the mechanical equivalent of heat as determined by this experiment.

11. If 300 gm. of water are placed in the drum of the Callendar apparatus whose dimensions are given in Question 10, how much should the temperature of the water and drum increase in 400 revolutions of the drum?

**97. The Electrical Circuit.** Let us consider some of the transformations of energy which take place in the electrical circuit shown in Fig. 85.

In the battery $A$ chemical energy is transformed to electrical energy, which in turn is changed into heat and light energy in the bank of lamps $B$ and to heat and mechanical energy in the motor $C$.

The current flows through the circuit because there is a difference in potential, or electrical pressure, between the terminals of the battery. Water flows in a pipe from a place of high pressure to one of low pressure; electricity flows from a place of high potential to one of low potential. Difference of pressure in the pipe corresponds to difference of potential in

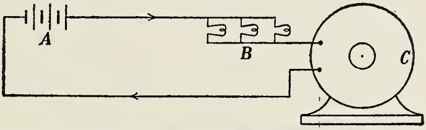


Fig. 85.—*A* is a battery, *B* a lamp rheostat, *C* a motor.

a conductor. Indeed the words pressure or tension are often used by electricians when they mean potential.

The strength of the current depends on this difference of potential and also upon the resistance of the circuit. A pipe of large section corresponds to a conductor of large section. There is little resistance to the flow in either case. A pipe of small section or one filled with sand corresponds to a conductor having a high resistance.

Current strength is measured in amperes, potential difference in volts and resistance in ohms. Ohm's Law defines the relation connecting these units:

$$\text{Resistance of a conductor} = \frac{\text{P.D. between ends of conductor}}{\text{Current flowing through conductor}}.$$

$$\text{Resistance of a circuit} = \frac{\text{Total E.M.F. in circuit}}{\text{Current flowing through circuit}}.$$

If $C$ is the measure of the current in amperes, $E$ the electromotive force in volts and $R$ the resistance in ohms,

$$R = \frac{E}{C}, \text{ or } E = CR, \text{ or } C = \frac{E}{R}.$$

Fig. 86 shows how the resistance of a wire $C$ may be measured by obtaining the P.D. between its ends by means

of the voltmeter $E$, and the current flowing through it by the ammeter $D$.

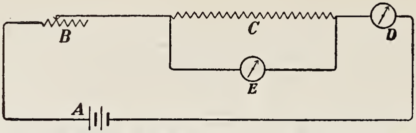



Fig. 86.—Measuring the resistance of the wire $C$ by means of a voltmeter $E$ and an ammeter $D$. $A$ is a battery and $B$ a rheostat.

**98. Measurement of Electrical Energy.** *Current Strength is rate of flow.* The amount of water delivered by a pipe depends on the rate of flow and upon the time of flow.

Quantity of water = rate of flow × time of flow.

Rate of flow may be measured in gallons-per-second, quantity in gallons.

Next consider the quantity of electricity which passes a cross-section of a wire carrying a current in a given time, and as before we have

Quantity of electricity = current strength × time of flow,
or Total amount = rate of flow × time of flow.

**If the current is expressed in amperes and the time in seconds, the quantity will be given in coulombs; a coulomb being defined to be the amount of electricity which passes a point in an electrical circuit in one second when the strength of the current is one ampere.**

The ampere corresponds to gallons-per-second, the coulomb to gallons.

The word coulomb is not often heard in ordinary electrical practice, but ampere-hour, a quite similar term, is commonly used in specifying the capacity of storage cells. A battery having a capacity of 100 ampere-hours is one which can deliver 100 amperes for 1 hour, or 50 for 2 hours, etc.

The work which a stream can do depends upon the quantity of water which flows and the distance through which it falls. The work done in an electric circuit depends upon the quantity

of electricity which passes through it and the P.D. (difference in potential) between the terminals of the circuit.

**The volt and the coulomb are so chosen that 1 coulomb of electricity passing between two points whose P.D. is 1 volt does 1 joule of work.**

Suppose $A$ and $B$ (Fig. 87) are the terminals of a dynamo which maintains a P.D. of 1 volt between $A$ and $B$ and causes a current of 1 ampere to flow through the circuit $D$. Then in 1 second 1 coulomb will flow from $A$ to $B$ and 1 joule of work will be done. The *power* required to perform 1 joule of work per second is 1 *watt*.

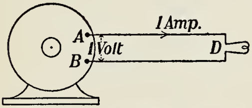


FIG. 87.—A generator driving electricity through a circuit $D$.

If the dynamo maintained a P.D. of 110 volts and supplied a current of 20 amperes, the power required (or the output of the dynamo) = 110 × 20 = 2200 watts.

**Hence power (in watts) = P.D. (in volts) × current (in amp.)**

$$1000 \text{ watts} = 1 \text{ kilowatt (k.w.)}$$

Also, 1 h.p. = 746 watts = $\frac{3}{4}$ k.w. (nearly, see Sec. 88),

$$\text{and power (in h.p.)} = \frac{\text{P.D. (in volts)} \times \text{current (in amps.)}}{746}.$$

Again, by Ohm's law $C = E/R$ or $E = CR$.

$$\text{Hence, power} = EC \text{ watts,}$$
$$= C^2R \text{ watts,}$$

where $R$ is the resistance of the circuit in ohms.

**99. Power Required for Electrical Appliances.** Let us consider how to find the power required to operate an electric toaster or any other appliance.

Connect the apparatus as shown in Fig. 88 and take the readings of the ammeter $A$ and voltmeter $V$. Let these be 5 amperes and 110 volts. Then electrical energy is being

transformed to heat energy in the toaster at the rate of $5 \times 110 = 550$ joules per second and the power being used is 550 watts.

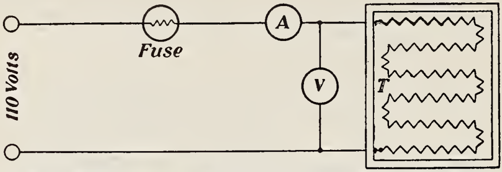



FIG. 88.—Finding the power required to operate an electric toaster. *A*, ammeter; *V*, voltmeter; *T*, toaster.

**100. Efficiency of Electric Lamps.** The efficiency of an electric lamp is usually determined in watts per candle-power.

The old 16-c.p. carbon lamp required $\frac{1}{2}$ ampere at 110 volts, the power being $\frac{1}{2} \times 110 = 55$ watts. The efficiency of such a lamp is $55 \div 16 = 3.4$ watts per c.p.

The ordinary 50-watt tungsten lamp gives about 32 c.p., and the efficiency $= 50 \div 32 = 1.6$ watts per c.p.

It is easy to understand why the tungsten lamps have almost entirely superseded those with carbon filaments. If the lamp is filled with an inert gas, such as nitrogen, the efficiency is still higher, namely about 1 c.p. per watt. In these lamps the filament can be raised to a higher temperature and a slight rise in temperature causes a large increase in radiating power.

**101. The Electrical Equivalent of Heat.** It is possible to determine the mechanical equivalent of heat by using an electric current. If the current is not effecting chemical change or doing mechanical work (such as driving an electric motor), it is simply heating the conductor through which it flows. If we measure the work required to cause the current to flow and at the same time measure the heat arising from it, we can deduce the mechanical work which is equivalent to one unit of heat.

A simple arrangement for determining the electrical equivalent of heat is shown in Fig. 89. The heating device is an 8-c.p. carbon lamp placed in a beaker or calorimeter containing water. The experiment is performed as follows:

Weigh the calorimeter, add enough water at a few degrees below room temperature to cover the glass part of the lamp when it is placed in the calorimeter and weigh again. Connect the lamp, ammeter and voltmeter as in the diagram and

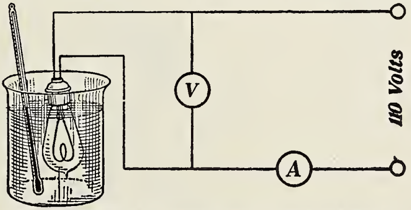



Fig. 89.—Finding the mechanical equivalent of heat.

place the lamp in position in the calorimeter. Take the temperature carefully, note the time and turn on the current. Keep the water stirred with the thermometer and let the current flow until the temperature of the water is as much above room temperature as it was below at the beginning of the experiment. Note the readings of the voltmeter and ammeter every minute and calculate the average power in watts. Note the temperature and the time when the current is turned off.

Calculate the number of calories of heat gained by the calorimeter and water and also the number of watt-seconds (joules) of electrical energy used. Divide the latter by the former.

**Numerical Example.**

| | |
|---|---|
| Weight of calorimeter (copper) | 80 gm. |
| Weight of water in calorimeter | 100 gm. |
| Initial temperature of water | 10°C. |
| Final temperature of water | 30°C. |
| Average current | 0.28 amperes |
| Average difference of potential | 110 volts |
| Time | 5 min. |

$$\text{Heat developed} = 80 \times 20 \times 0.094 + 100 \times 20,$$
$$= 2150.4 \text{ calories.}$$
$$\text{Electrical energy expended} = 110 \times 0.28 \times 300 = 9240 \text{ joules.}$$
$$\text{Electrical equivalent of heat} = \frac{9240}{2150.4} = 4.29 \text{ joules per calorie.}$$

**102. The Continuous Flow Calorimeter.** A more elaborate piece of apparatus for determining the electrical equivalent of heat is shown in Fig. 90.

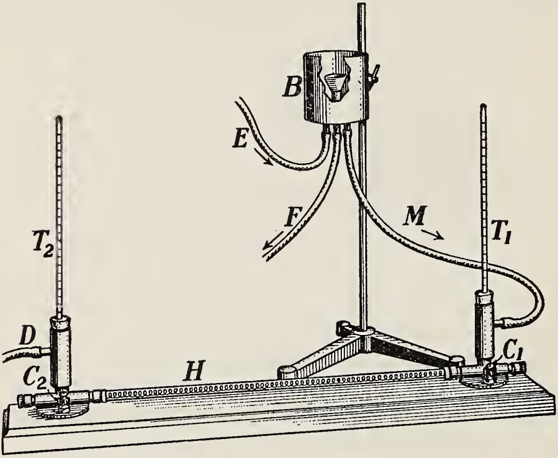



Fig. 90.—Callendar and Barnes's continuous flow calorimeter.

In the long glass tube $H$ a helix of resistance wire (manganin) is fixed with its ends connected to binding-posts $C_1$, $C_2$. Into the ends of the tube, thermometers $T_1$, $T_2$ are inserted. The aim is to have a constant current of water flowing through $H$ and heated by a constant current of electricity passing through the helix. The constant current of water is obtained from a cistern $B$, mounted on a stand. This contains two concentric chambers. The tube $E$ carries water from the tap (or other source) into the outer chamber, while the tube $M$ runs from the outer chamber to one end of the tube $H$. A third tube $F$ leads from the inner chamber to the sink. The supply of water through $E$ is adjusted so that the outer chamber is always full and a small amount flows over into the inner chamber and runs away through $F$. In this way a constant head of water is maintained and the flow through $H$ is steady.

The cistern can be raised to increase the current through $H$. With this apparatus the usual rate of flow is about 60 c.c. per minute, and it is desirable that the initial temperature of the water should be about as much below the temperature of the room as the final temperature is above it. If such is the case no correction for radiation need be made.

The water runs off through the tube $D$ and is collected in a graduated glass, from which the amount flowing in any interval may be read.

Fig. 91 shows the electrical connections. The strength of the current is measured by an ammeter $A$ placed in series with the helix, and the current may be obtained from a commercial source or from storage batteries, a suitable resistance $B$ being inserted to reduce the current sufficiently. A voltmeter $V$, with terminals joined to $C_1$, $C_2$ gives the P.D. between the ends of the helix.

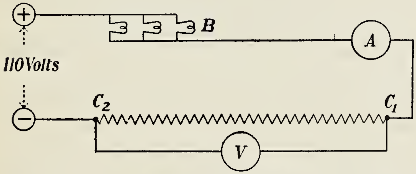



FIG. 91.—$A$, ammeter; $B$, lamp rheostat; $C_1C_2$, helix; $V$, voltmeter.

Let the P.D. = $E$ volts, the current = $C$ amperes, and the duration of an experiment = $t$ seconds.

Then the work done = $ECt$ joules,

$$= ECt \times 10^7 \text{ ergs.}$$

In the $t$ seconds suppose $M$ grams of water have been caught and let the rise in its temperature be $T°$ C.

The heat evolved, $H = MT$ calories, and if 1 calorie is equivalent to $J$ ergs

$$H = MTJ \text{ ergs.}$$

Hence, $$MTJ = ECt \times 10^7$$

or $$J = \frac{ECt \times 10^7}{MT} \text{ ergs per calorie.}$$

**Example.**—In the following table are given the readings in an experiment.

| Ammeter Reading $C$ | Voltmeter Reading $E$ | Flow of Water c.c. per min. | Temp. of Inflowing Water | Temp. of Outflowing Water | Rise in Temp. | Temp. of Room |
|---|---|---|---|---|---|---|
| 2.12 | 21.2 | 63.8 | 13.25° C. | 23.70° C. | 10.45° C. | 18.6° C. |
| 2.12 | 21.2 | 61.0 | 12.70 | 22.90 | 10.20 | 18.7 |
| 2.12 | 21.2 | 61.0 | 12.15 | 22.50 | 10.35 | 18.7 |
| Av. 2.12 | 21.2 | 61.9 | 12.70 | 23.00 | 10.33 | 18.7 |

$$\text{Hence, } J = \frac{2.12 \times 21.2 \times 60 \times 10^7}{61.9 \times 10.33} = 4.23 \times 10^7 \text{ ergs per calorie.}$$

## PROBLEMS

1. An electric motor which actually developed 2 h.p. required 16.5 amperes at an E.M.F. of 110 volts. Find the efficiency of the motor.

2. What is the resistance of a 40-watt, 110-volt lamp? How many such lamps can be operated by a 5 k.w. dynamo?

3. What horse-power engine would you order to operate a dynamo whose maximum load is to be 25 sixty-watt lamps?

4. In an electric furnace a current of 8000 amperes at 50 volts is used; find the calories generated per second.

5. A heating coil having a resistance of 50 ohms is connected to a 100-volt circuit and is placed in 1000 gm. water at 0°C. Find the temperature of the water 10 min. later.

6. A 16-c.p. electric lamp whose resistance is 220 ohms is connected to a 110-volt circuit and is immersed in 500 grams of water for 20 min. Find the increase in the temperature of the water.

7. A current of 5 amperes at 110 volts pressure is passed through an electrical heater on which is placed a beaker containing 100 gm. water. If it takes 10 minutes to bring the water from 10° C. to 100° C., what percentage of the energy passes into the water?

8. Water flows steadily at the rate of 30 c.c. per min. through a glass tube in which is a wire coil. The temp. of the water on entrance is 13.25° C., and on exit 20.65° C., and the p.d. between the ends of the wire is 25 volts. Find the resistance of the wire.

9. An electric tea-kettle requires 8.6 amperes at 110 volts and in 10 min. can raise 1.5 litres of water from 15° to 100° C. What percentage of the energy supplied is used in actually boiling the water?

10. In a power station 4 engines, each of 150 h.p., drive 2 dynamos, each of which delivers 150 amperes at 540 volts, and 2 others, each of which delivers 225 amperes at 270 volts. Calculate the efficiency of the arrangement.

**103. The Buying and Selling of Energy.** We are accustomed to dealing in flour, sugar, lumber, and other things which we can see and handle, but energy, though invisible and intangible, is quite as real a *thing* and can equally well be

bought and sold. Energy is *ability to do work,* and it is as reasonable that we should pay for any energy which is supplied to us as for the objects produced thereby.

Further, it is clear that the charge for energy should depend upon

(i) the rate at which it is supplied;
and (ii) the length of time it is supplied.

A man is a source of energy and the pay for his services should be according to his ability and the length of time he works, that is, per man-power-hour, or per man-power-day.

We have shown in Sec. 98 that in an electrical circuit the power (in watts) = P.D. (in volts) $\times$ current (in amperes).

The charge for the use of this power should evidently depend upon the time it is supplied. Power when used for a time does work. The unit of electrical work or energy is the **watt-hour** or (since that is rather small) the **kilowatt-hour. One kilowatt-hour is the electrical energy expended when 1 kilowatt power is used for 1 hour.**

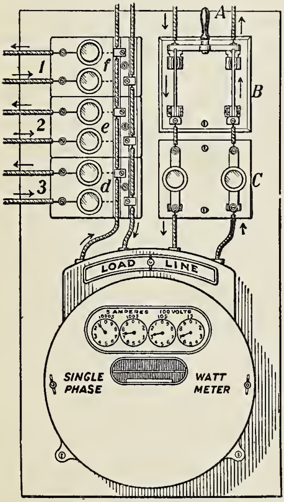


FIG. 92.– Meter connections; *A*, line wires; *B*, main switch; *C*, main fuses; *f*, *e*, *d*, fuse blocks; *1*, *2*, *3*, various circuits in house.

Near the place where the electric current enters the house a *watt-meter* (or more correctly, kilowatt-hour meter) is placed, and as the current passes through this it makes a light disc rotate. The number of rotations shown on the dials depends upon the voltage, the magnitude of the current and the length of time it is flowing. Consequently the instrument indicates the kilo-

watt-hours of energy supplied. An officer reads the dials periodically and the account for the energy used is then sent to the customer.

Figure 92 shows how the meter is connected to the line and load wires.

### PROBLEMS

1. Express 1 k.w.h. in joules.

2. An electric range takes 10 amperes at 220 volts pressure. Find the cost of using it for 2 hours at 5c. per k.w.h.

3. A watt-hour meter registered 2 k.w.h. in 2 hours when the E.M.F. was 110 volts and the current flowing through the load was 10 amperes. Find the error in the meter reading.

4. An electric fan requires 0.25 ampere at an E.M.F. of 110 volts. How much will it cost to run it for five days, 10 hours per day, in a town where electric energy costs 8c. per k.w.h.?

5. A 32-c.p., 120-volt lamp requires 1.25 watts per candle. Find the current which passes through the lamp and the cost of using it for 5 hours at 5c. per k.w.h.

6. In an advertisement for electric heaters the following phrases are found:—"615 watts per hour." "consumes only 960 watts per hour," "consumes only 1250 watts per hour." Criticise these statements.

Omit

**104. The Hydro-Electric System of Ontario.** The buying and selling of energy is well illustrated in the operations of the Hydro-Electric Power Commission of Ontario which, during recent years, has extended its distribution lines to many parts of the province. This great public utilities organization is managed for the benefit of the people, and is rapidly providing electric energy to the cities, towns, villages and farms of the country.

The Commission owns a number of power stations and transmission systems throughout the Province and controls others, and in some cases purchases power for distribution from independent supply corporations. Its largest source of energy is, of course, the Niagara River. A diagram representing the transformations of the current from the time it is

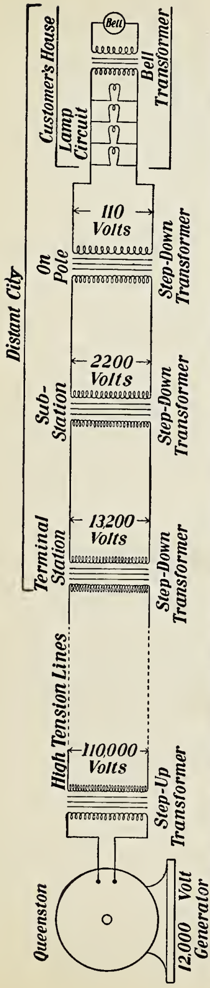



Fig. 93.—Diagram showing the different transformations which take place between Queenston and a house using Hydro-Electric Power.

generated at Queenston until it reaches the consumer is given in Fig. 93.

From the generating and transforming stations the energy is sent over the transmission or distribution lines built by the Commission, and is sold to the various municipalities at rates depending on the distance from the station and on the amount taken. Thus, Toronto pays about $24.55, London $24.66, Guelph $24.94, Owen Sound $30.10, Clinton $34.82, per horse-power per year.*

These places then sell the energy to the citizens for lighting, for heating (as in stoves, toasters, flat-irons, etc.), and for driving motors (as in manufacturing, operating elevators, etc.), the rate being adjusted to the service and the place. The charges are practically always per kilowatt-hour, and some typical accounts are given in the problems below.

The manner of charging for electrical energy varies somewhat with the companies supplying it. The Hydro-Electric Power Commission divides its rates under three headings:

*The figures given are for 1930. The rates vary slightly from year to year.

(*a*) *Domestic service,* which includes all energy used for domestic or household purposes.

(*b*) *Commercial service,* for stores, churches, hotels, etc.

(*c*) *Power service,* which includes all other services.

In each case there are certain fixed charges depending on the size of the installation or the amount of energy used.

## PROBLEMS

(The following examples, taken from the accounts of an Ontario town, will illustrate the methods used in charging for electrical energy.)

DOMESTIC SERVICE—

1. Name, *First Citizen.* Date, June 1, 1919.

| | | |
|---|---|---|
| Meter reading, June 1, 1919 | 5601 | k.w.h. |
| Previous reading, May 1, 1919 | 5314 | " |
| Consumption | 287 | " |

| | |
|---|---|
| Consumption charge | |
| 69 k.w.h. at 4.5c. per k.w.h. | $3.10 |
| 218 " " 2.25c. " " | 4.90 |
| Service charge | |
| 2300 sq. ft. at 3c. per 100 sq. ft. | .69 |
| Total bill | $8.69 |
| 10% discount if paid within 10 days | .87 |
| Net bill | $7.82 |

*Explanation.*—The floor space in this house, measured in a specified manner, was 2300 sq. ft. There is a fixed charge of 3c. per 100 sq. ft. per month. The charge for consumption is $4\frac{1}{2}$c. per k.w.h. for all up to 3 k.w.h. per 100 sq. ft. (or 69 k.w.h.) and $2\frac{1}{4}$c. per k.w.h. for all above this.

2. Name, *Second Citizen.* Date, June 1, 1919.
Meter reading, June 1, 1919 .......... 588 k.w.h.
Previous reading, May 1, 1919 .......... 576 "
Floor space, 1800 sq. ft. Make out the bill.

3. Make out the bills for the above persons at the Toronto rates, which are as follows:

| | |
|---|---|
| For first 3 k.w.h. per 100 sq. ft. | 2c. |
| For all additional k.w.h. | 1c. |
| Service rate 3c. per 100 sq. ft.; discount 10%. | |
| Minimum monthly bill | 83c. gross. |

Commercial Service—

4. Name, *Third Citizen.* Date, June 1, 1919.
Installed load, 2970 watts.

| | | |
|---|---|---|
| Meter reading, June 1, 1919 | 8007 | k.w.h. |
| Previous reading, May 1, 1919 | 7596 | " |
| Consumption | 411 | " |

Consumption charge

| | |
|---|---|
| 89 k.w.h. at 9c. per k.w.h. | $8.01 |
| 208 " " 4.5 " " | 9.36 |
| 114 " " .09 " " | .10 |
| Total bill | 17.47 |
| 10% discount if paid within 10 days | 1.75 |
| Net bill | $15.72 |

*Explanation*:—The "installed load" is the capacity of all the lights installed. A charge is made for the entire installed load for the first 30 hours ($30 \times 2.970 = 89$ k.w.h.) at 9c. per k.w.h., and for the next 70 hours at $4\frac{1}{2}$c. per k.w.h. For all additional consumption the charge is .09c. per k.w.h.

5. Name, *Fourth Citizen.* Date, June 1, 1919.
Installed load, 1042 watts.
Meter reading, June 1, 1919 .................. 2384 k.w.h.
Previous reading, May 1, 1919 ............... 2359 "
Make out the bill.

6. Name, *Fifth Citizen.* Date, June 1, 1919.
Installed load, 745 watts.
Meter reading, June 1, 1919 .................. 2924 k.w.h.
Previous reading, May 1, 1919 ............... 2890 "
Make out the bill.

7. Make out bills for the above three persons at the Toronto rates, as follows:

| | |
|---|---|
| For first 70 hours of installed load | 4c. per k.w.h. |
| For next 70 " " " " | 2c. " |
| For all additional consumption | 1c. " |
| Minimum monthly bill | 83c. gross |
| Discount, 10%. | |

Power Service—

8. Name, *Sixth Citizen.* Date, June 1, 1919.
Connected load, 50 h.p.

| | | |
|---|---|---|
| Meter reading, June 1, 1919 | 25,040 | k.w.h. |
| Previous reading, May 1, 1919 | 14,100 | " |
| Consumption | 10,940 | " |

Consumption charge

| | |
|---|---|
| First 50 hours use, 1865 k.w.h. at 4.7c. per k.w.h. | $87.66 |
| Second 50 " " 1865 " " 3.1 " " | 57.82 |
| Remaining consumption, 7210 k.w.h. at .15 per k.w.h. | 10.81 |
| Service charge | |
| 50 h.p. at $1 per h.p. per month | 50.00 |
| | 206.29 |
| 25% local discount | 51.57 |
| Total bill | 154.72 |
| 10% discount if paid in 10 days | 15.47 |
| Net bill | $139.25 |

*Explanation*:—In this case the "connected load or maximum demand" is 50 h.p. = 37.3 k.w. and the customer is charged for the full load for 50 hours ($37.3 \times 50 = 1865$) at 4.7c. per k.w.h. The charge for the full load for the second 50 hours is at 3.1c. per k.w.h. The charge for the remainder is .15c. per k.w.h.

In addition he pays a service charge of $1 per h.p. of "connected load" per month.

The "local discount" is a discount depending on the number of hours per day which the power may be used, in this case, 18 out of the 24 hours.

9. Name, *Seventh Citizen.* Date, June 1, 1919.
Meter reading, June 1, 1919 ................ 79,920 k.w.h.
Previous reading, May 1, 1919 ............. 77,450 "
Connected load, $19\frac{1}{2}$ h.p.
Local discount 10%; prompt payment discount 10%.
Make out the bill.

10. Make out bills for the above two persons at Toronto rates, as follows:

First 50 hours use per month, 1.5c. per k.w.h.
Second " " " .75c. " "
All additional .33c. " "
Service charge, $1.25 for first 10 h.p., $1 for all additional.
No local discount; discount for prompt payment, 10%.

# CHAPTER XI

## Composition of Forces

**105. Addition of Forces in the Same Direction.** When an automobile gets stuck on the road all hands step out and try to release it. Two may pull on the bumper in front while three may push at the rear, and thus move the car to the hard level track again. But in place of the five people we might get a good team of horses to do the job for us.

The forces exerted by the men are all in the same direction, all aiming to cause the car to move forward, and it is perfectly evident that the single force exerted by the team of horses is equal to all the forces exerted by the men added together. For instance, if the forces exerted by the men were 150, 125, 160, 100, 165 pounds, respectively, the force exerted by the horses must have been the sum, or 700 pounds.

**The resultant of a number of forces acting on a body is that single force which would produce the same effect as the other forces.**

We see then that the resultant of a number of forces acting **in the same direction** upon a rigid body is equal to the sum of the individual forces.

**106. Forces Inclined at an Angle.** But if several forces act upon a body in directions inclined to one another it is not so easy to see what the resultant force will be. As usual in a scientific problem, it is wise to study it experimentally.

Suspend two spring-balances $A$ and $B$ from nails or hooks in a horizontal bar (Fig. 94) which may conveniently be the frame above the blackboard.* Tie three strings together at $O$ and attach the other ends of two of them to the hooks of the

* If it is more convenient, pulleys may be used instead of the spring-balances. The strings pass over the pulleys and masses of $P$ and $Q$ pounds are attached to their ends. See Fig. 99.

balances. On the third string hang a weight $W$ pounds. This string will take a vertical direction and the tension in it will be $W$ pounds. The tensions in the other strings will be given by the readings on the spring-balances. Let $A$ show $P$ pounds and $B$ show $Q$ pounds. It is plain that the knot at $O$ is kept

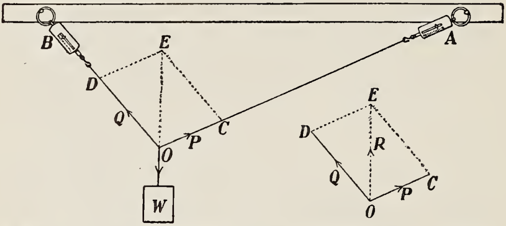



FIG. 94—How to demonstrate the law of Parallelogram of Forces.

in equilibrium by the three forces, $P$ acting along $OA$, $Q$ along $OB$ and $W$ acting vertically downwards.

The force $W$ may be looked upon as balancing the other forces $P$ and $Q$, and hence if $R$ is the resultant of $P$ and $Q$ (that is, the single force which is equivalent to $P$ and $Q$ acting together), it must be equal in magnitude to $W$ but be acting in the opposite sense, that is, upwards.

Now draw on the blackboard immediately behind the strings (or in some other convenient place), lines parallel to the strings $OA$, $OB$, and make $OC$, $OD$ as many units long as there are pounds shown on $A$, $B$, respectively.

Then carefully complete the parallelogram $OCED$ and measure the diagonal $OE$. It will be found to be in the vertical and to be $W$ units long.

Now we know that the resultant of $P$ and $Q$ acts vertically since it balances the vertical force $W$. Hence the line $OE$ represents the resultant of $P$ and $Q$ in both direction and magnitude.

A slightly different form of the experiment is as follows:

Fasten three cords (fish-line) to a small ring, and hook spring-balances on the other ends of the cords (Fig. 95). By means of pins in the top of the table, over which the rings of the balances may be placed, or in any other convenient way, exert force on the balances so that the cords are under considerable tension. The balances should move free of the table top.

Pin a sheet of paper under the strings and mark a dot precisely at $O$, the centre of the ring; also make dots exactly under each string and as far from $O$ as possible.

Read each balance. Then loosen them, and when they are lying on the table observe if the index returns to zero. If it does not, a correction to the reading on the balance must be made.

With great care draw lines from $O$ through the points under the cords, and on these lines take distances proportional to the tensions of the corresponding strings. Thus, if the tensions be 1000, 1500, 2000 grams, take lengths 10, 15, 20 cm. or 4, 6, 8 inches.

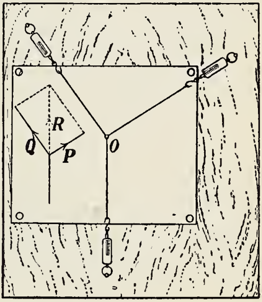


Fig. 95—Diagram illustrating the Parallelogram of Forces.

Using any two of these lines as adjacent sides, complete a parallelogram taking care to have the opposite sides accurately parallel. Draw the diagonal between these sides and carefully measure its length. Compare it as to length and direction with the third line.

From these experiments we deduce the proposition known as the *Parallelogram of Forces:* **If two forces acting at a point are represented in magnitude and direction by two sides of a parallelogram, then their resultant will be represented, in magnitude and direction, by the diagonal between the two sides.**

## PROBLEMS AND EXERCISES

1. Taking a line one centimetre in length to represent a gram-force, draw a line to represent a force of 12.3 grams acting (1) in a horizontal direction, (2) in a vertical direction, (3) in a direction making an angle of 45° with the horizontal.

2. Taking a line three-quarters of an inch long to represent a pound-force, draw a line which represents a force of $5\frac{3}{4}$ pounds acting in a direction making an angle of 60° with the vertical.

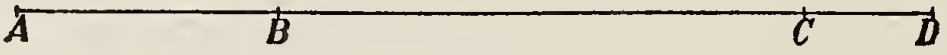



FIG. 96.

3. If $AB$ (Fig. 96) represents a force of 60 grams, what force will be represented by (1) $AC$, (2) $BC$, (3) $BD$, (4) $AD$, (5) $CD$? (Measure in centimetres).

4. If $BC$ (Fig. 96) represents a force of 24 pounds, what force will be represented by (1) $AB$, (2) $AC$, (3) $AD$, (4) $BD$, (5) $CD$?

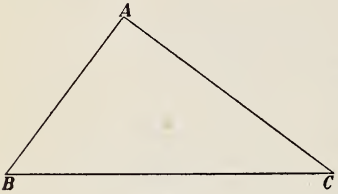



FIG. 97.

5. If $CD$ (Fig. 96) represents a force of 3 kilograms, what force will be represented by (1) $AB$, (2) $AC$, (3) $AD$, (4) $BC$, (5) $BD$?

6. If 2 cm. in length represents a force of 3 grams, what are the magnitudes of the forces represented by $AB$, $BC$, $CA$, the sides of the triangle $ABC$ (Fig. 97)? (Measure in centimetres).

7. If $AB$ (Fig. 98) represents a force of 4 pounds, what are the magnitudes of the forces represented by $AD$, $AE$ and $ED$? (Measure in inches).

8. Find the greatest and the least resultants of two forces whose magnitudes are 15 grams and 20 grams.

9. Find the greatest and least resultants of two forces whose magnitudes are $P + Q$ and $P - Q$.

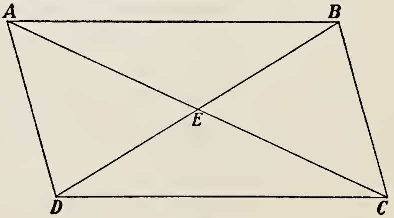



FIG. 98.

10. Find the resultant of forces of 15 pounds and 36 pounds, acting at right angles to each other.

11. Find the resultant of two forces of 12 kilograms and 35 kilograms acting at a point, the one acting north and the other east.

12. The resultant of two forces acting at right angles is 82 pounds. If one of the forces is 80 pounds, what is the other?

13. A force of $5P$ acts in a northerly direction, and the resultant of it and another force acting at the same point in an easterly direction is $13P$. What is the other force?

14. Determine the resultant of the following forces acting concurrently at the same point:—12 pounds N., 24 pounds E., 7 pounds S., and 36 pounds W.

15. A weight is supported by two strings. If the strings make an angle of 90° with each other, and the tension of the one is 9 pounds, while that of the other is 12, what is the weight?

16. A boat is moored in a stream by a rope fastened to each bank. If the ropes make an angle of 90° with each other, and the force of the stream on the boat is 500 pounds, find the tension of one of the ropes if that of the other is 300 pounds.

**107. Triangle of Forces.** Let us arrange again the apparatus used in demonstrating the Parallelogram of Forces (Fig. 99). Since the forces $P$, $Q$ and $W$ are in equilibrium, the resultant of $P$ and $Q$ must act vertically upward and must equal $W$ in magnitude.

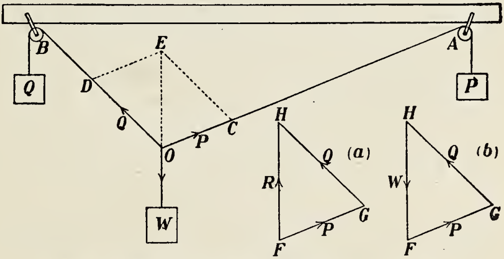



Fig. 99.—How to demonstrate the Triangle of Forces.

On the blackboard, or on a sheet of paper, draw a line $FG$ (Fig. 99$a$) parallel to $OA$ and make it $P$ units long. From $G$ draw $GH$ parallel to $OB$ and make it $Q$ units long. Then measure $FH$. It will be found to be $W$ units long and it will also be found to be parallel to $OW$. Hence $FH$ represents the resultant of $P$ and $Q$ in direction and magnitude.

Indeed it is evident that the triangle *FGH* is simply the half-parallelogram *OCE* and we may make the following statement:

**If two forces acting at a point are represented in direction and magnitude by two sides of a triangle taken in order, their resultant will be represented in direction and magnitude by the third side taken in the reverse order.**

Again, the three forces *P*, *Q* and *W* are in equilibrium and *FG*, *GH* and *HF* represent *P*, *Q* and *W*, respectively, in direction and magnitude (Fig. 99*b*).

Expressing this in general terms, we may state:

**If three forces acting at a point can be represented in direction and magnitude by the sides of a triangle taken in order, they will be in equilibrium.**

This is known as the ***Triangle of Forces.***

**108. Examples.**

1. Can a particle be kept at rest by a system of forces, 4, 3 and 8 pounds, acting on it?

If the particle can be kept in equilibrium by these forces a triangle can be drawn whose sides are 4, 3 and 8 units long.

But this is impossible since the sum of any two sides of a triangle must be greater than the third side.

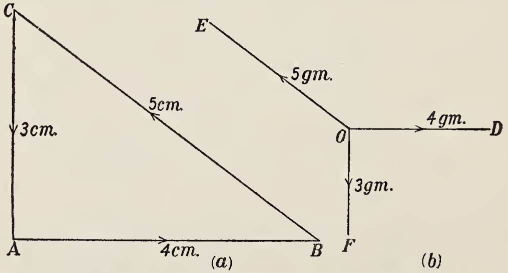



Fig. 100.—Lines showing the directions of three forces in equilibrium.

2. Draw lines to represent the direction of forces of 4, 5 and 3 gm. acting at a point and producing equilibrium.

Since the forces are in equilibrium a triangle with sides 4, 5 and 3 units long will show the directions in which the forces act.

Let $ABC$ (Fig. 100*a*) be this triangle (constructed geometrically). Let $O$ (Fig. 100*b*) be the point at which the forces actually act. Draw $OD$, $OE$ and $OF$ parallel to $AB$, $BC$ and $CA$, respectively. Then the 4-gm. force acts along $OD$, the 5-gm. force along $OE$ and the 3-gm. force along $OF$.

It should be clearly understood that the forces are actually acting at the point $O$ and not along the sides of the triangle $ABC$. These sides merely represent the forces in direction and magnitude.

## EXERCISES AND PROBLEMS

1. Can a particle be kept at rest by each of the following systems of forces acting on it?

   (1) 4 pounds, 3 pounds, 7 pounds.

   (2) 1 gram, 3 grams, 5 grams.

   (3) 4 pounds, 3 pounds, 2 pounds.

   (4) $P + Q$, $P - Q$, $P$, when $P$ is greater than $2Q$.

2. Draw lines to represent the directions of the following forces acting in one place at a point, when each system is in equilibrium:

   (1) 4 grams, 5 grams, 3 grams.

   (2) Three forces each equal to $P$.

   (3) $2P$, $P$, $\sqrt{3}P$.

   (4) 5 grams, 9 grams, 4 grams.

3. Forces $5P$, $12P$, $13P$ keep a particle at rest. Show that the directions of two of the forces are at right angles to each other.

4. Find the directions in which three equal forces must act at a point to produce equilibrium.

5. Forces $A + B$, $A - B$, and $\sqrt{2(A^2 + B^2)}$ keep a particle at rest. Show that the directions of two of the forces are at right angles to each other.

6. A mass of 10 pounds hangs at the end of a string 2 feet long. If the mass is held aside by a horizontal force so that the string makes an angle of 30° with the vertical, find the horizontal force and the tension of the string.

7. If the 10-pound mass in the preceding question is pulled aside until the string makes an angle of 60° with the vertical, find the horizontal force and the tension in the string. Does the length of the string affect the result?

8. A mass of 20 pounds is supported by a cord 5 feet long. What is the tension of the cord? If the mass is pulled aside by a horizontal force until it takes up a new position 1 foot higher than the original position, find the magnitude of the force and also the new tension in the cord.

**109. Calculation of Resultant.** Suppose two forces $P$ and $Q$, acting on a body, at a point $O$ in it, to be represented

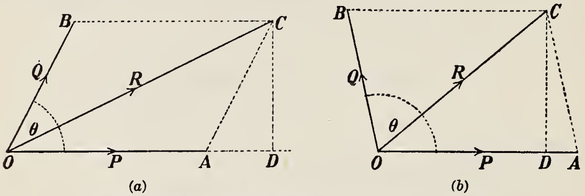



FIG. 101.—Calculating the value of $R$ in terms of $P$ and $Q$.

in direction and magnitude by $OA$, $OB$ (Fig. 101$a$, 101$b$), and let the angle between them be $\theta$.

Then completing the parallelogram, the resultant $R$ is represented by $OC$ and we wish to calculate its magnitude. From $C$ drop a perpendicular upon $OA$, meeting it (produced if necessary) in $D$.

In Fig. 101$a$ we have

$$\begin{aligned} OC^2 &= OD^2 + DC^2, \\ &= (OA + AD)^2 + DC^2, \\ &= OA^2 + 2\,OA.AD + AD^2 + DC^2, \\ &= OA^2 + AC^2 + 2\,OA.AD, \\ &= OA^2 + AC^2 + 2\,OA.AC \cos\theta, \\ &= OA^2 + OB^2 + 2\,OA.OB \cos\theta. \\ \therefore\ R^2 &= P^2 + Q^2 + 2\,PQ \cos\theta. \end{aligned}$$

In Fig. 101$b$ we have

$$\begin{aligned} OC^2 &= OD^2 + DC^2, \\ &= (OA - DA)^2 + DC^2, \\ &= OA^2 - 2\,OA.AD + AD^2 + DC^2, \\ &= OA^2 + AC^2 - 2\,OA.AD, \\ &= OA^2 + AC^2 - 2\,OA.AC \cos \angle\, DAC, \\ &= OA^2 + AC^2 - 2\,OA.AC \cos(180 - \theta), \\ &= OA^2 + OB^2 + 2\,OA.OB \cos\theta. \\ \therefore\ R^2 &= P^2 + Q^2 + 2\,PQ \cos\theta \text{ as before.} \end{aligned}$$

Hence the magnitude of the resultant of two forces $P$ and $Q$ whose lines of action make an angle $\theta$ with one another is given by the equation

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta.$$

## PROBLEMS

1. Find the resultant of the following forces:
   (1) 36 pounds and 60 pounds at an angle of 60°.
   (2) 10 pounds and 10 pounds at an angle of 45°.
   (3) 10 pounds and 10 pounds at an angle of 150°.
   (4) 30 pounds and 80 pounds at an angle of 120°.
   (5) 2 pounds and 7 pounds at an angle of 30°.
   (6) 2 pounds and 3 pounds at an angle of 135°.
   (7) 3 pounds and 16 pounds at an angle of 15°.
   (8) 4 pounds and 11 pounds at an angle of 75°.
   (9) $P$ acting toward the west and $P\sqrt{2}$ toward the northeast.

2. Prove that the resultant of two forces, $P$ and $P + Q$, acting at an angle of 120°, is equal to the resultant of two forces, $Q$ and $P + Q$, acting at the same angle.

3. Find the resultant of two forces represented by the side of an equilateral triangle and the perpendicular on this side from the opposite angle.

4. Six posts are placed in the ground so as to form a regular hexagon, and an elastic cord is passed around them and stretched with a force of 50 pounds. Find the magnitude and the direction of the resultant pressure on each post.

5. Two forces of two pounds each, acting at an angle of 60°, have the same resultant as two equal forces acting at right angles. What is the magnitude of these forces?

6. The resultant of two forces which act at an angle of 60° is 13 grams. If one of the forces is 7 grams, find the other.

7. A particle is acted upon by two forces, one of which is inclined at an angle of 80° to the vertical, and the other at an angle of 40° to the vertical and on the other side of it. If one of the forces is 10 pounds, and the combined effect of the two is $2\sqrt{31}$ pounds, find the other force.

8. If one of two forces acting on a particle is 5 kilograms, and the resultant is also 5 kilograms, and at right angles to the known force, find the magnitude and the direction of the other force.

9. The resultant of two forces, $P$ and $Q$, is $Q\sqrt{3}$, and its direction makes an angle of 30° with the direction of $P$. Show that $P$ is either equal to $Q$ or $2Q$.

10. Show that when two forces act at a point their resultant is always nearer the greater force, and the greater the angle between the forces the less is their resultant.

11. If a uniform heavy bar is supported in a horizontal position by a string slung over a smooth peg and attached to both ends of the bar, prove that the tension of the string will be diminished if its length is increased.

12. A weight is suspended by means of two strings of equal length attached to points in the same horizontal line. Show that if the lengths of the strings are increased their tension is diminished.

**110. Resolution of Forces.** Sometimes in felling a tree a rope is tied to the trunk, as high up as possible, and then pulled. This is done in order to make the tree topple over and also to cause it to fall in a safe direction (Fig. 102). Now

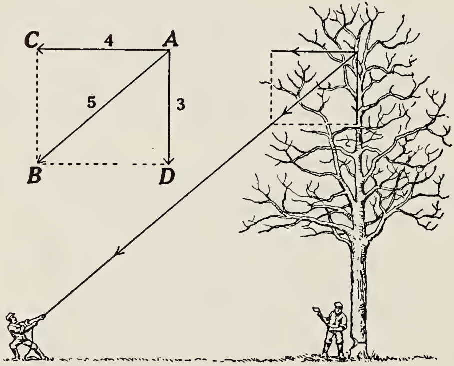



FIG. 102—A pull along the rope pulls the tree over and also pulls it vertically downwards.

it is evident that the tension in the rope has the effect not only of pulling the tree over but also of pulling it vertically downwards. This latter force does not help in the removal of the tree at all.

It seems, then, that the same effect upon the tree could be produced if we substituted for the force along the rope two other forces, one in the horizontal, tending to topple the tree over, and the other in the downward vertical direction.

The magnitudes of these **component** forces can be determined from a consideration of the parallelogram of forces. Suppose the pull on the rope is 100 pounds. Draw a line $AB$ parallel to the rope and make it 5 inches long, and from $B$ draw horizontal and vertical lines meeting the vertical and horizontal lines through $A$ in the points $D$ and $C$. The lengths of $AC$ and $AD$ will represent the magnitudes of the forces in the horizontal and vertical directions.

For example, if the lengths of $AC$, $AD$ are 4 and 3 inches, respectively, the horizontal force is 80 pounds and the vertical force is 60 pounds.

These two forces are said to be **components** of the 100-pd. force in the horizontal and vertical directions, and the force is said to be **resolved into these two components.**

It is well to observe, however, that a force can be resolved into components in any two directions. We need only represent the original force by the diagonal of a parallelogram, and the two components will be represented by the two adjacent sides.

**111. Calculation of Components.** A force $R$ is represented in magnitude and direction by the line $OC$ (Fig. 103); we wish to find the components $P$, $Q$ of the force in the directions $OA$, $OB$, where $OA$ is at right angles to $OB$.

Let angle $COA = \alpha$ and angle $COB = \beta$.

Draw $CA$ perpendicular to $OA$, and $CB$ perpendicular to $OB$.

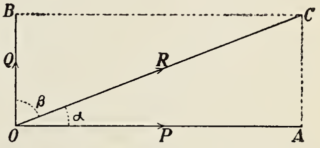


FIG. 103.—Finding the component of a force in any direction.

Then $OA$ and $OB$ will represent $P$ and $Q$, respectively, in magnitude.

Now $\frac{OA}{OC} = \cos \alpha$, or $OA = OC \cos \alpha$, whence $P = R \cos \alpha$;

and $\frac{OB}{OC} = \cos \beta$, or $OB = OC \cos \beta$, whence $Q = R \cos \beta$.

We thus see that **the component of a force resolved in any direction is equal to the product of the force into the cosine of the angle between the direction of the force and the new direction.**

But since $\alpha + \beta = 90°, \beta = 90 - \alpha,$

and $Q = R \cos \beta = R \cos (90 - \alpha) = R \sin \alpha.$

## PROBLEMS

1. Find the resolved part of a force of 10 pounds in a direction making an angle with the direction of the force of (1) 30°, (2) 45°, (3) 75°.

2. Find the horizontal and the vertical resolved parts of a force of 20 pounds, which makes an angle of 30° with the horizontal.

3. Find the resolved part S.W. of a force of 12 pounds S.

4. A force of 100 pounds is resolved into two equal forces at right angles to each other. What is the magnitude of either force?

5. The resultant of two forces acting at right angles is 16 pounds, and makes an angle of 30° with one of the components. Find the magnitude of the components.

6. The horizontal resolved part of a force making an angle of 30° with the horizontal is 4 pounds. Find the vertical resolved part.

7. A horse, in towing a canal boat, pulls with a force of 200 pounds. If the tow-rope is horizontal and makes an angle of 5° with the direction of the canal, find the magnitude of the force that would have to be applied in the direction of the canal to draw the boat.

8. The handle of a lawn mower is inclined at an angle of 30° to the ground. If a man pushes along the handle with a force of 20 pd., how much of the force is effective in moving the mower along the ground?

9. A block of wood is placed on a board inclined at an angle of 20° to the horizon. If the block weighs 10 pounds, how much of this can be considered as tending to make the block slip down the plane?

10. A boy pulls on the rope attached to his sled with a force of 30 pd. If the rope makes an angle of 25° with the ground, find the vertical and horizontal components of the force.

**112. Translation and Rotation.** It should be observed, however, that very generally when several forces act upon a body they tend nót only to cause the body to move as a whole or to give it a **motion of translation,** but also to make it turn about an axis as well, that is, to give it a **motion of rotation.** This turning effect of a force will be considered more fully in the next chapter.

Omit **113. The Sailing Ship.** The sailing of a ship in a direction almost opposite to that from which the wind is coming has long been considered an interesting example of the resolution of forces.

Let the ship be moving with a velocity of $V$ ft. per sec. in the direction $CD$, and let the sail $AB$ make with $CD$ the angle $\theta$. Let the velocity of

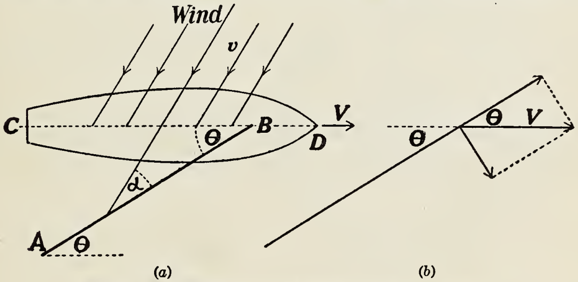



Fig. 104.—Finding the force propelling a sailing vessel.

the wind $= v$ ft. per sec. in a direction making an angle $\alpha$ with the sail. (Fig. 104$a$).

It is assumed that the keel or centre-board prevents drifting sideways.

Resolving the velocity of the wind, we have (Fig. 104$a$)

$$v \cos \alpha \text{ along the surface of the sail,}$$

and $$v \sin \alpha \text{ at right angles to the sail.}$$

Again, the sail's motion in the direction $CD = V$ ft. per sec.

This may be resolved into two components (Fig. 104$b$),

$$V \cos \theta \text{ along the plane of the sail,}$$

and $$V \sin \theta \text{ at right angles to the sail.}$$

Consequently the velocity of the wind with respect to the sail, in the direction at right angles to the sail,

$$= v \sin \alpha - V \sin \theta \text{ ft. per sec.;}$$

and the force at right angles to the sail

$$= k \times \text{area} \times \text{density} \times (\text{velocity})^2 \qquad \text{(Sec. 56)}$$

$$= kA \times 0.08\,(v \sin \alpha - V \sin \theta)^2 \text{ poundals,}$$

where $k$ is a constant depending on the shape of the sail and $A$ is its area in sq. ft.

Now the only part of this which is effective in driving the ship forward is that component in the direction $CD$, which

$$= kA \times 0.08\,(v \sin \alpha - V \sin \theta)^2 \sin \theta \text{ pdl.}$$

$$= kA \times 0.0025\,(v \sin \alpha - V \sin \theta)^2 \sin \theta \text{ pd.}$$

**Example.**—Find the total force on a sail 120 sq. ft. in area if the sail is set at 30° to the central line of ship, the wind is directly across the ship with a velocity of 5 mi. per hr., and the ship is moving at rate of 7 mi. per hr. Take $k = 0.7$.

Here, $\alpha = 60°$, $\theta = 30°$, $v = 7.33$ ft. per sec., and $V = 10.27$ ft. per sec.

Total force $= 0.7 \times 120 \times 0.0025\,(7.33 \sin 60° - 10.27 \sin 30°)^2$ pd.

$= 0.309$ pd.

This example shows that the velocity of a ship may be greater than the velocity of the wind, which causes the motion. This has often been remarked in the case of an ice-boat, which meets with little resistance and can make great speed.

# CHAPTER XII

## Moment of a Force

**114. Moment of a Force.** If you have to turn a nut which is rusted tight you can exert the greatest turning effort by using a wrench with a long handle. Again if you wish to turn a wheel which is hard to move you do not take hold of the hub, but of the rim (*i.e.*, as far as possible from the axis), and you exert a force at right angles to the spoke where you take hold. Similarly, in stormy weather, in order to keep the ship on her course the wheelsman grasps the wheel by the pins at the rim and exerts a force at right angles to the line joining the axis to the point where he takes hold (Fig. 105). If a machine is driven by a crank the longer the crank is the greater is the turning effort which can be exerted.

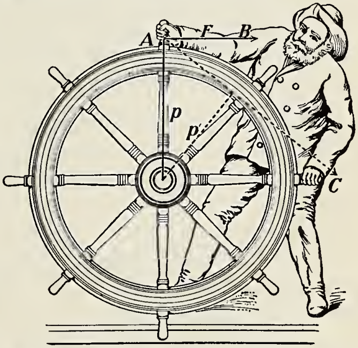



Fig. 105.—The moment of a force depends on the force applied and its distance from the axis of rotation.

From our experience we know that the turning effect upon the wheel is proportional to the force exerted and also to the distance from the axis of the point where the force is applied.

Let $F$ = the force applied,

$p$ = the perpendicular distance from the axis to the line $AB$ of the applied force.

By experience we know that the power to turn the wheel depends directly on $F$ and on $p$, and is therefore proportional to $Fp$. This product $Fp$ is called the **moment of the force $F$** about the axis. **The moment of a force about a point is the turning effect of the force about the point. It is measured by the product of the force and the perpendicular distance drawn from the point to the line of action of the force.**

If the direction of the force $F$ is not perpendicular to the line joining its point of application to the axis, the moment is clearly not so great, since part of the force is spent uselessly in pressing the wheel against its axis. In Fig. 105, if $AC$ is the new direction of the force, then $p'$, the new perpendicular, is shorter than $p$, and hence the product $Fp'$ is smaller.

**115. Experiment on Moments.** The tendency of forces to produce rotation about a point may be studied experimentally by using the apparatus shown in Fig. 106. $AB$ is a metre stick provided with a slider $F$ which carries a knife edge by which the stick is supported. The slider is moved until the metre stick just balances in a horizontal position. The masses $P$ and $W$ are then suspended from the stick by loops of thread and adjusted until the stick balances once more.

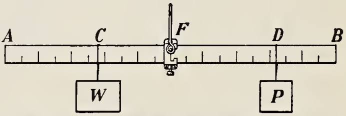


Fig. 106.—Testing the principle of moments.

Five or six experiments should be made, changing the masses and their distances from $F$ and tabulating the results as follows:

| $P$ | Arm of $P$ $FD$ | Moment of $P$ $P \times FD$ | $W$ | Arm of $W$ $FC$ | Moment of $W$ $W \times FC$ |
|---|---|---|---|---|---|
| 200 gm. | 35 cm. | 7,000 | 500 gm. | 14 cm. | 7,000 |
| 300 " | 40 " | 12,000 | 400 " | 30 " | 12,000 |

It will be found that in every case the moment of $P$ about $F$ equals the moment of $W$ about $F$.

The experiment may be varied by attaching a third mass $Q$ on the same side of $F$ as $P$. In this case both $P$ and $Q$ will tend to produce rotation in a clockwise direction, while $W$ will tend to cause contra-clockwise rotation. On taking moments it will be found that the sum of the clockwise moments equals the sum of the contra-clockwise moments.

**116. The Principle of Moments.** In the experiments just described the lines of action of the forces were parallel. Fig. 107 shows how apparatus may be arranged so that the forces are not parallel.

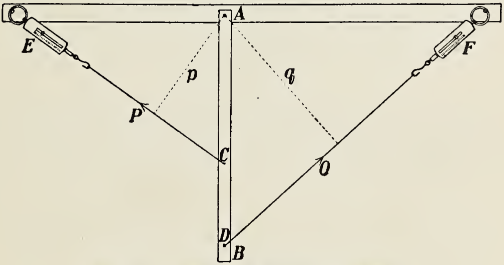



FIG. 107.—Testing the principle of moments.

$AB$ is a wooden strip about 100 cm. long pivoted at $A$ to the top of the blackboard. A chalk mark is made along one edge of the strip when it is hanging freely. Cords fastened to the hooks of the spring-balances $E$ and $F$ are then attached at $C$ and $D$ and adjusted until the strip takes up its original position again. $P$ and $Q$, the readings of the spring-balances, are then taken and the perpendiculars $p$ and $q$ measured.

The moment of $P$ about $A$ is $Pp$, tending to make the rod rotate in a clockwise direction; while the moment of $Q$ about $A$ is $Qq$, tending to produce contra-clockwise rotation. Since the rod is in equilibrium $Pp$ should be found equal to $Qq$.

**Example.**—In an experiment

$$P = 700 \text{ gm.}, p = 73 \text{ cm.}; Pp = 51{,}100,$$
$$Q = 1{,}000 \text{ gm.}, q = 51 \text{ cm.}; Qq = 51{,}000.$$

A slightly different arrangement of the experiment is shown in Fig. 108.

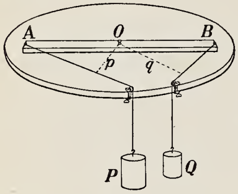


Fig. 108.—Alternative apparatus for testing the principle of moments.

$AB$ is a wooden strip pivoted at its centre $O$ on a horizontal board. To reduce friction a greased washer is placed under the strip at the pivot. Cords are attached at $A$ and $B$ and passed over smooth-running pulleys at the edge of the board. The weights $P$ and $Q$ are fastened to the ends of the cords and the perpendiculars $p$ and $q$ measured after the system has taken up a position of equilibrium. The moments $Pp$ and $Qq$ will be found equal within the limits of experimental error, as before.

These and similar experiments lead us to a conclusion called the *Principle of Moments:* **When a body free to turn about a point is in equilibrium, the sum of all the clockwise moments about that point must equal the sum of all the contra-clockwise moments about the point; or, the algebraic sum of the moments about the point equals zero.**

Clockwise moments are usually considered negative and contra-clockwise moments positive.

**117. The Wheel and Axle.** The principle of moments is well illustrated in the apparatus known as the wheel and axle (Fig. 109$a$).

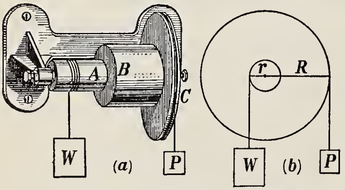


Fig. 109.—The wheel and axle: ($a$) general appearance; ($b$) diagram to explain its action.

A mass $W$, fastened to one end of a cord which is wound about a cylinder $A$ (or $B$), called the axle, is raised by a force $P$ applied to a cord which is wound about the circumference of a wheel $C$, joined to the axle.

If $P$ is just sufficient to balance $W$, its moment about the axis of the cylinder must be equal to the moment of $W$ about the axis (Fig. 109*b*).

Let $r$ and $R$ be the radii of the axle and of the wheel, respectively.

Then, $$W \times r = P \times R,$$
or $$W/P = R/r.$$

Thus by taking a large wheel and a small axle the mass $W$ can be lifted by a small force $P$.

Before hay-forks were invented this principle was used by many farmers to facilitate the unloading of hay and grain. The rack of the loaded wagon was attached to ropes wound around a strong axle resting on beams near the top of the barn and was raised to the level of the mow by the team pulling on a rope passing over a large wheel fastened to the axle.

## PROBLEMS

1. A metre stick just balances at the 50-cm. mark. Masses of 50 and 100 gms. are then attached on opposite sides of the fulcrum and the stick balances once more. If the 50-gm. mass is at the 10-cm. mark, where is the 100-gm. mass?

2. A boy pushes on the pedal of his bicycle with a force of 30 pounds. If the crank, which is 8 inches long, is horizontal and if the push is vertical, what is the moment of the force? Find the moment if the direction of the push makes an angle of 60° with the crank.

3. A uniform metre stick just balances at the 50-cm. mark when masses of 50, 100 and $x$ grams are attached at the 10, 20 and 90-cm. marks, respectively. Find $x$.

4. A uniform plank is pivoted at its centre and just balances when two boys weighing 100 and 120 lb. are on opposite sides of the fulcrum. If the heavier boy is 5 feet from the fulcrum, where is the other?

5. If the wheel (Fig. 109) has a diameter of 6 inches and the axle a diameter of 1.5 inches, what force applied to the cord which passes around the wheel will support a mass of 10 lb. attached to the cord wound around the axle?

6. A force of 12 acts along a median of an equilateral triangle whose side is 18. Find the measure of the moment of the force about each angle of the triangle.

7. A force of 6 acts along one side of an equilateral triangle whose side is 10. Find the measure of its moment about the opposite angle.

8. A force of 20 acts along a diagonal of a square whose side is $8\sqrt{2}$. Find the measure of its moment about each of the four angles.

9. The connecting-rod of an engine is inclined to the crank-arm at an angle of 30°. Compare the moment of the force to turn the shaft when in this position with the moment when in the most favourable position.

10. $ABCD$ is a square, whose side is 2 ft. long. Find the moments about both $A$ and $D$, of the following forces:—(1) 3 pounds along $AB$, (2) 9 pounds along $CB$, (3) 2 pounds along $DA$, (4) 11 pounds along $AC$, (5) 1 pound along $DB$, (6) 20 pounds along $DC$.

11. $ABCD$ is a rectangle, the side $AB$ being 12 cm. and the side $BC$ 5 cm. long. $O$ is the intersection of the diagonals. Find the algebraic sum of the moments about (1) $A$, (2) $O$, of the following forces:—14 dynes along $BA$, 19 dynes along $BC$, 3 dynes along $CD$, 4 dynes along $AD$, 10 dynes along $AC$, and 9 dynes along $DB$.

12. At what point of a tree must one end of a rope whose length is 50 feet be fixed, so that a man pulling at the other end may exert the greatest moment tending to pull it over?

**118. The Centre of Gravity of a Body.** In performing the experiment described in Sec. 115, care was taken to balance the metre stick before attaching the weights. A uniform rod or stick will balance on a pivot or fulcrum placed at its centre. A non-uniform rod, such as a fishing-rod, balances at a point closer to the thicker end. **This point of balance is called the *Centre of Gravity* of a body and the action of gravity on the body produces no turning effect or moment about this point. From the standpoint of moments we can consider the whole weight of the body as being concentrated at the centre of gravity.**

The problem of determining the centres of gravity of various bodies will be considered in a later chapter.

**119. Resultant of Parallel Forces in the same Direction.** This may be illustrated by the following experiment:

Weigh a metre stick and find $C$, its centre of gravity. Attach it to spring-balances by loops of thread placed at $A$ and $B$ (Fig. 110). Suspend a weight $W$ from the rod by a thread tied at $C$.

See that the rod is horizontal and take the readings $P_1$, $P_2$ on the balances; measure also the distances $l_1$ and $l_2$ of the weight from $P_1$ and $P_2$. Repeat for different positions of $A$ and $B$.

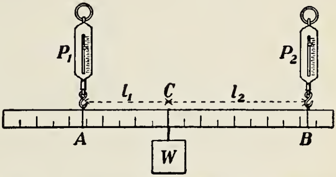


FIG. 110.—Finding the resultant of parallel forces.

Since $W$ is suspended from the centre of gravity of the rod, it is evident that we can consider the total weight at $C$ as being $W + w$ where $w$ is the weight of the rod.

Tabulate the results as follows:—

| $P_1$ | $P_2$ | $P_1 + P_2$ | $W + w$ | $P_1 \times l_1$ | $P_2 \times l_2$ |
|---|---|---|---|---|---|
| 200 gm. | 100 gm. | 300 gm. | 300 gm. | 4000 | 4000 |
| 300 " | 200 " | 500 " | 500 " | 6000 | 6000 |

It will be found that $P_1 + P_2 = W + w$ and that $P_1 \times l_1 = P_2 \times l_2$ in every case. Now $W + w$ balances $P_1$ and $P_2$; hence the resultant of $P_1$ and $P_2$ must be equal to $W + w$ and must act at $C$ vertically upward, that is, parallel to $P_1$ and $P_2$.

Also $P_1 \times l_1$ is the moment of $P_1$ about $C$ and $P_2 \times l_2$ is the moment of $P_2$ about $C$, and these moments have been found equal. We conclude then that **the resultant of two parallel forces acting in the same direction is equal to the sum of the forces and its point of application is so situated that the moments of the two forces about the point are equal.**

Another method of performing the experiment is shown in Fig. 111.

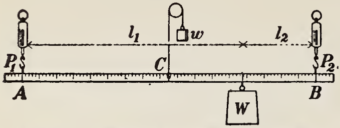



FIG. 111.—Finding the resultant of parallel forces.

Support the rod at its centre of gravity $C$ by a weight $w$ passing over a pulley. Attach spring-balances at $A$ and $B$ and hang a weight $W$ at any point of the stick.

Take the readings $P_1$, $P_2$ of the spring-balances and read the distances $l_1$, $l_2$ of the balances from the weight.

No matter where $W$ is placed, we shall find

$$P_1 + P_2 = W,$$

and
$$P_1 \times l_1 = P_2 \times l_2.$$

Next, arrange that the metre stick shall not be horizontal, but in the position $AB$ (Fig. 112). It will be found to be in equilibrium still, with the balances showing the same readings.

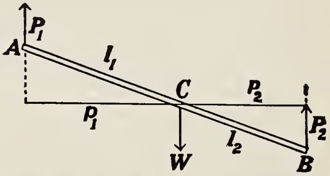



FIG. 112.—Forces on a rod not horizontal.

From $C$ drop perpendiculars on the line of action of $P_1$, $P_2$ and let their lengths be $p_1$, $p_2$.

Then moment of $P_1$ about $C = P_1 \times p_1$,

and moment of $P_2$ about $C = P_2 \times p_2$,

and these are equal.

But from similar triangles $p_1/p_2 = l_1/l_2$,

and so $P_1 \times l_1 = P_2 \times l_2$, as before.

Hence the resultant of $P_1$, $P_2$ has the same magnitude as before and its point of application is unchanged.

**120. Couples.** Attach a string to each end of a rod lying on a table, and pull on these with equal forces $P$ in parallel directions (Fig. 113). The rod moves forward in the direction of the force.

Next, pull with equal forces but in opposite senses (Fig. 114). Now the rod simply turns about a vertical axis without moving forward as a whole.

**Two equal unlike parallel forces are called a couple.**


Fig. 113.— Motion of translation only.

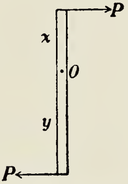



Fig. 114.— Motion of rotation only.

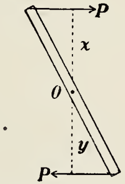



Fig. 115.— Motion of rotation only.

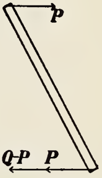



Fig. 116.—Translation and rotation.

Let us calculate the moment of this couple about **any** point $O$ in the rod (Figs. 114 and 115). It is evident that the total turning effect is

$$Px + Py = P\,(x + y) = Pd,$$

where $d$ is the perpendicular distance between the lines of action of the forces. Moreover it is evident that the magnitude of the moment is independent of the position of $O$.

Next, pull one end of the rod with a force $P$, and the other with a greater force $Q$ (Fig. 116). This force $Q$ may be considered as made up of two components,

$$P, \text{ and } Q - P.$$

The two forces $P$, $P$ will form a couple and will produce rotation of the rod, while the force $Q - P$ will produce a motion of the rod as a whole, or a translation, in the direction of the force.

**121. Examples.** 1. Find the magnitude and point of application of the resultant of two parallel forces of 5 pd. and 10 pd. acting in the same direction at points 30 inches apart.

Let the forces act at $A$ and $B$ as indicated in Fig. 117, and let $C$ be the point at which a force $W$ must act to produce equilibrium.

Let $AC = x$; then $CB = 30 - x$.

Now $W$ must equal $5 + 10 = 15$ pd. and therefore $R$, the resultant, $=$ 15 pd. (upwards). Moreover, taking moments about $C$,

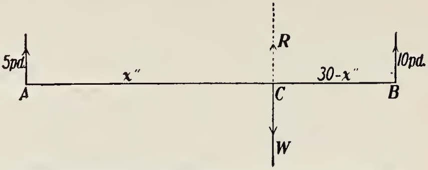



FIG. 117.—Finding the resultant of like parallel forces.

$$5 \times x = 10\,(30 - x),$$

or $$15x = 300,$$

whence $$x = 20 \text{ inches.}$$

It is instructive to obtain $x$ by taking moments about $A$ or $B$.

Since the rod is not turning about $A$, the clockwise moments about $A$ must equal the contra-clockwise moments.

Hence $$Wx = 10 \times 30,$$

or $$15x = 300,$$

or $$x = 20 \text{ inches, as before.}$$

2. Find the magnitude and point of application of the resultant of two parallel forces of 4 pd. and 7 pd. acting in opposite directions at points 30 inches apart.

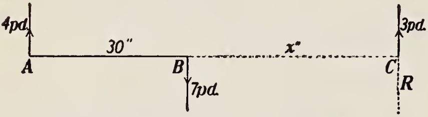



FIG. 118.—Finding the resultant of unlike parallel forces.

Let the forces act at $A$ and $B$ (Fig. 118) in the directions indicated. Then it is evident that to produce equilibrium there must be acting an additional upward force of $7 - 4 = 3$ pd. If we can determine where this force must act to produce equilibrium, we shall know where the resultant acts.

If we place this force between $A$ and $B$ and consider moments about $B$, there will be an unbalanced clockwise moment, since both the 4-pd. force and the 3-pd. force will tend to produce clockwise rotation about $B$. There would also be an unbalanced clockwise moment about $B$ if the force were applied to the left of $A$.

Hence the force must be applied to the right of $B$, at some point $C$. Let $BC = x$.

Since we have equilibrium, the rod is not turning about $B$ (or any other point). Taking moments about $B$,

$$4 \times 30 = 3 \times x,$$

whence $$x = 40 \text{ inches.}$$

We could choose the point $A$ equally well.

Taking moments about $A$,

$$7 \times 30 = 3\,(30 + x),$$

whence $$3x = 120,$$

and $$x = 40 \text{ inches, as before.}$$

Hence $R = 3$ pd. acting vertically *downwards* at $C$ which is 40 inches from $B$.

In these examples the following rules have been followed:

1. Construct a diagram showing the forces acting, representing each force by a straight line and its direction by an arrow.

2. Indicate on the diagram where a force must be applied and in what direction it must act to produce equilibrium. The resultant will act at the same point but in the opposite direction.

3. Solve for the unknown quantities by writing the equations which must hold in order that there may be no translation and no rotation.

## PROBLEMS

1. Find the magnitude and point of application of the resultant of two parallel forces of 3 dynes and 2 dynes acting in the same direction at points 5 cm. apart.

2. Two men of the same height carry on their shoulders a pole 6 feet long, and a mass of 120 pounds is slung on it, 30 inches from one of the men. What portion of the weight does each man support?

3. Two men support a weight of 112 pounds on a pole of negligible weight which rests on the shoulder of each. The weight is twice as far from the one as from the other. Find what weight each supports.

4. A man carries two buckets of water by means of a pole which he holds in his hand at a point three-fifths of its length from one end. If the total weight carried is 40 pounds, how much does each bucket weigh?

5. Two men, one stronger than the other, have to remove a block of stone weighing 270 pounds by means of a light plank whose length is 6 feet; the stronger man is able to carry 180 pounds. How must the stone be placed on the plank so as to allow him that share of the weight?

6. A plank weighing 10 pounds rests on a single prop at its middle point; if the single prop is replaced by two others, one on each side of it, 3 feet and 5 feet from the middle point, find the pressure on each.

7. A light rigid rod 20 feet long is supported in a horizontal position on two posts 9 feet apart, one post being 4 feet from the end of the rod; from the middle point of the rod a weight of 63 pounds is suspended. Find the pressures on the posts.

8. Find the magnitude and point of application of the resultant of two opposite parallel forces of 17 dynes and 25 dynes acting at points 8 centimetres apart.

9. The resultant of two parallel forces is 15 pounds, and acts at a distance of 4 feet from one of them whose magnitude is 7 pounds. Find the position and magnitude of the second force, when (1) the forces are in the same direction, (2) when opposite.

10. Find the magnitude and point of application of the resultant of two opposite parallel forces of 10 pd. and 15 pd. acting at points 4 feet apart.

# CHAPTER XIII

## Equilibrium of a Rigid Body

**122. Translation and Rotation.** As has been remarked (Sec. 112) when forces act upon a body they may tend to give it a translation or a rotation or both at the same time. A body is translated when its centre of gravity (see Chap. XV) is displaced; it is rotated when any lines drawn in the body change their directions.

Consider a body acted upon by forces $P, Q, R, S$, etc., acting in one plane, as in Fig. 119.

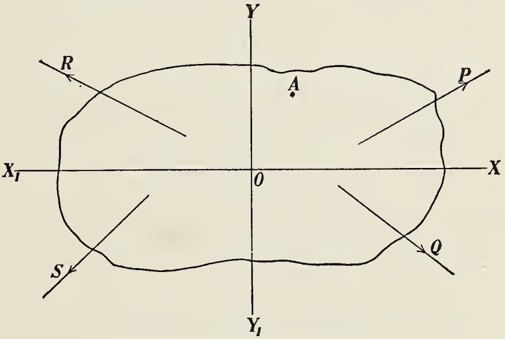



Fig. 119.—Equilibrium of a rigid body.

If the body is in equilibrium, of course there is no translation. Consequently if the forces are resolved along any two axes $XOX_1$ and $YOY_1$ at right angles to each other, the components in the direction $OX$ must balance those in the direction $OX_1$ and the components in the direction $OY$ must balance those in the direction $OY_1$.

Also, there is no rotation. Consequently the clockwise moments about *any* point $A$ must balance the contra-clockwise moments about this point.

We have then two general conditions for the equilibrium of a body acted upon by a number of forces in one plane:—

**1. If all the forces acting on the body be resolved along any two directions at right angles, the algebraic sum of the resolved parts along each of these directions must equal zero.**

**2. The algebraic sum of the moments of the forces about any point must equal zero.**

By making use of these two general principles many problems of equilibrium may be solved.

**123. First Example.** A uniform plank 13 feet long, weighing 40 pounds, rests on two trestles placed 12 feet apart, each trestle being placed six inches from an end of the plank. Find the weight carried by each trestle when a man weighing 150 pounds stands on the plank 4 feet from one trestle.

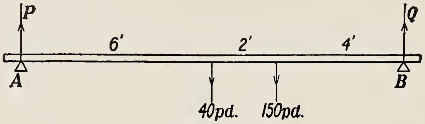



FIG. 120.

(1) Fig. 120 is a diagram of the system of forces *acting on the plank* and keeping it in equilibrium. $P$ and $Q$ are the *reactions* of the trestles on the plank.

(2) Since all of the forces are parallel we do not need to resolve them. We see immediately that the up forces must equal the down forces.

Hence $$P + Q = 190.$$

(3) Since there is no rotation the contra-clockwise moments about any point must equal the clockwise moments about the same point.

Taking moments about $A$,

$$Q \times 12 = 40 \times 6 + 150 \times 8,$$

whence $$Q = 120 \text{ pd.}$$

and $$P = 190 - 120 = 70 \text{ pd.}$$

In calculating moments the point $A$ was chosen so that $P$ would not appear in the equation. The point $B$ would be equally satisfactory. If any other point were taken, both $P$ and $Q$ would appear in the equation but we could still solve for $P$ and $Q$ by using the other equation, namely, $P + Q = 190$.

**Experiment.**—Weigh a metre stick and find its centre of gravity. Support it by means of two spring balances attached to loops of thread placed at the 10 and 90-cm. marks. Attach a mass of 50 gm. to a loop placed at the 60-cm. mark and adjust the balances until they are vertical and the stick horizontal. Take the readings of the balances and compare with results calculated mathematically as in the above example.

## PROBLEMS

1. A uniform beam is of length 12 metres and mass 50 kg., and from its ends are suspended bodies of masses 20 and 30 kg. respectively. At what point must the beam be supported that it may remain in equilibrium?

2. A lever with a fulcrum at one end is 3 feet in length. A mass of 24 lb. is suspended from the other end. If the mass of the lever is 2 lb. and acts at its middle point, at what distance from the fulcrum will an upward force of 50 pd. preserve equilibrium?

3. Masses of 7 lb., 1 lb., 3 lb., and 5 lb. are placed on a rod, supposed weightless, 1 foot apart. Find the point on which the rod will balance.

4. A bar 16 cm. long is balanced on a fulcrum at its middle. On the right arm are suspended 4 grams and 3 grams at distances of 5 cm. and 7 cm., respectively, from the middle, and on the left arm 5 grams at a distance 5 cm. from the middle and $w$ at the end. Determine $w$.

5. A light rigid bar 30 feet long has suspended from its middle point a mass of 700 lb., and rests on two walls 24 feet apart, so that 1 foot of it projects over one of them. A mass of 192 lb. is suspended from a point 2 feet from the other end. What is the pressure borne by each of the walls?

6. Six parallel forces of 7 dynes, 6 dynes, 5 dynes, 4 dynes, 3 dynes and 2 dynes are applied to a rigid rod at points 1 cm. apart. Find the magnitude and position of the resultant.

7. Five parallel forces 1, 6, 3, 4, 8 dynes act 1 cm. apart on a straight horizontal rod. How much must be added to the 1-dyne force in order that if the rod is supported where the force of 3 dynes acts it may remain horizontal?

8. Four parallel forces 3, 2, 5, 7 dynes act at right angles to a straight rod, at points 6 cm. apart. Where must a force of 17 dynes act in order to maintain equilibrium?

9. A straight uniform heavy rod of length 6 feet has masses of 15 and 22 lb. attached to its ends, and rests in equilibrium when placed across a fulcrum distant $2\frac{1}{2}$ feet from the 22-lb. mass. Find the mass of the rod.

10. A straight rod of negligible weight 2 feet long rests in a horizontal position between two fixed pegs, placed 3 inches apart, one of

the pegs being at one end of the rod. If a mass of 5 lb. is suspended at the other end, find the pressure on each of the pegs.

11. A heavy uniform beam, whose mass is 40 kg., is suspended in a horizontal position by two vertical strings attached to the ends, each of which can sustain a tension of 35 kg. How far from the centre of the beam must a body, of mass 20 kg., be placed so that one of the strings may just break?

12. A heavy tapering rod, having a mass of 20 lb. attached to its smaller end, balances about a fulcrum placed at a distance of 10 feet from the end. If the mass of the rod is 200 lb., find the point about which it will balance when the attached mass is removed.

**124. Three Forces in Equilibrium.** Let us consider the special case of three co-planar forces, not parallel, acting on a body and keeping it in equilibrium.

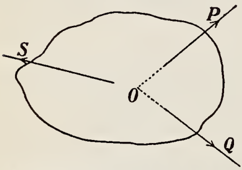



Fig. 121.—Three forces in equilibrium meeting in a point.

Let *P*, *Q* and *S* (Fig. 121) represent the lines of action of the forces.

Any two of these lines of action will meet in a point. Let the lines of action of *P* and *Q* meet in the point *O*.

Then the resultant of *P* and *Q* will also pass through the point *O* and consequently the line of action of *S*, which just balances this resultant, must also pass through the point *O*.

We arrive then at the following conclusion:

**When three co-planar forces, not parallel, are in equilibrium, their lines of action must meet in a point.**

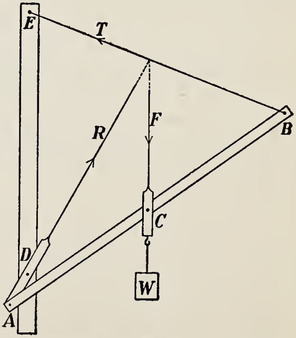



Fig. 122.—A rigid body in equilibrium under the action of three co-planar forces.

**125. Experiment.** The apparatus shown in Fig. 122 illustrates the principle just stated.

*AB* is a metre stick with holes drilled at *A*, *B* and *C*. *AD* is a light metal strap pivoted to the metre stick at *A* and to the wall at *D* and carrying a pointer on the end remote from *A*. The weight *W* is suspended from the centre of the stick *C* by a metal strap to which another pointer is attached as shown in the figure. A cord *BE* keeps the stick in position.

The forces acting on the stick are:

(1) the tension of the string acting in the direction *BE*;

(2) the combined weight of stick and *W* acting at *C* in a direction indicated by the pointer *FC*;

(3) the reaction of the pivot *A* acting at *A* in a direction indicated by the pointer *DR*.

It will be found that the directions of the pointers meet at a point on *BE* no matter what the length of *BE* is made.

## 126. Reactions of Smooth Surfaces.

Let *AB* be a heavy bar pivoted at *A* and having the end *B* resting against a perfectly smooth surface *BD*.

It is evident that *BD* is exerting a force on the bar at *B* since it is preventing gravity from turning the bar about *A*. We call this force the *reaction* of the surface.

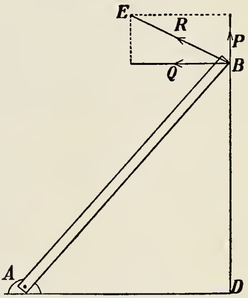


Fig. 123.—Study of the reaction of a smooth surface. *AB* is a heavy bar pivoted at *A*. The end *B* rests against a smooth surface *BD*.

**Since the surface is smooth this reaction must be at right angles to the surface.**

For, suppose it could act in any other direction, *BE* for example.

Then the reaction *R* could be resolved into a component *P* along the surface and a component *Q* at right angles to the surface.

But the component *P* along the surface would mean that friction exists, which is contrary to the assumption that the surface is smooth.

**127. General Rules.** In the solution of problems the following general rules will be found useful.

1. Construct a diagram of the system of forces which keep the body at rest, representing each force by a straight line and its direction by an arrow. In drawing lines to represent the lines of action of the various forces the following facts should be observed:

(*a*) The reactions of smooth surfaces are at right angles to these surfaces.

(*b*) When three forces, not parallel, are in equilibrium their lines of action must meet in a point.

2. Equate to zero the algebraic sum of the components of the forces in each of two convenient directions at right angles. These relations will furnish two equations.

In choosing the directions for resolution, the solution is generally simplified by resolving along and at right angles to the directions of unknown forces. Forces not to be determined may thus be eliminated.

3. Equate to zero the algebraic sum of the moments of the forces about some convenient point. A third equation is thus furnished. If additional equations are required, they are obtained from the geometrical relations of the figure.

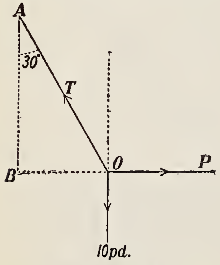


FIG. 124.—Equilibrium of a weight on the end of a string.

In choosing the point about which moments are to be taken, it is generally advisable to choose a point common to the directions of as many forces as possible. In this way also unknown forces not to be determined may be eliminated.

**128. Second Example.** A weight of 10 lb. hangs at the end of a string attached to a peg. If the weight is held aside by a horizontal force so that the string makes an angle of 30° with the vertical, find the horizontal force and the tension of the string.

The forces act as in Fig. 124.

*Solution I.*—Resolving vertically and horizontally, we obtain the following equations:

$$T \cos 30° = 10 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$P = T \cos 60° \ldots\ldots\ldots\ldots (2)$$

whence $$T \times \frac{\sqrt{3}}{2} = 10, \text{ or } T = \frac{20}{\sqrt{3}} \text{ pd.},$$

and $$P = T \times \tfrac{1}{2} = \frac{10}{\sqrt{3}} \text{ pd.}$$

*Solution II.*—Taking moments about $A$ and taking $AO = 2l$, we have

$$P \times AB = 10 \times OB,$$

whence $$P \times l\sqrt{3} = 10 \times l, \text{ and } P = \frac{10}{\sqrt{3}} \text{ pd.}$$

Also $$T^2 = P^2 + 10^2,$$

$$= \tfrac{100}{3} + 100,$$

whence $$T = \frac{20}{\sqrt{3}} \text{ pd.}$$

*Solution III.*—The sides $OA$, $AB$ and $BO$ of the triangle $OAB$ represent $T$, 10 and $P$ in direction, and must consequently represent them in magnitude also.

$$l\sqrt{3} \text{ represents } 10 \text{ pd.},$$

whence $$l \text{ represents } \frac{10}{\sqrt{3}} \text{ pd.},$$

and $$2l \text{ represents } \frac{20}{\sqrt{3}} \text{ pd.}$$

Therefore $$P = \frac{10}{\sqrt{3}} \text{ pd. and } T = \frac{20}{\sqrt{3}} \text{ pd.}$$

**Experiment.**—Verify the results obtained in the above example by setting up the apparatus shown in Fig. 125. The cords may very conveniently be attached to hooks at the top and side of the blackboard. The 30° angle may be obtained by using a large protractor or by making $AO = 2AC$. Use a plumb-line to fix $CO$ and a protractor, set square, or level for $OD$.

Take the readings of the spring balances and compare with the values of $F$ and $T$ obtained mathematically.

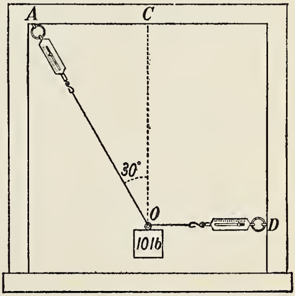



Fig. 125.—Experimental verification of results obtained mathematically.

**129. Third Example.** $AB$ (Fig. 126) is a rod 10 feet long pivoted at $A$ to a vertical wall and kept in position by a horizontal cord $BC$ 6 feet long. Neglecting the weight of the cord and rod, find the tension of the cord and the reaction of the point $A$ when a mass of 10 pounds is suspended from $B$.

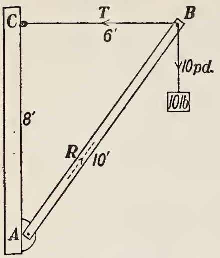


Fig. 126.—Equilibrium of forces acting on a rod.

The forces acting at $B$ and producing equilibrium are $T$ pounds in a horizontal direction, 10 pounds in a vertical direction and $R$ pounds acting along $AB$.

*Solution I.*—The sides of the triangle $BCA$ are parallel to the lines of action of the forces and will consequently represent the forces in magnitude.

In this case 8 feet represents 10 pd.,
and hence 6 " " $7\frac{1}{2}$ "
and 10 " " $12\frac{1}{2}$ "
Therefore $T = 7\frac{1}{2}$ pd. and $R = 12\frac{1}{2}$ pd.

*Solution II.*—Taking moments about $A$,

$$10 \times 6 = T \times 8,$$

whence $T = 7\frac{1}{2}$ pd.;

also $R$ = resultant of $7\frac{1}{2}$ pd. and 10 pd. acting at right angles to each other,

whence $R^2 = (7\frac{1}{2})^2 + 10^2$

and $R = 12\frac{1}{2}$ pd.

*Solution III.*—Since we know the sides of the triangle $BCA$,

$\sin \angle ABC = 0.8$ and $\cos \angle ABC = 0.6$.

Resolving the forces at $B$ horizontally and vertically and applying the conditions for equilibrium,

$$T = R \cos \angle ABC = 0.6\,R \quad \ldots\ldots (1)$$
$$10 = R \sin \angle ABC = 0.8\,R \quad \ldots\ldots (2)$$

From (2) $R = \frac{100}{8} = 12\frac{1}{2}$ pd.,

and from (1) $T = \frac{6}{10} \times \frac{25}{2} = 7\frac{1}{2}$ pd.

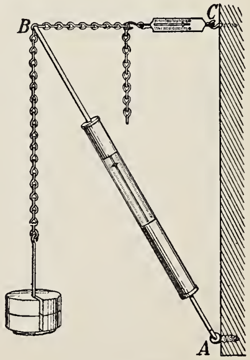


Fig. 127.—Experimental crane for verifying the equilibrium of forces acting on a rod.

**Experiment.**— Set up the experimental crane shown in Fig. 127. $AB$ is a compression balance by which the thrust

along $AB$ may be measured. Set $AB$ at an angle of about 30 degrees with the wall and make $BC$ horizontal. Read the balances before attaching the 10-lb. mass to $B$ and after attaching it, keeping $AB$ at the same inclination and keeping $BC$ horizontal for both readings. Find the increase in each reading due to attaching the 10-lb. mass and compare these values with the values of $R$ and $T$ calculated mathematically as in the above example.

## PROBLEMS

1. A mass of $10\sqrt{3}$ lb. hangs at the end of a string attached to a peg. If the mass is held aside by a horizontal force, so that the string makes an angle of 30° with the vertical, find the horizontal force and the tension of the string.

2. A mass is hung at the end of a string attached to a peg. If the mass is held aside by a horizontal force, so that the string makes an angle of 60° with the vertical, compare the tension of the string and the weight of the mass.

3. A mass of 10 lb. is supported by two strings, one of which makes an angle of 30° with the vertical. If the other string makes an angle of 45° with the vertical, what is the tension of each string?

4. A string fixed at its extremities to two points in the same horizontal line supports a smooth ring weighing 2 pounds. If the two parts of the string contain an angle of 60°, what is the tension of the string?

5. A mass of 12 lb. is supported by two strings, each of which is 4 feet long, the ends being tied to two points in a horizontal line 4 feet apart. What is the tension of each string?

6. A picture hangs symmetrically by means of a string passing over a nail and attached to two rings fixed to the picture. What is the tension of the string, if the picture weighs 6 pounds and the angle contained by the two parts of the string is 90°?

7. A string is tied to two points. A ring, mass $W$, can slip freely along the string, and is pulled by a horizontal force $P$. If the parts of the string when in equilibrium are inclined at 90° and 45° respectively to the horizon, find the value of $P$.

8. A uniform bar, the weight of which is 100 pounds, is supported in a horizontal position by a string slung over a peg and attached to both ends of the bar. If the two parts of the string contain an angle of 120°, find the tension of the string.

9. A ball weighing 20 pounds slides along a perfectly smooth rod inclined at an angle of 30° with the vertical. What force applied in the

direction of the rod will sustain the ball, and what is the pressure on the rod?

10. A body, the weight of which is 20 pounds, rests on a smooth plane, inclined to the horizon at an angle of 60°. Find (1) what force acting horizontally will keep the body at rest, (2) the reaction of the plane.

11. A spar $AB$, 10 feet long, is freely hinged to a mast at its lower end $A$. The upper end $B$ is fastened to a horizontal rope attached to a point on the mast 6 feet above $A$. Find the tension set up in the rope and the thrust produced along the spar when a mass of 1200 pounds is suspended from $B$.

12. A horizontal boom $AB$ is hinged at $A$ and is kept in position by a cable attached to $B$ and to a point vertically above $A$. The angle between the cable and boom is 30°. Find the tension in the cable and the thrust along the boom produced by suspending a mass of one ton from $B$.

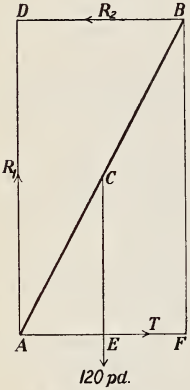



FIG. 128.—A beam resting upon a smooth wall and a smooth floor.

**130. Fourth Example.** A uniform beam $AB$, 17 feet long, whose mass is 120 lb., rests with one end against a smooth vertical wall, and the other end on a smooth horizontal floor, this end being tied by a cord 8 feet long to a peg at the bottom of the wall. Find (1) the tension of the cord, (2) the reaction of the wall, (3) the reaction of the floor.

The forces acting on $AB$ (Fig. 128) are

(*a*) Its weight, 120 pd., acting vertically downwards at its middle point $C$.

(*b*) The reaction of the floor, $R_1$, acting perpendicularly to the floor at $A$. (The reaction of a smooth surface is at right angles to itself).

(*c*) The reaction of the smooth wall, $R_2$, acting perpendicularly to the wall at $B$.

(*d*) The tension of the cord, $T$, acting parallel to the floor at $A$.

In this case there are four forces acting and we cannot conclude that they meet at a point.

Equating to zero the algebraic sum of the horizontal forces,

$$T - R_2 = 0 \ldots\ldots\ldots\ldots (1)$$

Equating to zero the algebraic sum of the vertical forces,

$$R_1 - 120 = 0 \ldots\ldots\ldots\ldots (2)$$

or $$R_1 = 120 \text{ pd.}$$

Equating to zero the algebraic sum of the moments of the forces about $A$,

$$R_2 \times AD - 120 \times AE = 0 \quad\quad (3)$$

or $$R_2 \times 15 - 120 \times 4 = 0,$$

whence $$R_2 = 32 \text{ pd.}$$

From (1) $$T = R_2 = 32 \text{ pd.}$$

**131. Fifth Example.** A uniform beam $AB$, 17 feet long, whose mass is 120 lb., rests with one end $B$ against a smooth vertical wall and is prevented from slipping by a peg driven into the ground at its lower end $A$, which is 8 feet from the bottom of the wall. Find (1) the reaction of the wall and (2) the reaction of the peg-ground corner.

The forces acting on $AB$ (Fig. 129) are

(1) Its weight, 120 pd., acting vertically downwards at its middle point $C$.

(2) The reaction of the wall $R_2$ acting perpendicularly to the wall at $B$;

(3) The reaction of the peg-ground corner acting through $D$, the point where the line of action of the weight of the beam meets $R_2$.

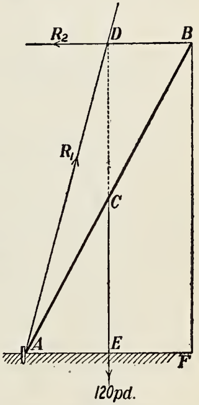



Fig. 129.—A beam resting against a smooth wall and kept from slipping by a peg.

*Solution I.*—Taking moments about $A$,

$$R_2 \times 15 = 120 \times 4,$$

whence $$R_2 = 32 \text{ pd.}$$

Also, since $R_2$ and 120 are at right angles,

$$R_1^2 = R_2^2 + 120^2,$$

whence $$R_1 = 8\sqrt{241} \text{ pd.}$$

*Solution II.*—Since $DE$, $EA$ and $AD$ are parallel to the lines of action of 120, $R_2$ and $R_1$, respectively, the triangle $DEA$ can be considered a triangle of forces and its sides will represent 120, $R_2$, and $R_1$ in magnitude.

But $DE = 15$ ft., $EA = 4$ ft. and $AD = \sqrt{241}$ ft.

Hence

| | | |
|---|---|---|
| 15 ft. | represents | 120 pd. |
| 1 ft. | " | 8 " |
| 4 ft. | " | 32 " |
| $\sqrt{241}$ ft. | " | $8\sqrt{241}$ pd. |

Therefore $$R_2 = 32 \text{ pd. and } R_1 = 8\sqrt{241} \text{ pd.}$$

*Solution III.*—The horizontal component of $R_1 = R_1 \cos \angle DAE$, the vertical component of $R_1 = R_1 \sin \angle DAE$.

$$\text{But } \cos \angle DAE = \frac{4}{\sqrt{241}}, \qquad \sin \angle DAE = \frac{15}{\sqrt{241}};$$

$$\text{Hence} \qquad R_1 \times \frac{4}{\sqrt{241}} = R_2, \qquad \text{and } R_1 \times \frac{15}{\sqrt{241}} = 120,$$

$$\text{whence} \qquad R_1 = 8\sqrt{241} \text{ pd.},$$

$$\text{and} \qquad R_2 = 32 \text{ pd.}$$

## PROBLEMS

1. A uniform beam 32 feet long, whose mass is 200 lb., rests with one end on a smooth horizontal plane and the other end against a smooth vertical wall. If a cord 16 feet long connects the lower end with the foot of the wall, find (1) the tension of the cord, (2) the pressure against the wall, (3) the pressure on the plane.

2. A ladder, the weight of which is 90 pounds, acting at a point one-third of its length from the foot, is made to rest against a smooth vertical wall, and inclined to it at an angle of 30°, by a force applied horizontally at the foot. Find the force.

3. A uniform ladder, 40 feet long, whose mass is 180 lb., rests with one end against a smooth vertical wall and is prevented from slipping by a peg in the ground. Find the pressure against the wall and at the ground if the inclination of the ladder to the horizon is 60°.

4. A uniform beam, 12 feet long, whose mass is 50 lb., rests with one end $A$ at the bottom of a vertical wall, and a point $C$ in the beam 10 feet from $A$ is connected by a horizontal cord $CD$ with a point $D$ in the wall 8 feet above $A$. Find (1) the tension of the cord, (2) the pressure against the wall-ground corner.

5. A ladder, 14 feet long, whose mass is 50 lb., rests with one end against the foot of a vertical wall; and from a point 4 feet from the upper end a cord which is horizontal runs to a point 6 feet above the foot of the wall. Find the tension of the cord and the reaction at the lower end of the ladder.

6. A uniform heavy beam $AB$, whose mass is $W$, rests against a smooth horizontal plane $CA$ and a smooth vertical wall $CB$, the lower extremity $A$ being attached to a string which passes over a smooth pulley at $C$ and sustains a mass $P$. Find the pressure on the plane and the wall.

## MISCELLANEOUS PROBLEMS

1. A uniform rod is suspended from a peg by two strings, one attached to each end. The strings are of such lengths that the angles between them and the rod are 30° and 60° respectively. Find the tensions of the strings, the mass of the rod being one kilogram.

2. A straight lever is inclined at an angle of 60° to the horizon, and a mass of 360 lb. hung freely at the distance of 2 inches from the fulcrum is supported by a force acting at an angle of 60° with the lever, at the distance of 2 feet on the other side of the fulcrum. Find the force.

3. A rod $AB$ movable about a hinge $A$ has a mass of 20 lb. attached at $B$. $B$ is tied by a string to a point $C$ vertically above $A$ and such that $CB$ is six times $AC$. Find the tension of the string $BC$.

4. A heavy uniform rod $AB$ whose mass is $W$ is hinged at $A$ to a fixed point, and rests in a position inclined at 60° to the horizon, being acted on by a horizontal force $F$ applied to the lower end $B$. Find the reaction of the hinge and the magnitude of $F$.

5. A light rod is hinged at one end and loaded at the other end with a weight of 6 pounds. The rod is supported in a horizontal position by a string which is attached to the loaded end, and which makes an angle of 30° with the rod. Find the tension of the string and the reaction of the hinge.

6. A carriage wheel, whose mass is $W$ and radius $r$, rests upon a level road. Show that the least horizontal force $F$ applied at the centre which will be on the point of drawing the wheel over an obstacle of height $h$ is

$$F = \frac{W\sqrt{(2rh - h^2)}}{r - h}.$$

Interpret the result, ($a$) when $r = h$, ($b$) when $r$ is less than $h$.

7. A body, the weight of which is 100 pounds, rests on a smooth plane inclined to the horizon at an angle of 30°. What force acting at an angle of 30° to the plane will keep the body at rest? What is the pressure on the plane?

8. Two weights of 2 pounds and $\sqrt{6}$ pounds, respectively, rest, one on each of two inclined planes which are of the same height and are placed back to back. The weights are connected by a string which passes over a smooth pulley at the common apex of the planes. If the first plane makes an angle of 60° with the horizon, find (1) the tension of the string, (2) the pressure on each plane, (3) the inclination of the second plane to the horizon.

9. A ring, mass 9 lb., slides freely on a string of length $a\sqrt{2}$ whose ends are fastened to two points at a distance $a$ apart in a line making an angle of 45° with the horizon. Find the tension of the string in the position of equilibrium.

# CHAPTER XIV

## Friction

**132. Friction is Resistance to Motion.** The word **friction** has already been used a number of times in the preceding chapters, as it is hardly possible to discuss the motion of a body without taking friction into account. In this chapter we shall study more closely some of its effects.

A heavy railway train may be running on a level track at the rate of a mile a minute, but if the steam is shut off the train will slow down and at last will come to rest. This is due to the friction in the bearings of the wheels and in the rolling of the wheels on the rails.

The machinery in a great factory may be "humming," but immediately after the "power" is turned off at twelve o'clock the wheels slow down and come to rest in a few seconds.

***Friction* is the resistance to the motion of a body when it slides or rolls over another body.**

**133. Friction Depends on the Surfaces in Contact.** It is a common observation that the friction between two bodies depends upon the nature of the substances and the conditions of the surfaces which are in contact.

A sleigh may be drawn easily on a good road but when going over the planks or the rails at a railway crossing the horses have to exert much more of their strength, and if the ground is bare in some places the passengers may have to get out, to diminish the weight and so reduce the friction.

To slide a heavy plank over another requires more force if the surfaces are rough than if they are planed smooth.

**134. The Cause of Friction.** In the case of many surfaces the irregularities on them can be seen with the unaided eye, but even the smoothest surface when examined with a good microscope is seen to be covered with little projections with hollows between them. Hence when two surfaces are pressed together there is a kind of interlocking of these projections, which resists the motion of one surface over the other (Fig. 130).

Fig. 130.—Section through two surfaces showing roughness as seen under a microscope.

**135. Experimental Study of Friction.** A simple apparatus like that shown in Fig. 131, enables us to investigate the laws of friction.

A flat block $M$ rests on a board, which should be made as nearly horizontal as possible, and a cord attached to $M$ passes over a pulley and bears a pan on the end of it. The block can be loaded to any desired amount and weights can be put on the pan.

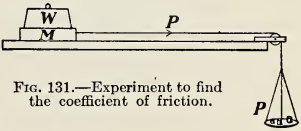



Fig. 131.—Experiment to find the coefficient of friction.

Let the horizontal board and the block $M$ be both of dry pine. Clean the surfaces by rubbing with fine sand-paper and then wipe the dust off carefully. Also rub the block back and forth upon the board. These operations are to give the surfaces a clean and permanent condition.

Weigh $M$ and also the pan. Put a weight $W$ on $M$ and a smaller one on the pan, and let the weight of the pan and the mass on it be $P$. Then the block is pulled by a force $P$ and (supposing there is no motion) sufficient friction between $M$ and the board is called into action to balance $P$.

Continue adding weights to the pan, doing it carefully and avoiding jerks, until at last the block begins to move. Record the weight of the pan and its contents. This gives us the **limiting value of friction** for this particular experiment. Repeat the experiment several times, and take the average of the several weights of the pan and its contents.

Let this average $= F$; also let $w =$ weight of the block $M$, and $W =$ the weight upon it.

Then find the value of $\frac{F}{W+w}$. This is called the **static coefficient of friction.**

By increasing the weight upon $M$ and obtaining the corresponding values of the force required to start the motion we secure a series of values of the coefficient of friction.

Next, try the same experiments, but, instead of being very careful in placing the weights on the pan, gently tap the board or give a slight jerk each time a new weight is put on. Continue to adjust the weights on the pan until the block moves forward with approximately uniform motion.

As before, obtain several values of $F$ with each value of $W$, and then calculate the value of $\frac{F}{W+w}$ for each average value of $F$. This quantity is now called the **kinetic coefficient of friction,** the word 'kinetic' meaning 'producing motion.'

The kinetic is considerably smaller than the static coefficient, which simply indicates that it is harder to start a body moving than to keep it moving when once motion has begun.

In the following table are given sample results for pine on pine, the grain of the block being parallel to that of the board.

COEFFICENT OF FRICTION $\frac{F}{W + w}$, PINE ON PINE.

Block 15 cm. square, weight, 0.21 kg.

| $W + w$ in Kg. | STATIC | | KINETIC | |
|---|---|---|---|---|
| | $F$ in Kg. | Coeft. | $F$ in Kg. | Coeft. |
| 0.71 | 0.26 | 0.36 | 0.20 | 0.28 |
| 1.21 | .47 | .38 | .33 | .27 |
| 1.71 | .58 | .34 | .38 | .22 |
| 2.21 | .88 | .40 | .53 | .24 |
| 2.71 | .87 | .32 | .70 | .26 |
| 3.21 | .96 | .30 | .72 | .22 |
| 3.71 | 1.41 | .38 | .93 | .25 |
| 4.21 | 1.47 | .35 | 1.18 | .28 |
| 4.71 | 1.51 | .32 | 1.08 | .23 |
| 5.21 | 1.94 | .37 | 1.30 | .25 |
| Average............ | | 0.35 | | 0.25 |

**136. The Laws of Friction.** From the results in the table above we deduce

(1) **The limiting friction varies directly as the normal force (often called the thrust) between the surfaces in contact.**

(2) **The static coefficient is considerably greater than the kinetic coefficient of friction;** or the friction at starting is greater than when uniform motion is maintained.

Next, take a block of the same material of different area, either larger or smaller. We find that the force required to cause motion in the case of a given value of $W$ is the same as before, and we conclude that

(3) **The limiting friction is independent of the extent of the surfaces in contact.**

In this case the pressure per square inch is different but the total force of the block upon the board is the same as before when the area of the block was different.

There is another law which we cannot investigate with this apparatus but which is important. It is as follows:

**(4) For moderate speeds the friction is independent of the rate of motion.**

This means that whether a machine is running rapidly or slowly the friction of the rubbing surfaces is approximately the same. However, this law is only approximately correct since, in general, friction with high speed is less than with slow speed of the moving surfaces. Thus when the engine driver sets the brakes on a train moving 60 miles per hour, the 'grip' on the wheels is not so powerful as when the speed is reduced to 20 miles or less per hour.

**137. Meaning of Coefficient of Friction (Static).** In the experiments described in Sections 135 and 136, the surfaces were being pressed together by the pull of gravity on the block and its load. If we were to place the block against a vertical plane, however, the force of gravity would no longer be effective in holding the surfaces together to cause friction. For friction to exist in this case, the block would have to be pressed against the plane by some other force.

**In every case, however, the coefficient of friction is the ratio of the limiting friction to the normal force between the two surfaces.**

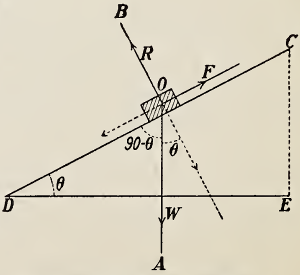



Fig. 132.—Finding the coefficient of friction with the inclined plane.

**138. Another Method of Determining the Coefficient of Friction.** Place the block with its load upon the board and slowly raise one end of the board until the block slides

down with uniform motion, the board being tapped to allow the block to start freely. Let the inclination be $\theta$ degrees (Fig. 132).

Consider the forces acting *on the block* and producing equilibrium when the block is just on the point of moving. We have

(1) $W$ the weight of the block acting vertically downwards along $OA$.

(2) $R$ the *normal* reaction of the board acting along $OB$.

(3) $F$ the force of friction acting along $OC$.

Resolving $W$ along the board and at right angles to it and applying the conditions of equilibrium,

$$\begin{aligned} F &= W \cos (90 - \theta), \\ &= W \sin \theta. \\ R &= W \cos \theta. \end{aligned}$$

But $R$ is equal to the normal force or thrust between the block and the board.

$$\text{Hence coefficient of friction} = \frac{F}{R} = \frac{W \sin \theta}{W \cos \theta} = \tan \theta.$$

Now measure the inclination with a protractor and the **tangent of the angle = the coefficient of friction.**

The tangent of the angle $\theta$ can also be found by measuring the height $CE$ and the horizontal distance $DE$, since $\tan \theta = EC/DE$.

The angle $\theta$ is called the **angle of friction** or the **angle of repose.**

**Example.**—Pine on Pine. By experiment it is found that $\theta = 14°$. From the mathematical tables (see page 378) $\tan 14° = 0.25$ = coefficient of friction.

**139. Magnitude of the Coefficients of Friction.** Since the surfaces of the bodies rubbing together are continually changing, the following values of the coefficients of friction must be considered as only roughly approximate:

TABLE OF COEFFICIENTS OF FRICTION

| | |
|---|---|
| Wood on wood, dry | 0.25 to 0.50 |
| " " " soapy | 0.20 |
| Metals on oak, dry | 0.50 to 0.60 |
| " " " soapy | 0.20 |
| Leather on oak | 0.27 to 0.38 |
| Metals on metals, dry | 0.15 to 0.20 |
| " " " wet | 0.30 |
| Iron on stone | 0.30 to 0.70 |
| Wood on stone | 0.40 |

The familiar stone-boat (Fig. 133) is made of wood or iron turned up in front, and is used for transporting stones, a barrel of water or perhaps a plough from one part of the farm to another. When drawn over a dirt road the coefficient of friction is from 0.5 to 0.7. This is a large fraction of the weight, but yet the stone-boat is found useful for such jobs as those mentioned.

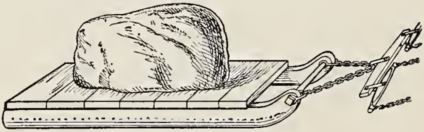


FIG. 133.—A stone-boat.

In launching a ship an abundance of soft soap is placed on the wooden ways down which the wooden cradle carrying the ship slides. From the above table the coefficient in this case is seen to be 0.20, which = tan $11\frac{1}{3}°$. Hence, if the ways make this angle with the horizontal the ship will slide down of itself when once the statical friction at starting has been overcome.

**140. Rolling Friction.** The resistance experienced by a body when rolling upon a surface is called **rolling friction** although in nature it is quite different from the resistance due to sliding.

Rolling friction is much smaller in magnitude than sliding friction. One may not be able to slide a heavy box over the floor and yet may move it without difficulty if rollers are put under it.

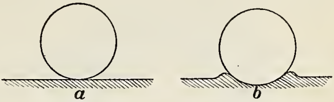


Fig. 134.—Illustrating rolling friction.

Consider the rolling of a wheel on a soft substance like india-rubber; it does not simply touch it (Fig. 134*a*) but sinks down, making a hollow with a mound on each side (Fig. 134*b*). As the wheel moves forward the mound behind practically disappears, but that in front continues there, and indeed is somewhat larger than when the wheel is at rest. The wheel is all the time trying to climb out of the hollow but it never succeeds in doing so.

In the case of hard substances, like steel, the mound is small, but it exists, nevertheless; and indeed under a heavy load the wheel itself is slightly flattened. An examination of railway tracks will give sufficient proof of the deformation of steel under heavy loads.

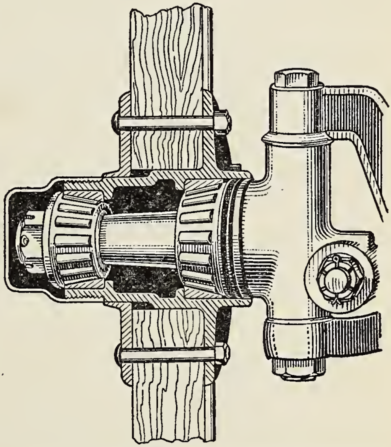


Fig. 135.—Front wheel of an automobile equipped with tapered roller bearings.

The indentation in the surface can be reduced by increasing the diameter of the wheel and by widening its tire. As self-binders and farm tractors have to pass over soft soil their driving wheels are large and broad.

In the ball bearings and roller bearings now so commonly used in automobiles and other high-class machines the surfaces

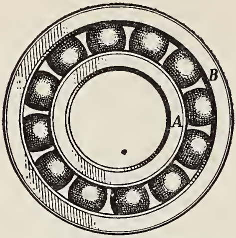



Fig. 136*a*.—Ball bearing. *A*, inner race which fits on axle. *B*, outer race, which fits inside hub.

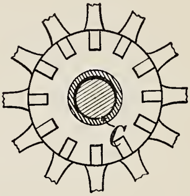



Fig. 136*b*.—Section through a carriage hub, showing an ordinary bearing.

in contact are made very hard, thus reducing the rolling friction. In Fig. 135 is shown the front wheel of an automobile equipped with roller bearings, while Fig. 136*a* illustrates a well made ball bearing.

It is to be noted, however, that while an ordinary carriage wheel rolls over the ground, there is sliding friction in the hub—at the point *C* in Fig. 136*b*.

**141. Lubrication.** The amount of friction can be greatly reduced by lubricating the surfaces in contact. The oil or other substance used forms a thin film between the surfaces and in place of one solid rubbing upon another one layer of liquid moves over another and there is much less friction.

**142. Utility of Friction.** But we must not think of friction simply as a waster of energy and an unmixed evil. As a matter of fact we make great use of it.

If it were not for friction we could not drive pulleys by means of belts, and nails and screws would be useless. Thread and yarn would not hold together and textiles could not be woven. The experience of walking on smooth ice or on a polished floor suggests the difficulty of moving about if friction were altogether absent. Drivers of automobiles on

slippery streets have good reason to appreciate the value of friction as an aid to locomotion.

**143. Examples.** 1. A mass of 20 lb. resting on a rough horizontal plane is on the point of moving when acted upon by a force which makes an angle of 30° with the plane. If the coefficient of friction is 0.2, find the force.

In the diagram (Fig. 137) $P$ is the applied force, $F$ is the force of friction and $R$ is the *normal* reaction of the plane on the body.

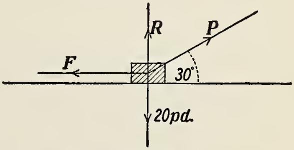



Fig. 137.—Friction on a horizontal surface.

Resolving $P$ vertically and horizontally and applying the conditions of equilibrium,

$$F = P \cos 30°,$$

or $$F = P\frac{\sqrt{3}}{2} \text{ ......(1)}$$

$$R + P\cos 60° = 20,$$

or $$R + \frac{P}{2} = 20 \text{ ..........(2)}$$

Since the coefficient of friction = 0.2,

$$F = 0.2\,R, \text{ or } F = R/5, \text{ or } R = 5F \text{ ............(3)}$$

Substituting this value of $R$ in (2),

$$5F + \frac{P}{2} = 20.$$

But $$F = P\frac{\sqrt{3}}{2}.$$

Hence $$P\frac{5\sqrt{3}}{2} + \frac{P}{2} = 20,$$

or $$P = \frac{40}{5\sqrt{3}+1} = \frac{40\,(5\sqrt{3}-1)}{74} = 4.14 \text{ pd.}$$

2. A mass of 10 lb. rests on a rough plane inclined at an angle of 30° to the horizon. What force must be applied parallel to the plane so that the body may be on the point of moving *up* the plane, the coefficient of friction being 0.25.

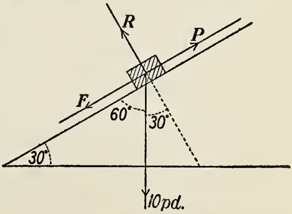



Fig. 138—Friction on an inclined plane.

In the diagram (Fig. 138) $P$ is the applied force, $F$ is the force of friction, acting *down* the plane since the body is on the point of moving *up* the plane, and $R$ is the normal reaction of the plane.

Resolving along the plane and at right angles to it,

$$R = 10 \cos 30°, \text{ or } R = 5\sqrt{3} \ldots\ldots\ldots\ldots (1)$$

$$P = F + 10 \cos 60°, \text{ or } P = F + 5 \ldots\ldots\ldots (2)$$

Also, since the coefficient of friction = 0.25,

$$F = 0.25\, R, \text{ or } F = R/4 \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

From (1) and (3) $F = \dfrac{5\sqrt{3}}{4}.$

Substituting this value of $F$ in (2)

$$P = \frac{5\sqrt{3}}{4} + 5 = \frac{5\,(\sqrt{3} + 4)}{4} = 7.165 \text{ pd.}$$

## QUESTIONS AND PROBLEMS

1. Explain the utility of friction in
   (*a*) Locomotive wheels on a railway track.
   (*b*) Leather belts for transmitting power.
   (*c*) Brakes to stop a moving car.

2. The current of a river is less rapid near its banks than in mid-stream. Can you suggest a reason for this?

3. What horizontal force is required to drag a trunk weighing 150 pounds across a floor, if the coefficient of friction between trunk and floor is 0.3 ?

4. Give two reasons why it is more difficult to start a heavily-laden cart than keep it in motion after it has started?

5. A brick, 2 x 4 x 8 inches in size, is slid over ice. Will the distance it moves depend on what face it rests upon?

6. A mass of 10 pounds rests on a rough horizontal plane. If the coefficient of friction is 0.2, find the least horizontal force which will move the mass. Find also the reaction of the plane. (The reaction is the resultant of the normal reaction and the force of friction).

7. A force of 5 pounds is the greatest horizontal force that can be applied to a mass of 75 pounds resting on a rough horizontal plane without moving it. What is the coefficient of friction?

8. A mass of 10 pounds is resting on a rough horizontal plane, and is on the point of moving when acted on by a force which makes an angle of 45° with the plane. If the coefficient of friction is 0.5, find the force.

9. A body resting on a rough horizontal plane is on the point of moving when acted on by a force equal to one-half its own weight inclined to the plane at an angle of 30°. Find the coefficient of friction.

10. A body placed on a rough plane is just on the point of sliding down when the plane is inclined to the horizon at an angle of (1) 60°, (2) 45°, (3) 30°. What is the coefficient of friction in each case?

11. A body placed on a rough inclined plane is on the point of sliding when the plane rises 3 feet in 6 feet measured along the slope. What is the coefficient of friction?

12. A mass of 20 lb. rests on a rough plane inclined at an angle of 30° to the horizon. What force must be applied parallel to the plane that it may be on the point of moving up the plane, the coefficient of friction being 0.1?

13. A body, the mass of which is 30 lb., rests on a rough inclined plane, the height of the plane being $3/5$ of its length. What force must be applied to the body parallel to the plane that it may be on the point of moving up the plane, the coefficient of friction being 0.75?

14. The load on the driving wheels of a locomotive is 60 tons and the coefficient of friction between rails and wheels is $1/6$. What is the greatest force the locomotive can exert? If its entire mass is 75 tons, what is the greatest speed it can give to itself in 10 sec.?

15. A mass of 14 lb. when placed on a rough plane inclined to the horizon at an angle of 60° slides down unless a force of at least 7 pounds acts up the plane. What is the coefficient of friction?

16. A mass of 20 lb. is on the point of moving up a rough plane inclined to the horizon at an angle of 45° when a horizontal force is applied to it. Find the horizontal force, if the coefficient of friction is 0.1.

17. A body, the mass of which is 4 lb., rests in limiting equilibrium when the inclination of the plane to the horizon is 30°. Find the force which, acting parallel to the plane, will support the body when the inclination of the plane to the horizon is 60°.

18. A body placed on a rough plane inclined to the horizon at an angle of 30° is just on the point of moving upward when acted upon by a horizontal force equal to its own weight. Find the coefficient of friction.

19. If the smallest horizontal force which will move a mass of 3 lb. along a horizontal plane is $\sqrt{3}$ pounds, find the greatest angle at which the plane may be inclined to the horizon before the mass begins to slide.

20. An automobile weighing 3500 lb. climbs a hill rising 1 in 7 at 30 mi. per hr. If the coefficient of friction is 0.01, find the horse-power developed.

21. Show that in order to relieve a horse in drawing a sleigh the traces should be so inclined as to make the angle of friction with the ground.

# CHAPTER XV

## Centre of Gravity

**144. A Unique Central Point in Every Body.** The meaning of centre of gravity has already been briefly explained (Sec. 118); it is discussed more fully in the present chapter.

When one side of a carriage is somewhat lower than the other we experience an uncomfortable sensation, as we know that there is a definite position beyond which we must not go or the carriage will upset.

Next, consider a rectangular block of wood, or a brick, resting on a flat surface. Gradually raise one side until the middle point of the block is just over the line along which it touches the surface. This is a critical position, and if the body is turned any more it will topple over on another face. Try with the new face. When the central point gets beyond the line of support, over the block falls to a new position of rest.

Again, push a book or a piece of board slowly over the edge of a table. It rests safely on the table until it reaches a certain definite position, when it is seen to totter, and if pushed any farther it falls. When in the tottering position draw a line on the underside along the edge of the table. Now turn the object about and push it over the edge again, drawing another line when it is in its critical position. Repeat this several times and then look at the lines drawn. They all meet very approximately in a point, and thus it is seen that as soon as that particular point gets beyond the line of support the body falls over into a new position.

Our everyday experience leads us to believe that there is a unique central point in a body, and if that point goes beyond a certain position the body moves over into a new position of rest.

**145. Experiments with a Thin Flat Body.** Support an irregular-shaped sheet of metal, or a piece of cardboard, at $A$ (Fig. 139*a*), by hanging it on a pin or in some other convenient way. Have a cord attached to the pin, with a small weight on the end of it. Chalk the cord and snap it on the plate, thus making a straight white line across it. Next support the body at $B$ (Fig. 139*b*), and obtain another chalk line. Let it cut the first line at $G$. Support the plate at other places and get other lines on it. All the lines cut at a single point—the point $G$—which must be a unique point in the plate.

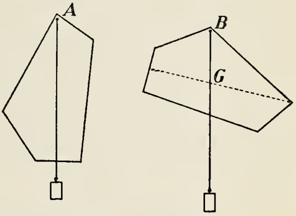



Fig. 139.—How to find the centre of gravity of a flat body.

Now try to balance the plate on the end of a finger. You find the plate balances if it is supported at $G$. But it is simply the weight of the plate that the finger has to overcome, and we conclude, then, that **the entire weight of the body may be considered as concentrated at a point.** This point is called the **Centre of Gravity** of the body. The abbreviation C.G. will be used for Centre of Gravity.

**146. Composition of Forces due to Gravity.** A body consists of a very great number of particles, and according to the principle of Universal Gravitation the earth attracts every particle with a force which we call its weight. The lines of action of these forces are directed to the centre of the earth, but since that point is 4000 miles away the directions of the forces may be taken to be parallel.

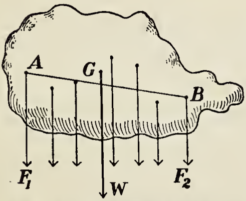



Fig. 140.—The weight of a body acts at its centre of gravity.

These forces will have a single resultant acting at a definite point fixed in the body as can be seen in the following way:

The two forces $F_1$, $F_2$, acting on particles at $A$, $B$ (Fig. 140), will have a resultant, $F_1 + F_2$, acting at a point in $AB$ so situated that the moment of $F_1$ about the point equals the moment of $F_2$ about the point. Further, if the body moves into another position the magnitude and point of action of this resultant will be unchanged. (Sec. 119).

Next, combine this resultant with a third force $F_3$, and obtain the point of action of $F_1 + F_2 + F_3$, the resultant of the three forces.

Then combine this resultant with a fourth force; and continuing in this way we at last reach a single resultant of all the forces acting at a definite point in the body.

The sum of all these separate forces is the weight of the body and the point of application of the resultant force is the centre of gravity of the body.

**147. To Find the Centre of Gravity of a Body of any Form.** Suspend the body by a cord attached to any point $A$ (Fig. 141) in it. Then there are two forces acting on the body, namely, the weight acting downwards at $G$ and the tension of the string acting upwards at $A$. These are equal in magnitude and form a couple. They cause the body to rotate until $G$ is directly beneath $A$, in which case the line of action of the weight coincides with the direction of the string, and the tension of the string will just balance the weight of the body. The body will then be in equilibrium.

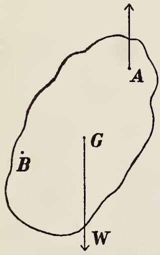



Fig. 141.—How to find the centre of gravity of a body of any form.

Thus, if the body is suspended at $A$ and allowed to come to rest the direction of the supporting string will pass through the centre of gravity.

Next attach a cord at $B$ and hang up the body as before.

The direction of the cord will again pass through the C.G.; that point, therefore, will be where the two lines intersect.

This experimental method may be employed to determine the C.G. of any kind of body at all, and indeed in many cases it is the only available method. But when the body is of simple form it is often easy to determine the position of the C.G. from geometrical considerations. (Sec. 150).

**148. Centre of Gravity of Weights on a Rod.** Let $AB$ (Fig. 142) be a light rod, of negligible weight and 40 cm. long,


FIG. 142.—To find centre of gravity of three weights in a straight line.

with 1 kg. at $A$, 4 kg. at $C$, 30 cm. from $A$, and 5 kg. at $B$ (Fig. 142). We have to find the C.G. of the system.

Let the centre of gravity be at $G$, $x$ cm. from $A$.

Then for the three weights, 1, 4, 5 kg. at $A$, $C$, $B$, respectively, we can substitute their sum 10 kg. at the C.G. Now if we apply an upward force of 10 kg. to the rod at $G$ the rod will be in equilibrium. Since the rod is in equilibrium the sum of the clockwise moments about *any* point is equal to the sum of the contra-clockwise moments about the same point.

Taking the point $A$ as our imaginary turning point, we have the following equation of equilibrium:

$$10x = 1 \times 0 + 4 \times 30 + 5 \times 40,$$
$$= 320;$$

whence $$x = 32 \text{ cm.}$$

(*To the student*: $G$ might appear a more natural turning point. If $G$ is taken, however, difficulty will be encountered in determining which moments are positive and which are negative, since the exact location of $G$ is not known.)

## PROBLEMS

1. Masses of 2 lb., 4 lb., 6 lb., 8 lb., are placed so that their centres of gravity are in a straight line, and six inches apart. Find the distance of their common centre of gravity from that of the largest mass.

2. Two masses of 6 lb. and 12 lb. are suspended at the ends of a uniform horizontal rod, whose mass is 18 lb. and length 2 ft. Find the centre of gravity.

3. A uniform rod, 1 ft. in length and mass 1 oz., has an ounce of lead fastened to it at one end, and another ounce fastened to it at a distance from the other end equal to one-third of its length. Find the centre of gravity of the system.

4. Four masses of 3 lb., 2 lb., 4 lb., and 7 lb., respectively, are at equal intervals of 8 in. on a lever supposed weightless, 2 ft. in length. Find where the fulcrum must be, in order that they balance.

5. A uniform bar, 3 ft. in length and of mass 6 ounces, has three rings, each of mass 3 ounces, at distances 3, 15 and 21 in. from one end. About what point of the bar will the system balance?

6. A ladder, 50 ft. long and of mass 100 lb., is carried by two men; one lifts it at one end, and the other at a point 2 ft. from the other end. The first carries two-thirds of the weight which the second does. Where is the centre of gravity of the ladder?

7. A pole, 10 ft. long and mass 20 lb., has a mass of 12 lb. fastened to one end. The centre of gravity of the whole is 4 ft. from that end. Where is the centre of gravity of the pole?

8. Four masses, 1 lb., 4 lb., 5 lb., and 3 lb., respectively, are placed 2 ft. apart on a rod 6 ft. long, whose mass is 3 lb. and centre of gravity 2 ft. from the end at which the 1 lb. is placed. Find the centre of gravity of the whole.

9. A cylindrical vessel whose mass is 4 lb. and depth 6 in. will just hold 2 lb. of water. If the centre of gravity of the vessel when empty is 3.39 in. from the top, determine the position of the centre of gravity of the vessel and its contents when full of water. (Think of the vessel as lying on its side, when taking moments.)

10. A cylindrical vessel, without lid, one foot in diameter and one foot in height, is made of thin sheet metal of uniform thickness. If it is half filled with water, where will be the common centre of gravity of the vessel and the water, assuming the mass of the vessel to be one-fifth the mass of the contained water?

11. A uniform iron bar weighs 4 pounds per foot of its length. A weight of 5 pounds is hung from one end and the rod balances about a point which is 2 ft. from that end. Find the length of the bar.

**149. Centre of Gravity of Weights in a Plane.** The method of moments may be applied to masses distributed over a plane.

Let $ABCD$ (Fig. 143) be a uniform square board with sides 26 inches long and of mass 8 lb., and let masses 4, 6, 5, 3 lb. be placed at the corners $A$, $B$, $C$, $D$. We wish to find the C.G. of the system.

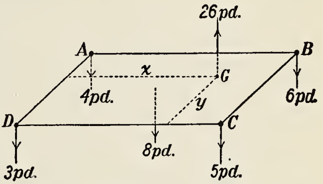


FIG. 143.—Masses at the corners of a board.

Let the C.G. be at $G$, and be $x$ inches from $AD$ and $y$ inches from $DC$. The total mass = 26 lb. If, then, we apply an upward force of 26 pd. at $G$ we shall obtain equilibrium.

First, think of the board as being hinged along the line $AD$.

Taking moments about $AD$, the masses 4 and 3 are on the line $AD$ and give no moments about $AD$.

| | | |
|---|---|---|
| Moment of 6 | $= 6 \times 26 =$ | 156 |
| Moment of 5 | $= 5 \times 26 =$ | 130 |
| Moment of board | $= 8 \times 13 =$ | 104 |
| | Moment of whole = | 390 |

But moment of whole $= 26 \times x = 26\,x$

Therefore $26\,x = 390,$

and $x = 15$ inches.

Next, think of the board as being hinged along the line $DC$.

Taking moments about $DC$,

| | | |
|---|---|---|
| Moment of 4 | $= 4 \times 26 =$ | 104 |
| Moment of 6 | $= 6 \times 26 =$ | 156 |
| Moment of board | $= 8 \times 13 =$ | 104 |
| | Moment of whole = | 364 |

But moment of whole $= 26 \times y = 26\,y.$

Therefore $26\,y = 364,$

and $y = 14$ inches.

Hence, the C.G. is at $G$, 15 in. from $AD$ and 14 in. from $CD$.

In this example moments were taken about $AD$ and $DC$, but any other lines might be chosen. As an exercise, solve the problem by taking moments about $BC$, $CD$.

## PROBLEMS

1. Masses of 1, 1, 1 and 2 lb., are placed at the angular points of a square. Find the position of their centre of gravity with reference to the 2-lb. mass.

2. Masses of 2 lb., 1 lb., 2 lb., 3 lb., are placed at $A$, $B$, $C$, $D$ respectively, the angular points of a square. Find the distance of the centre of gravity from the centre $O$.

3. Masses of 1, 4, 2, 3 lb., are placed at the corners $A$, $B$, $C$, $D$ of a rectangle; a mass of 10 lb. is also placed at the intersection of the diagonals. If $AB = 7$ in. and $BC = 4$ in., find the distance of the centre of gravity of the whole from $A$.

4. At the angular points of a square, taken in order, there act parallel forces in the ratio 1 : 3 : 5 : 7. Find the distance from the centre of the square of the point at which their resultant acts.

5. Masses 5, 7, 10 are placed at three angles of a square whose sides are 4 ft. Find the distance of their centre of gravity from 5.

6. Three masses 3, 4, 5 lb. are placed at the angles of an equilateral triangle whose sides are 12 inches. Find the distance of the centre of gravity of the whole from the least mass.

7. $ABC$ is a triangle right-angled at $A$, $AB$ being 12 and $AC$ 15 inches in length. Masses in the ratio 2 : 3 : 4 are placed at $A$, $C$, and $B$ respectively. Find the distances of their centre of gravity from $B$ and $C$.

8. Prove that the centre of gravity of an equilateral triangular lamina coincides with that of three equal masses placed at its angular points.

**150. Centre of Gravity of Simple Geometrical Figures. (1) A uniform straight bar.** For a uniform straight bar $AB$ (Fig. 144) it seems evident that the C.G. is at its mid-point.

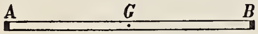



Fig. 144.—Centre of gravity of a uniform rod.

We may, however, consider it as made up of equal particles uniformly distributed from one end to the other. The C.G. of equal particles at $A$ and $B$ will be half-way between, at $G$. In the same way the C.G. of the particles next to $A$ and $B$

will be at $G$; and continuing in this way we find the C.G. of all to be at $G$.

(2) **A uniform parallelogram.** This may be considered as made up of uniform thin rods, such as $LM$, parallel to the side $AB$ (Fig. 145). The C.G. of $LM$ is at its mid-point $g$, and the mid-points of all such rods will be on a line $EF$, mid-way between the sides $AD$, $BC$. The C.G. of the parallelogram is evidently in this line.

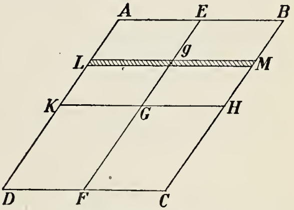


Fig. 145.—Finding the centre of gravity of a parallelogram.

In the same way we may consider the parallelogram as made up of uniform rods parallel to $AD$, and the C.G. of each will be on the line $KH$, midway between $AB$, $DC$; and the C.G. of the parallelogram will be somewhere in $KH$.

Hence, the C.G. of the parallelogram is where $EF$ and $KH$ intersect, that is, at $G$. This is the geometrical centre of the parallelogram, and is where the diagonals meet.

(3) **A triangular plate.** The plate may be considered to be made up of a series of thin rods like $LM$, parallel to the side $BC$ (Fig. 146), and the C.G. of each rod is at its mid-point $g$. The median line $AE$ (that is, the line joining $A$ to the mid-point of the opposite side $BC$) bisects all such rods, and hence the C.G. of the triangle is on this line.

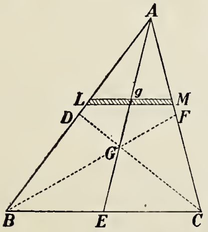


Fig. 146.—Finding the centre of gravity of a triangle.

In the same way it can be shown that the C.G. is on the median line $BF$ and also on the median line $CD$. Consequently it must be at $G$ where these three lines intersect.

From geometry we know that $EG = \frac{1}{3}EA$, $DG = \frac{1}{3}DC$ and $FG = \frac{1}{3}FB$.

(4) **Other geometrical forms.** It may be of interest to know the positions of the C.G. in some familiar geometrical forms. The C.G. of a pyramid or a cone is on the line joining the vertex to the C.G. of the base and one-fourth of the way up. The C.G. of a solid hemisphere is $\frac{3}{8}$ of its radius from the flat face, that is, from its geometrical centre. The C.G. of a hemispherical shell is half-way between centre and circumference.

**151. Example.** In Fig. 147 $ABCD$ is a rectangle whose middle point is $E$. The sides $AB$ and $BC$ are 6 in. and 4 in. long, respectively. If the triangle $BEC$ is removed, find the centre of gravity of the remainder.

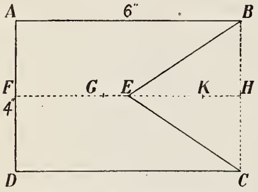



Fig. 147.—Finding the C.G. of $ABECD$.

Let $F$ and $H$ be the middle points of $AD$ and $BC$, respectively.

Since the figure $ABECD$ is symmetrical with respect to $FH$, it is evident that the C.G. lies somewhere on this line. Let $G$ be the C.G.

The C.G. of the rectangle $ABCD$ is at $E$ and the C.G. of the triangle $BEC$ is at $K$ where $HK = 1$ in. (Sec. 150).

Taking $AD$ as an imaginary axis of rotation, it is evident that the moment of the triangle $BEC$ about $AD$ + moment of $ABECD$ about $AD$ = moment of rectangle $ABCD$ about $AD$.

The masses of these figures are proportional to their areas. Let $m$ = mass of the triangle $BEC$ and let $FG = x$. We have then as our equation of moments,

$$m \times 5 + 3m \times x = 4m \times 3,$$

whence $$x = 2\tfrac{1}{3} \text{ in.}$$

## PROBLEMS

1. An isosceles triangle has its equal sides of length 5 cm. and its base of length 6 cm. Find the distance of the centre of gravity from each of the angular points.

2. If the angular points of one triangle lie at the middle points of the sides of another, show that the centres of gravity of the two are coincident.

3. The equal sides of an isosceles triangle are 10 ft.; and the base is 16 ft. in length. Find the distance of its centre of gravity from each of the sides.

4. The sides of a triangle are 3, 4, and 5 ft. in length. Find the distance of the centre of gravity from each side.

5. The sides of a triangular lamina are 6, 8, and 10 ft. in length. Find the distance of the centre of gravity from each of its angular points.

6. The sides $AB$, $AC$ of a triangle $ABC$, right-angled at $A$, are respectively 18 and 12 in. long. Find the distance of the centre of gravity from $C$.

7. An equilateral triangle is described upon one side of a square whose side is 16 in. Find the distance of the centre of gravity of the figure so formed from the vertex of the triangle, the vertex being without the square.

8. The length of one side of a rectangle is double that of an adjacent side, and on one of the longer sides an equilateral triangle is described externally. Find the centre of gravity of the whole.

9. A piece of cardboard is in the shape of a square $ABCD$ with an isosceles right-angled triangle described on the side $BC$ as hypotenuse. If the side of the square is 12 in., find the distance of the centre of gravity of the cardboard from the line $AD$.

10. An isosceles right-angled triangle is described externally on the side of a square as hypotenuse. Find the centre of gravity of the whole figure.

11. A square is described on the base of an isosceles triangle. What is the ratio of the altitude of the triangle to its base when the centre of gravity of the whole figure is at the middle point of the base?

12. $ABCD$ is a square whose middle point is $E$ and whose side $= a$. If the triangle $ECD$ is removed, find the centre of gravity of the remainder.

13. $E$ and $F$ are the middle points of the sides $AB$, $AC$ of an equilateral triangle $ABC$. If the portion $AEF$ is removed, find the centre of gravity of the remainder.

14. $ABCD$ is a square, $O$ its centre, $E$ and $F$ the middle points of $AB$. $AD$. If $AEF$ is cut away, find $G$, the centre of gravity of the remainder,

15. From a square piece of paper $ABCD$ a portion is cut away in the form of an isosceles triangle whose base is $AB$ and altitude equal to one-third $AB$. Find the centre of gravity of the remaining portion.

16. $ABCD$ is a rectangle, $E$ the middle point of $CD$; the triangle $ADE$ is cut away. Find the centre of gravity of the remainder.

**152. The three States of Equilibrium.** The centre of gravity of a body will always descend to as low a position as possible, since the potential energy of a body tends to become a minimum.

Consider a body in equilibrium, such as the cone $A$ (Fig. 148), and suppose that by a slight motion this equilibrium is disturbed. Then, since the body tends to return to its former position, its equilibrium is said to be **stable.** In this case the slight motion raises the centre of gravity, and on letting it go the body tends to return to its original position.

If, however, a slight disturbance lowers the centre of gravity, as in $B$ (Fig. 148), the body will not return to its original position, but will take up a new position in which the centre of gravity is lower than before. In this case the equilibrium is said to be **unstable.**

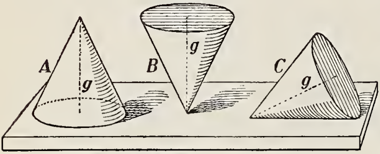


**Fig. 148.**—Stable, unstable and neutral equilibrium illustrated by a cone.

Sometimes a body, such as a cone resting on its curved surface, ($C$ Fig. 148), rests equally well in any position in which it may be placed. In this case the equilibrium is said to be **neutral.**

An egg standing on end is in unstable equilibrium; if resting on its side, the equilibrium is stable as regards motion in an oval section and neutral as regards motion in a circular section (Fig. 149). A uniform sphere rests anywhere it is placed on a level surface; its equilibrium is neutral.

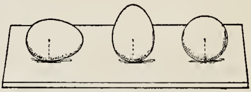


Fig. 149.—An egg in stable, unstable and neutral equilibrium.

A round pencil lying on its side is in neutral equilibrium; balanced on its end, it is unstable. A cube, or a brick, lying on a face, is stable.

The degree of stability possessed by a body resting on a horizontal plane varies in different cases. It increases with the distance through which the centre of gravity has to be raised in order to make the body tip over. Thus, a brick

lying on its largest face is more stable than when lying on its smallest.

**153. Condition for Equilibrium.** In the case of a body resting on a surface there are two forces acting on the body,—

(i) The weight of the body acting vertically downwards through its centre of gravity;

(ii) The reaction of the surface, which is the resultant of the various forces upwards exerted by the surface upon the body.

If the body is in equilibrium it is evident that these two forces must be equal in magnitude and must act in the same line but in opposite directions.

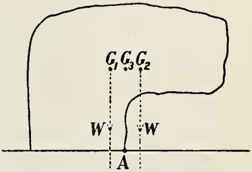



FIG. 150.—If the C.G. is at $G_1$, the body is stable, if at $G_2$, the body will topple over.

Consider the body in Fig. 150. The reaction of the surface on which it is resting acts upwards somewhere within the area of the base. This area is called the supporting base. If the centre of gravity of the body is at $G_1$, then the vertical line through the centre of gravity falls within the supporting base and the body is in stable equilibrium. For if the body is tipped slightly in a clockwise direction about the point $A$, the weight of the body, acting vertically downwards through $G_1$, produces a contra-clockwise moment which tends to restore the body to its original position when it is released.

If, however, the centre of gravity is at $G_2$, the weight of the body acting vertically downwards through $G_2$ produces an unbalanced clockwise moment which makes the body topple over.

If the centre of gravity is at $G_3$, vertically above $A$, the body will be *just on the point of toppling.*

Consider the stool $C$ in Fig. 151. The reactions of the surface are at the points where the feet rest on the surface, and the resultant of these reactions must be a single force within the

area formed by a cord drawn closely about the legs. The stool is in stable equilibrium because a vertical line through its centre of gravity falls within the supporting base.

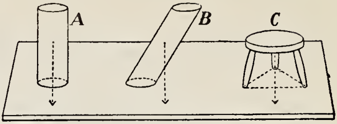



FIG. 151.—*A* and *C* are in stable equilibrium; *B* is not, it will topple over.

This is seen to be the case also in *A* (Fig. 151), but it is not so in *B*, and consequently the cylinder *B* will topple over.

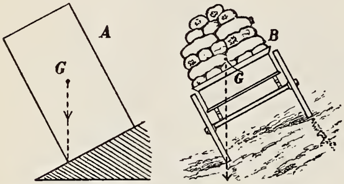



FIG. 152.—The rectangle in *A* and the wagon in *B* are just on the point of toppling over.

In Fig. 152 both the rectangle in *A* and the wagon in *B* are in the critical position since the vertical lines from their respective centres of gravity are just passing through the boundaries of the supporting bases.

We see then that a body in stable equilibrium may be tipped without toppling over, until the vertical line through its C.G. just passes through the boundary of the supporting base.

The famous Leaning Tower of Pisa (Fig. 153) is an interesting case of stability of equilibrium. It is circular in plan, 51 feet in diameter and 172 feet high, and has eight stages, including the belfry. Its construction was begun in 1174. It was founded on wooden piles driven in boggy ground, and when it had been carried up 35 feet it began to settle to one side. The tower overhangs the base upwards of 13 feet, but the centre of gravity is so low down that a vertical through it falls within the base and hence the equilibrium is stable.

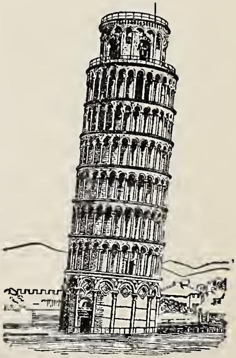


FIG. 153.—The Leaning Tower of Pisa. It overhangs its base more than 13 feet, but it is stable. (Drawn from a photograph.)

## QUESTIONS AND PROBLEMS

1. Why is a pyramid a very stable structure?

2. Why is ballast used in a vessel? Where should it be put?

3. Why should a passenger in a canoe sit on the bottom?

4. A pencil will not stand on its point, but if two pen-knives are fastened to it (Fig. 154) it will balance on one's finger. Explain why this is so.

5. If a heavy uniform lamina, in the shape of an equilateral triangle, is suspended from any of its angles, show that the opposite side is always horizontal.

6. If a right-angled triangle is suspended from either of the points of trisection of the hypotenuse, show that it will rest with one side horizontal.

7. The wheels of a hay cart are 5 ft. apart and the centre of gravity of the cart and load is 6 ft. above the ground and midway between the wheels. How much could either wheel be raised without the cart falling over?

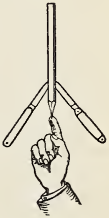


Fig. 154.—Why is the pencil in equilibrium?

8. How many coins of the same size, having the thickness $\frac{1}{20}$ the diameter, can stand in a cylindrical pile on an inclined plane of which the height is $\frac{1}{6}$ the base, if there is no slipping?

9. A number of cent pieces are cemented together so that each just laps over the one below it by the ninth part of its diameter. How many may be thus piled without falling?

10. A square table, whose mass is 10 kg., stands on four legs placed respectively at the middle points of its sides. Find the greatest mass which can be put at one of the corners without upsetting the table.

11. A circular table, of mass 50 lb., rests on three legs attached to three points in the circumference at equal distances apart. When the table rests on a horizontal plane what is the least mass which when placed on it will be on the point of upsetting it?

12. A brick is laid with a quarter of its length projecting over the edge of a wall; a brick and a quarter are laid on the first with a quarter of its length over the edge of the first brick; a brick and a half laid on this and so on. Prove that four such courses can be laid, but that if the fifth course is added the mass will topple over.

# CHAPTER XVI

## Machines

**154. Object of a Machine.** It is frequently necessary to raise the axle of an automobile in order to renew a tire, and everyone knows how easy this is when you have a suitable machine.

Or perhaps a barrel of oil or of flour is to be loaded on a wagon. It is too heavy to lift, but it can easily be put in place by rolling it up a plank.

Again, an electric current may be at your disposal. By suitable contrivances you can make it sew your clothes or separate the cream, or print the newspaper, or do a thousand other tasks.

In each case we use a suitable machine, and the function of the machine is to transfer energy from one place to another, or transform it from one kind to another.

The six simple machines, usually known as the **mechanical powers,** are the lever, the pulley, the wheel and axle, the inclined plane, the wedge and the screw. All other machines, no matter how complicated, are only combinations of these.

Since energy cannot be created or destroyed, but is simply changed from one form to another, it is evident that, neglecting friction, the amount of work put into or done upon a machine is equal to the amount which it will perform. **Furthermore, since in every machine which man can make some friction is unavoidably present, it is clear that more work must be done in driving the machine than will be its output.** Many attempts have been made to invent a machine which will continue to deliver as much work as is spent upon it, and indeed sometimes more work has been expected from a machine than has been spent upon it. Such attempts have always

failed, and if the law of Conservation of Energy is true such efforts cannot possibly succeed. If there is only five gallons of gasoline in your tank, that is all you can use,—unless you put more in. It is the same with stores of energy.

**The efficiency of a machine is the ratio of the output to the input, usually expressed as a percentage.**

**155. The Lever; First Class.** The lever is a rigid rod movable about a fixed axis called the fulcrum. Levers are of three classes.

In Fig. 155 is shown a lever of the first class. By applying a force $P$ at $A$ a force sufficiently great to balance the force $W$ is obtained at $B$, the lever turning about the fulcrum $F$.

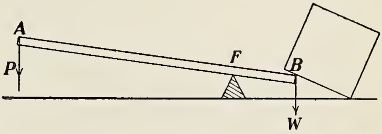



Fig. 155.—A lever of the first class. The fulcrum $F$ is between the applied force $P$ and the weight lifted $W$.

The relation between the forces $P$ and $W$ follows from the principle of moments, and it can be determined experimentally as follows:

Lay a metre rod on a prism with the 50 cm. mark exactly over the edge of the prism (Fig. 156). If it does not balance add bits of lead or plasticine to the lighter end until it does. Put blocks under the ends to reduce the vibrations.

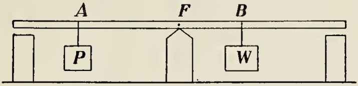



Fig. 156.—Investigating the law of the lever of the first class.

Suspend a mass $W$ from some graduation, noting its distance from $F$. This distance $BF$ is one arm of the lever, and the product $W \times BF$ is the moment of $W$ about $F$.

Move the mass $P$ until it just balances $W$ and note the length of the arm $AF$. The moment of $P = P \times AF$. Make 5 or 6 readings, changing masses and distances each time.

Then compare the values of $W \times BF$ and $P \times AF$ for each set of readings. They will be found to be equal.

Applying this result to either figure we see that

$$\textbf{Force obtained} \times \textbf{its arm} = \textbf{Force applied} \times \textbf{its arm},$$

or $$\frac{\textbf{Force obtained}}{\textbf{Force applied}} = \textbf{inverse ratio of length of arms.}$$

This is called the **Law of the Lever,** and the ratio W/P is called the **Mechanical Advantage.**

Suppose, for instance, $AF = 36$ inches, $BF = 4$ inches.

Then $W/P = AF/BF = 36/4 = 9$, the mechanical advantage.

It is evident that the mechanical advantage of a lever of the first class may be greater than, equal to, or less than 1 according to the position of the fulcrum.

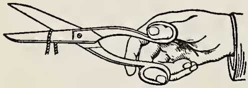


Fig. 157.—Shears, lever of the first class.

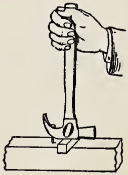


Fig. 158.—Claw-hammer, used as a lever of the first class.

There are many examples of levers of the first class. Among them are the common balance, a pump handle, a pair of scissors (Fig. 157), a claw-hammer (Fig. 158).

The law of the lever can be deduced by applying the principle of energy.

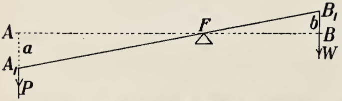



Fig. 159.—The principle of energy applied to the lever.

Suppose the end $A$ (Fig. 159) to move through a vertical distance $a$ and the end $B$ through a vertical distance $b$. It is evident that

$$a/b = A_1F/B_1F = AF/BF.$$

Now the work done by the force $P$, acting through a distance $a$ is $P \times a$, while the work done on raising $W$ a distance $b$ is $W \times b$.

Neglecting all considerations of friction or of the weight of the lever, the work done by the applied force $F$ must be equal to the work accomplished.

Hence, $$P\,a = W\,b,$$

and the mechanical advantage $W/P = a/b = AF/BF$, which is the law of the lever.

**156. The Lever; Second Class.** In levers of the second class the weight to be lifted, or the resistance to be overcome, is placed between the point where the force is applied and the fulcrum.

A lever of this class is shown in Fig. 160. The force $P$ is applied at $A$, and the force obtained, or the resistance overcone, is at $B$, between $A$ and the fulcrum $F$.

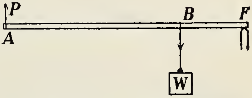


FIG. 160.—A lever of the second class.

The law in this case can be determined experimentally as follows:

Find the position of the centre of gravity $F$ of a metre stick by balancing it on the adjustable knife-edge shown in Fig. 161. Support it at this point.

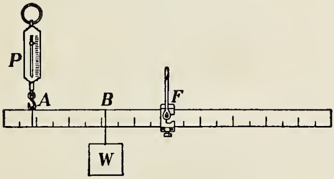


FIG. 161.—Demonstrating the law for a lever of the second class.

Now attach a weight $W$ to the rod, noting its distance from the fulcrum $F$ and observe the reading $P$ of the spring-balance when the stick is horizontal. Make 5 or 6 readings, varying the value of $W$ and the point where it is placed.

Compare the products $P \times AF$ and $W \times BF$: they will be found equal; and we have, as in the first class,

Mechanical Advantage $W/P = AF/BF$, a ratio which is greater than 1.

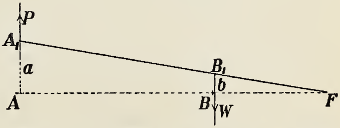


FIG. 162.—Applying the principle of energy.

If we apply the principle of energy we have (Fig. 162):

Work done by $P = P\,a$; work done on $W = W\,b$,

and these must be equal, or $P\,a = W\,b$.

Hence, $W/P = a/b = A_1F/B_1F = AF/BF$, the law of the lever.

Examples of levers of the second class: nut crackers (Fig. 163), trimming board (Fig. 164), safety valve (Fig. 165), wheel-barrow, oar of a row-boat.

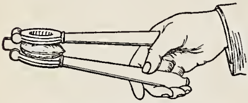


FIG. 163.—Nut-crackers, lever of the second class.

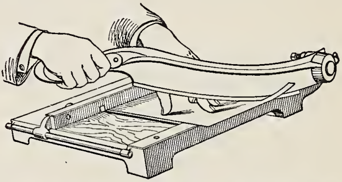


FIG. 164.—Trimming board for cutting paper or cardboard; a lever of the second class.

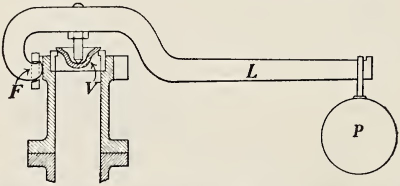



FIG. 165.—A safety-valve of a steam boiler. (Lever of the second class). $L$ is the lever arm, $V$ the valve against which the force of the steam acts, $P$ the applied force which keeps the steam from escaping, $F$ the fulcrum.

**157. The Lever; Third Class.** In this case the force $P$ is applied between the fulcrum and the weight to be lifted. (Fig. 166).

To investigate this arrangement experimentally the apparatus shown in Fig. 167 may be used. A wire loop is placed around the metre stick at its centre of gravity and is fastened to the table as indicated.

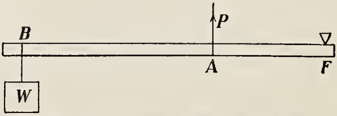



FIG. 166.—A lever of the third class.

As before, compare the products $P \times AF$ and $W \times BF$ for various values and positions of $W$.

These will be found to be equal, and

$$W/P = AF/BF, \text{ the law of the lever.}$$

Notice that the weight lifted is always less than the force applied, or the mechanical advantage is less than 1.

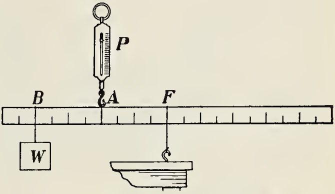



Fig. 167.—Demonstrating the law of the lever of the third class.

Examples of levers of this class: sugar-tongs (Fig. 168), the human forearm (Fig. 169); treadle of a lathe or a sewing machine.

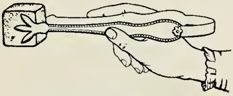


Fig. 168.—Sugar-tongs, lever of the third class.

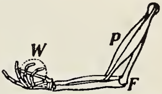



Fig. 169.—Human forearm, lever of the third class. One end of the biceps muscle is attached at the shoulder, the other is attached to the radial bone near the elbow, and exerts a force to raise the weight in the hand.

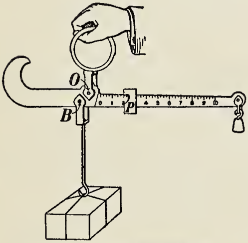



Fig. 170.—The steelyards.

### PROBLEMS

1. Explain the action of the steelyards (Fig. 170). To which class of lever does it belong? If the distance from $B$ to $O$ is $1\frac{1}{2}$ in., and the sliding weight $P$ when at a distance 6 in. from the zero mark balances a mass of 5 lb. on the hook, what must be the weight of $P$?

If the mass of the hook is too great to be balanced by $P$, what additional attachment would be required in order to weigh it?

2. A hand-barrow (Fig. 171), with the mass loaded on it, weighs 210 pounds. The centre of gravity of the barrow and load is 4 feet

from the front handles and 3 feet from the back ones. Find the amount each man carries.

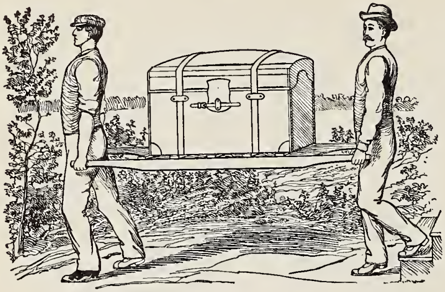


FIG. 171.—The hand-barrow.

3. To draw a nail from a piece of wood requires a pull of 200 pounds. A claw-hammer is used, the nail being $1\frac{1}{2}$ inches from the fulcrum $O$ (Fig. 158) and the hand being 8 inches from $O$. Find what force the hand must exert to draw the nail.

4. A cubical block of granite, whose edge is 3 feet in length and which weighs 4500 lb., is raised by thrusting one end of a crow-bar 40 inches long under it to the distance of 4 inches, and then lifting on the other end. What force must be exerted?

5. A wheelbarrow with its load (Fig. 172), weighs 120 lb.; the horizontal distances of the handles and of the C.G. of the loaded barrow from the centre of the wheel are 4 ft. and 18 in., respectively. Find the force which must be applied to each handle to lift the legs of the barrow off the ground.

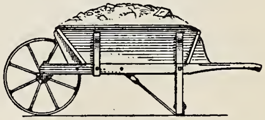


FIG. 172.—A wheel-barrow.

6. In the safety valve (Fig. 165) the distance from the fulcrum to the valve is 4 in. and from the fulcrum to the weight 18 in. If the weight $P$ is 20 lb., what force must the steam exert to be just on the point of escaping?

7. In using the "triple-beam" balance shown in Fig. 173, the slider $D$ gives grams from 0 to 100 in steps of 10 gm.; $E$ gives single grams from 0 to 10 and $F$ centigrams from 0 to 100. The removable hanger $G$ is equivalent to 100 gm. added to the pan. $AB$ measures 7.5 cm. and $BC$ 25 cm.

(1) What must be the mass of the hanger $G$ to balance 100 gm. placed on the pan?

(2) If the slider $D$ has a mass of 50 gm. how far must it be moved from its zero position to balance 80 gm. placed on the pan? What must be the distance between the 0 and 100 gm. marks?

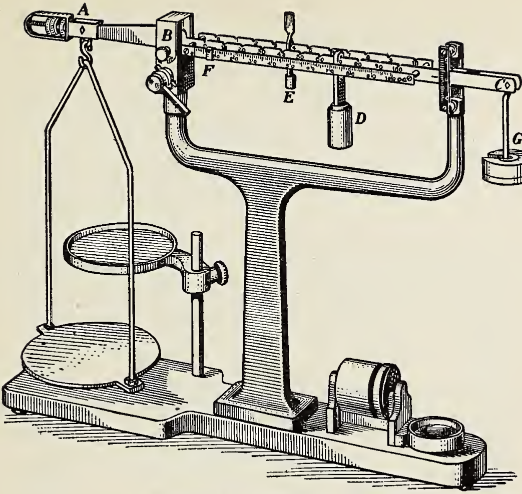



FIG. 173.—A "triple-beam" balance.

**158. The Pulley.** The pulley is used sometimes to change the direction in which a force acts, sometimes to gain mechanical advantage, and sometimes for both purposes.

The pulleys used in experiments should be of very light construction and with well-made bearings, in which there is little friction.

A single fixed pulley, such as is shown in Fig. 174, can change the direction of a force but cannot give a mechanical advantage greater than 1. *P*, the force applied, is equal to the weight lifted, *W*.

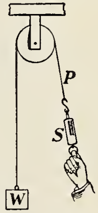



FIG. 174.—A fixed pulley simply changes the direction of force.

By this arrangement a lift is changed into a pull in any convenient direction. It is often used in raising materials during the construction of a building.

By inserting a spring-balance, $S$, in the rope, between the hand and the pulley, one can show that the force $P$ is equal to the weight $W$.

In order to apply the principle of energy, suppose the hand to move through a distance $a$, then the weight rises through the same distance.

Hence, $$P \times a = W \times a,$$ $$\text{or } P = W,$$

as tested by the spring-balance.

If the friction is not negligible, pull on the balance until $W$ rises slowly and uniformly. Then the difference between the weight $W$ and the reading on the balance will give the magnitude of the friction.

**159. A Single Movable Pulley.** Here the weight $W$ (Fig. 175) is supported by the two portions $B$ and $C$, of the rope,

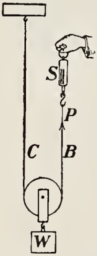


Fig. 175.—With a movable pulley the force exerted is only half as great as the weight lifted.

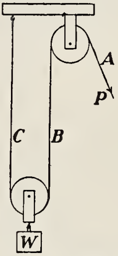


Fig. 176.—With a fixed and a movable pulley the force is changed in direction and reduced one-half.

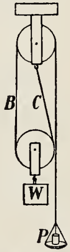


Fig. 177.—One fixed and one movable pulley. Usual arrangement.

and hence each portion supports half of it.

Thus the force $P$, which is indicated by the balance $S$, is equal to $\frac{1}{2}$ $W$, and the mechanical advantage is 2.

This result can also be deduced from the principle of energy.

Let $a$ be the distance through which $W$ rises. Then each portion, $B$ and $C$, of the rope, will be shortened a distance $a$, and so $P$ will be applied through a distance $2a$.

Then, since $P \times 2a = W \times a$,

$$W/P = 2, \text{ the mechanical advantage.}$$

For convenience a fixed pulley is generally used in addition as in Figs. 176 or 177.

Here when the weight rises 1 inch, $B$ and $C$ each shorten 1 inch and hence $A$ lengthens 2 inches. That is, $P$ is exerted through twice the distance through which $W$ rises, and $W/P = 2$, as before.

**160. Other Systems of Pulleys.** To obtain greater mechanical advantage various combinations of pulleys may be used. Two are shown in Figs. 178, 179, the latter one being very common. In Fig. 178 the pulleys are arranged in tandem while in Fig. 179 they are mounted side by side as shown in greater detail in Fig. 180.

Fig. 178.—Combination of 6 pulleys; 6 times the force lifted.

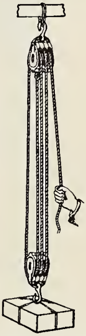


Fig. 179.—A familiar combination for multiplying the force 6 times.

Here there are six portions of the rope supporting $W$, and hence the tension in each portion is $\frac{1}{6}$ $W$.

$$\text{Hence, } P = \tfrac{1}{6} W,$$

or a force equal to $\frac{1}{6}$ $W$ will hold up $W$. This entirely neglects friction, which in such a system is often considerable, and it therefore follows that, to prevent $W$ from descending, less than $\frac{1}{6}$ $W$ will be required. On the other hand, to actually lift $W$ the force $P$ must be greater than $\frac{1}{6}$ $W$. In every case friction acts to prevent motion.

Let us apply the principle of energy to this case. If $W$ rises 1 foot, each portion of the rope supporting it must shorten 1 foot and the force $P$ will act through 6 feet.

Then, work done on $W = W \times 1$ foot-pounds.

" " by $P = P \times 6$ "

These are equal, and hence

$$W = 6\,P$$

or $W/P = 6$, the mechanical advantage.

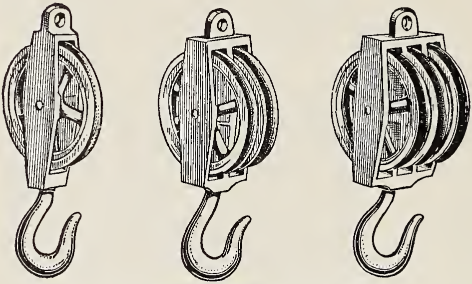


Fig. 180.—Single, double and triple pulleys.

## PROBLEMS

1. A clock may be driven in two ways. First, the weight may be attached to the end of the cord; or secondly, it may be attached to a

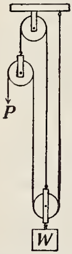



Fig. 181.—The Spanish Burton.

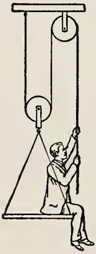


Fig. 182.—An easy method to raise one's self.

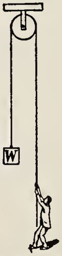



Fig. 183.—Find pressure of the feet on the floor.

pulley, movable as in Fig. 203, one end of the cord being fastened to the framework, and the other being wound about the barrel of the driving wheel. Compare the weights required, and also the length of time the clock will run in the two cases.

2. Find the mechanical advantage of the system shown in Fig. 181. This arrangement is called the Spanish Burton.

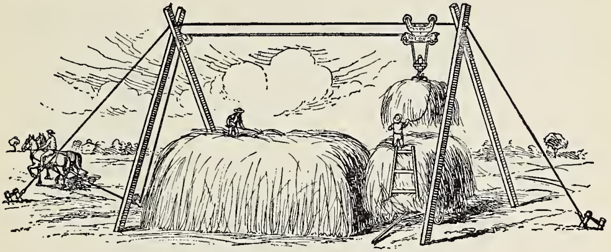


Fig. 184.—A hay-fork being used in building a stack.

3. What fraction of his weight must the man shown in Fig. 182 exert in order to raise himself?

4. A man weighing 140 pounds pulls up a weight of 80 pounds by means of a fixed pulley, under which he stands (Fig. 183). Find his pressure on the floor.

5. Show how you would thread the rope through the pulleys in Fig. 178 to obtain a mechanical advantage of 5. (If necessary leave a pulley idle).

6. Fig. 184 shows a hay-fork being used to lift hay from a wagon to a stack. If the load on the fork weighs 500 pd. what pull must the team exert? If the distance from the wagon to the car is 25 feet, what work is done by the team in raising the loaded fork? (Neglect friction.)

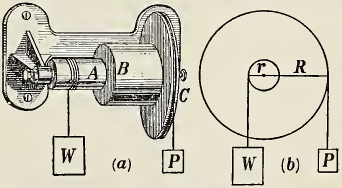



Fig. 185.— The wheel and axle. (a) general appearance; (b) diagram to explain its action.

**161. The Wheel and Axle.** This machine (Fig. 185) has already been considered from the standpoint of moments in Sec. 117. Let us now apply the principle of energy to its action. It is evident that in one complete rotation the weight $P$ will descend a distance equal to the circumference of the wheel,

while the weight $W$ will rise a distance equal to the circumference of the axle.

Hence $P \times$ circumference of wheel $= W \times$ circumference of axle. Let the radii be $R$ and $r$, respectively; the circumferences will be $2\pi R$ and $2\pi r$, and therefore

$$P \times 2\pi R = W \times 2\pi r,$$

or

$$PR = Wr,$$

and the mechanical advantage, $W/P = R/r$, as before.

**162. Examples of Wheel and Axle.** The windlass (Fig. 186) is a common example, but in place of a wheel, handles are used. Forces are applied at the handles and the bucket is lifted by the rope, which is wound about the axle.

Fig. 186.—Windlass used in drawing water from a well.

If $P$ = applied force, and $W$ = weight lifted, $\dfrac{W}{P} = \dfrac{\text{length of crank}}{\text{radius of axle}}$.

The capstan, used on board ships for raising the anchor is another example (Fig. 187).

The sailors apply the force by pushing against bars thrust into holes near the top of the capstan. Usually the rope is too long to be all coiled up on the barrel, so it is passed about it several times and the end $A$ is held by a man who keeps that portion taut. The friction is sufficient to prevent the rope from slipping. Sometimes the end $B$ is fastened to a post or a ring on the dock, and by turning the capstan this portion is shortened and the ship is drawn in to the dock.


Fig. 187.—Raising the ship's anchor by a capstan.

**163. Differential Wheel and Axle.** This machine is shown in Fig. 188. It will be seen that the rope winds off one axle and on the other. Hence, in one rotation of the crank the rope is lengthened (or shortened) by an amount equal to the difference in the circumferences of the two axles; but since the rope passes round a movable pulley the weight to be lifted, attached to this pulley, will rise only one-half the difference in the circumferences.

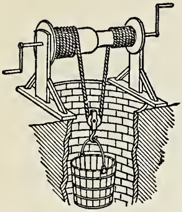


FIG. 188.—Differential wheel and axle.

Let $R$ and $r$ be the radii of the two axles and let $l$ be the length of the crank.

$$\text{Then } P \times 2\pi l = \frac{W\,(2\pi R - 2\pi r)}{2},$$

$$\text{or } \frac{W}{P} = \frac{2l}{R - r}.$$

Thus by making the two drums which form the axles nearly equal in size we can make the difference in their circumferences as small as we please, and the mechanical advantage will be as great as we desire.


FIG. 189.—Explanation of the action of the differential pulley.

**164. Differential Pulley.** This is somewhat similar to the last described machine (Figs. 189, 190).

Two pulleys, $A$ and $B$, of different radii (Fig. 189), are fastened together and turn with the same angular velocity. Grooves are cut in the pulleys so as to receive an endless chain and prevent it from slipping.

FIG. 190.—The actual appearance of the differential pulley.

Suppose the chain is pulled by a force $P$ until the two

pulleys have made a complete rotation. Then $P$ will have moved through a distance equal to the circumference of $A$, and it will have done work $= P \times$ circumference of $A$.

Also, the chain between the upper and the lower pulley will be shortened by the circumference of $A$, but lengthened by the circumference of $B$, and the net shortening is the difference between these two circumferences.

But the weight $W$ will rise only half of this difference. Hence, work done on $W$

$$= W \times \tfrac{1}{2} \text{ difference of circumferences of } A \text{ and } B,$$

and therefore $$\frac{W}{P} = \frac{\text{circumference of } A}{\tfrac{1}{2} \text{ difference of circumferences of } A \text{ and } B}.$$

## PROBLEMS

1. A man weighing 160 pounds is drawn up out of a well by means of a windlass (Fig. 186) the axle of which is 8 in. in diameter, and the crank 24 in. long. Find the force required to be applied to the handle.

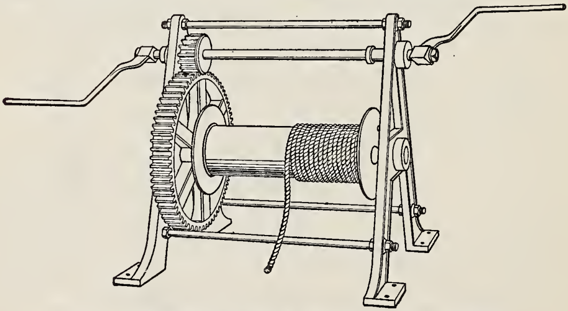


Fig. 191.—Windlass, with gearing, used to raise a heavy weight.

2. Calculate the mechanical advantage of the windlass shown in Fig. 191. The length of the crank is 16 in., the small wheel has 12 teeth and the large one 120, and the diameter of the drum about which the rope is wound is 6 in.

If a force of 60 pounds be applied to each crank how great a weight can be raised? (Neglect friction).

3. In the experimental crane shown in Fig. 192, the small gear wheel has 12 teeth and the large one 72. The diameter of the drum is 5 in. and of the grooved wheel 10 in. Neglecting friction, what force $P$ must be applied to the cord around the grooved wheel to support 200 lb. attached to the rope passing around the drum?

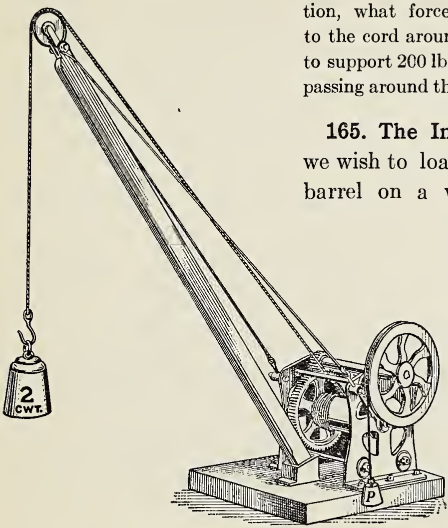


FIG. 192.—An experimental crane.

**165. The Inclined Plane.** If we wish to load a heavy box or barrel on a wagon it is often convenient to slide or roll it up a plank which has one end on the ground and the other on the wagon. The relation between the force exerted and the resistance overcome can be investigated by means of the apparatus shown in Fig. 193, which has already been discussed in Sec. 80.

If $W$ is the weight of the car $C$ and $P$ the applied force which will just make $C$ move up the plane without acceleration, it is evident from the principle of energy that $Pl$ should equal $Wh$ if there were no friction. An experiment to verify this relation has been described fully in Sec. 80.

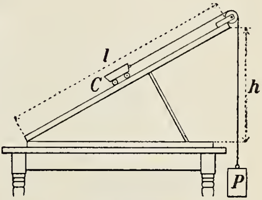


FIG. 193.—To show that $Pl = Wh$.

Hence, $$W/P = l/h,$$

or the theoretical mechanical advantage is the ratio of the length to the height of the plane.

Taking friction into account, the mechanical advantage is not so great, and to reduce the friction as much as possible the body may be rolled up the plane.

**Exercise.**—Obtain the relation $W/P = l/h$ by considering the forces acting on $C$ when it is just on the point of moving up the plane. Resolve the forces along the plane and at right angles to it and write the equations of equilibrium. (Neglect friction).

**166. The Wedge.** The wedge is designed to overcome great resistance through a small space. Its most familiar use is in splitting wood. Knives, axes and chisels are also examples of the wedge.

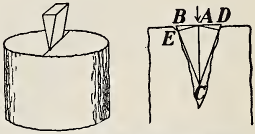


Fig. 194.—The wedge, an application of the inclined plane.

The resistance $W$ (Fig. 194) to be overcome is considered to act at right angles to the slant sides $BC$, $DC$, of the wedge, and when the wedge has been driven in, as shown in the figure, the work done in pushing back one side of the split block will be $W \times AE$, and hence the work for both sides is $W \times 2\,AE$.

But the applied force $P$ acts through a space $AC$, and thus does work $P \times AC$.

Hence, $$W \times 2AE = P \times AC,$$
and $$W/P = AC/2AE.$$
This is the mechanical advantage, and it is evidently greater the thinner the wedge is.

This result is of little practical value, as we have not taken friction into account, nor the fact that the force $P$ is applied as a blow, not as a steady pressure. Both of these factors are of great importance.

**167. The Screw.** The screw consists of a grooved cylinder which turns within a hollow cylinder or nut which it just fits. The distance from one thread to the next is called the *pitch*.

The law of the screw is easily obtained. Let $l$ be the length of the handle by which the screw is turned (Fig. 195) and $P$ the force exerted on it. In one rotation of the screw the end of the handle describes the circumference of a circle with radius $l$, that is, it moves through a distance $2\pi l$, and the work done is therefore

$$P \times 2\pi l.$$

Let $W$ be the force exerted upwards as the screw rises, and $d$ be the pitch. In one rotation the work done is

$$W \times d.$$

Hence, $$W \times d = P \times 2\pi l,$$

or $$W/P = 2\pi l/d,$$

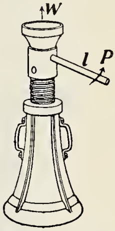


Fig. 195.—The jack-screw.

or the mechanical advantage is equal to the ratio of the circumference of the circle traced out by the end of the handle to the pitch of the screw.

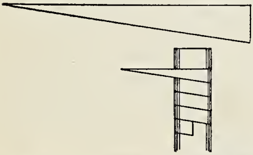


Fig. 196.—Diagram to show that the screw is an application of the inclined plane.

In actual practice the advantage is much less than this on account of friction.

The screw is really an application of the inclined plane. If a triangular piece of paper, as in Fig. 196, be wrapped about a cylinder (a lead pencil, for instance), the hypotenuse of the triangle will trace out a spiral like the thread of a screw.

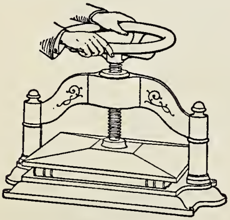


Fig. 197.—The letter press.

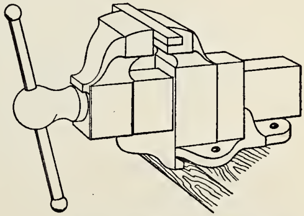


Fig. 198.—The mechanic's vice.

Examples of the screw are seen in the letter press (Fig. 197), and the vice (Fig. 198).

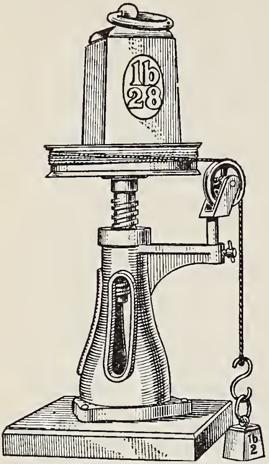



Fig. 199.—An experimental jack-screw.

Fig. 199 shows an experimental jack-screw by which the actual mechanical advantage may be measured and compared with the theoretical value.

## ILLUSTRATIVE PROBLEMS

1. Why should shears for cutting metal have short blades and long handles?

2. In the driving mechanism of a self-binder, shown in Fig. 200, the driving-wheel *A* has a diameter of 3 ft., the sprocket-wheels *B* and *C* have 40 teeth and 10 teeth, respectively. The large gear-wheel *D* has 37 teeth and the small one *E* has 12 teeth, and the crank *G* is 3 in. long. Neglecting friction, what pull on the driving-wheel will be required to exert a force of 10 pounds on the crank *G*? (Find the number of

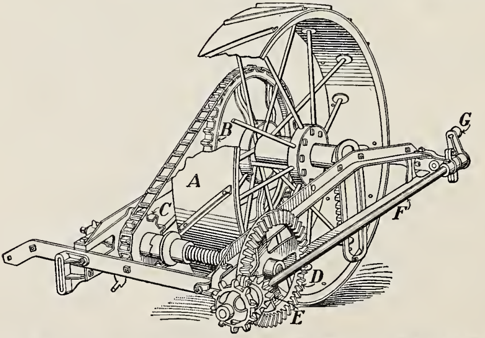



Fig. 200.—The driving part of a self-binder. The driving-wheel *A* is drawn forward by the horses. On its axis is the sprocket-wheel *B*, and this, by means of the chain, drives the sprocket-wheel *C*. The latter drives the cog-wheel *D* which, again, drives the cog-wheel *E*, and this causes the shaft *F* with the crank *G* on its end to rotate.

revolutions of the crank for one revolution of the driving-wheel and apply the principle of energy.)

3. Explain the action of the levers in the scale shown in Fig. 201.

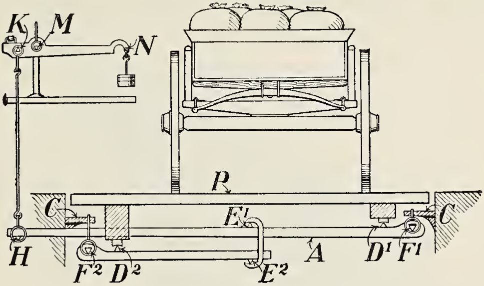



Fig. 201.—Diagram of multiplying levers in a scale for weighing hay, coal and other heavy loads. In the figure is shown one-half of the system of levers, as seen from one end. The platform $P$ rests on knife-edges $D^1$, $D^2$, the former of which is on a long lever, the latter on a short one. The knife-edges $F^1$, $F^2$ at the end of these levers are supported by suspension from the brackets $C$, $C$ which are rigidly connected with the earth.

If $HF^1 = 12$ ft., $F^1D^1 = 4$ in., $MN = 36$ in., $KM = 3$ in., what weight on $N$ would balance 2000 pounds of load (wagon and contents)? In the scale $E^1F^1 = E^2F^2$, and $F^1D^1 = F^2D^2$, so the load is simply divided equally between the two levers.

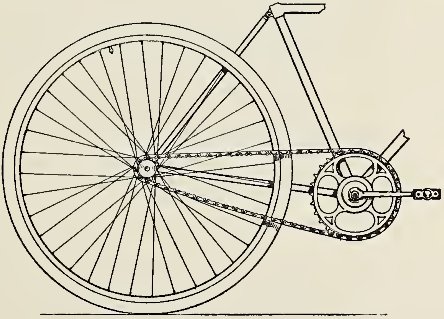


Fig. 202.—The driving mechanism of a bicycle.

4. If the crank arm of the bicycle (Fig. 202) is 7 in. long, and if the wheel has a diameter of 28 in., find the tangential force exerted on the road by

the tire when the rider pushes downwards with a force of 50 pd. upon the arm when it is horizontal, the numbers of the teeth on the sprocket-wheels being 28 and 8, respectively. Find the force when the arm makes 30° with the horizontal. How far does the bicycle travel during one revolution of the crank?

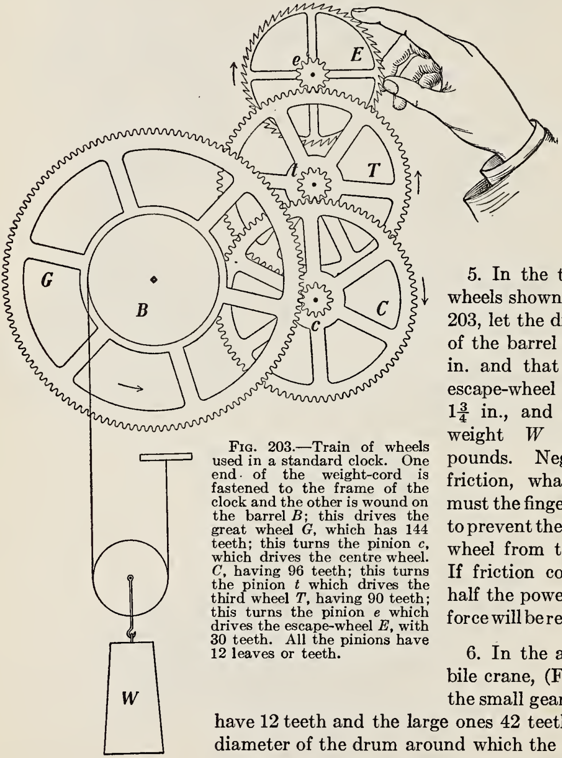



Fig. 203.—Train of wheels used in a standard clock. One end of the weight-cord is fastened to the frame of the clock and the other is wound on the barrel $B$; this drives the great wheel $G$, which has 144 teeth; this turns the pinion $c$, which drives the centre wheel. $C$, having 96 teeth; this turns the pinion $t$ which drives the third wheel $T$, having 90 teeth; this turns the pinion $e$ which drives the escape-wheel $E$, with 30 teeth. All the pinions have 12 leaves or teeth.

5. In the train of wheels shown in Fig. 203, let the diameter of the barrel $B$ be 2 in. and that of the escape-wheel $E$ be $1\frac{3}{4}$ in., and let the weight $W$ be 10 pounds. Neglecting friction, what force must the fingers exert to prevent the escape-wheel from turning? If friction consumes half the power, what force will be required?

6. In the automobile crane, (Fig. 204) the small gear wheels have 12 teeth and the large ones 42 teeth. The diameter of the drum around which the chain is wound is 5 in. and the length of the handle is 14 in. The two lower wheels are rigidly attached to a common shaft; the crank and small upper wheel are also attached to a common shaft about which the drum and large upper gear connected to it can turn freely on

roller bearings. Neglecting friction, what force must be applied to the handle to support one ton attached to the hanging pulley?

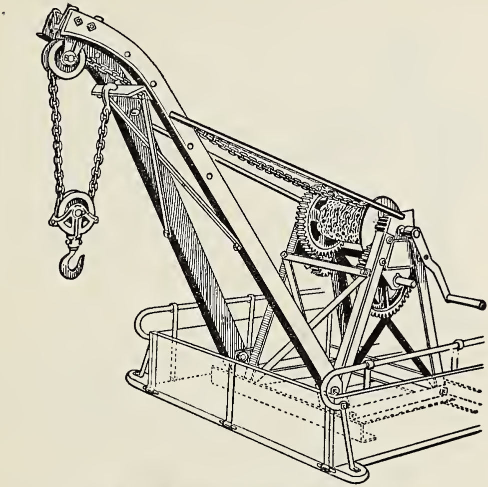


FIG. 204.—A crane for raising disabled automobiles.

7. Fig. 205 shows a three-horse evener used when three horses are to be attached to a binder or other farm implement.

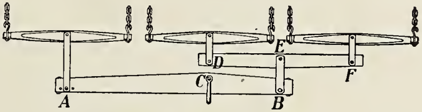



FIG. 205.—A three-horse evener.

(1) If $AC = 40$ in. and $CB = 19$ in., find the pulls exerted by the single horse at $A$ and by the team at $B$ to overcome a resistance of 295 pd. at $C$.

(2) If $DE = 21$ in. and $EF = 19$ in., find the pulls exerted by the horses attached at $D$ and $F$.

8. Find the mechanical advantage (neglecting friction) of the automobile jack shown in Fig. 206. The crank arm $A$ is 10 in. long, the small gear wheel has 10 teeth, the large one 26 teeth and the pitch of the screw is $\frac{1}{4}$ in.

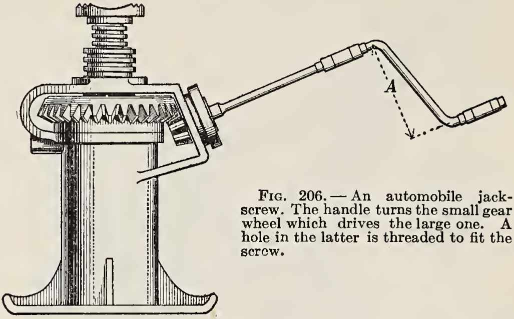



FIG. 206. — An automobile jack-screw. The handle turns the small gear wheel which drives the large one. A hole in the latter is threaded to fit the screw.

**168. Automobile Transmission.** The automobile, considered as a whole, is an excellent example of a high-class machine, but some parts of it are especially interesting. Two of these are the 'transmission' and the 'differential.'

By means of the transmission the driver of the car can go forward with different speeds, can go backwards, or can stand still while the engine continues to run.

not

**169. Selective Type of Transmission.** When it is desired to have three (or more) speeds forward the selective type of transmission illustrated in Fig. 207 is used.

The shaft which comes from the engine enters the square opening in the shaft $H$, which, beyond the bearing, has the gear $M$ fixed upon it. This shaft terminates just beyond $M$, but in the same line with it is the shaft $E$, which for some distance has a square section and which at $D$ is attached to the driving shaft leading back to the rear wheels.

Mounted parallel to $E$ is the countershaft $C$ on which are four gears $D$, $S$, $L$, $R$. The gear $D$ is always in mesh with $M$ and consequently always rotates when the shaft $H$ does.

The gears $A$ and $B$ can be shifted forward or backward on the square shaft $E$, by means of the sliding rods $F$, $G$. In order to obtain first, or low, speed, the gear $B$ is shifted forward until it meshes with $L$. Thus $M$ turns $D$ and $L$ turns $B$ which turns the driving shaft. For second, or intermediate, speed the gear $A$ is shifted backwards until it meshes with $S$. The speed will now be greater since $S$ is larger than $L$, and $A$ is smaller than $B$. For third, or high, speed the gear $A$ is shifted forward until the little projections or 'dogs' $d$ fit between similar dogs on $M$. In this case the shaft $E$, and hence the driving shaft, turns at the same rate as the shaft $H$.

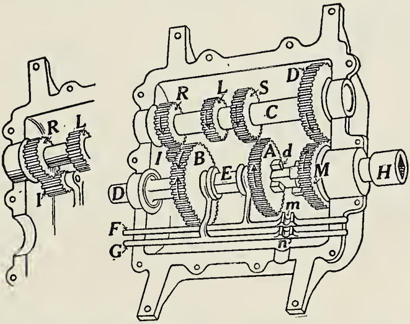



Fig. 207.—The selective type transmission, by which three forward speeds can be obtained.

In order to reverse, the gear $B$ is shifted backwards until it meshes with a small idle gear $I$ (seen better in the left-hand portion of the figure). In this case $M$ turns $D$, $R$ turns $I$, and $I$ turns $B$ in the opposite direction.

The shifting of the gears is accomplished by moving a lever, the lower end of which fits into the notches $m$, $n$ according to the way the lever is moved.

**170. The Differential.** This is placed on the rear axle and its object is to permit the two rear wheels to turn independently. Such an arrangement is very necessary, since in

turning around one of the rear wheels moves much farther than the other. Without the differential it would be almost impossible to turn sharply as one wheel would have to slide on the road.

Fig. 208 gives a view of the differential as seen from above, and facing the front of the car. The pinion *A* on the end of the driving shaft *S* meshes with the large gear wheel. Upon this latter wheel is a strong metal case or 'housing' *C* which rotates with the wheel. In the walls of this are pins *d*, *d* upon which gears *D*, *D* can turn, and these mesh with gears *E*, *E*, one of which is fixed on the left axle *L*, the other on the right axle *R*.

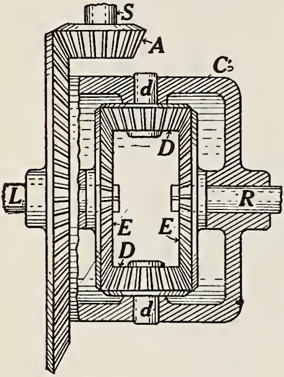


Fig. 208.—The "differential" of an automobile.

Imagine the housing *C* to be turning in such a way that the upper part of the figure is moving from the observer. Then the gears *D*, *D* will drag *E*, *E* in this direction and the two axles *L* and *R* will drive the car forward. *D*, *D* do not rotate on their axes *d*, *d* at all.

But suppose the large left wheel is fast and cannot move. Then the left gear *E* does not move, and as *D*, *D* are carried about by the housing they must rotate on their axes, and this rotation will simply double the rate of rotation of the *R* axle.

## PROBLEMS

1. If the pinion *A* has 11 teeth and the larger gear into which it meshes has 40 teeth, compare the revolutions per minute of the wheels with those of the driving shaft. (Fig. 208).

2. If the wheels of the car are 30 in. in diameter find the revolutions per minute of the engine when the car is going forward at 30 miles per hour.

# CHAPTER XVII

## Pressure and Its Transmission

**171. Pressure: How Measured.** With the idea of pressure we are all familiar. In a pile of books each presses the one below it; or when a piece of wood or metal is held in a vice the jaws of the vice press upon the surface of the object. Many other illustrations could be given, and it is to be observed that in every case **pressure acts upon a surface.**

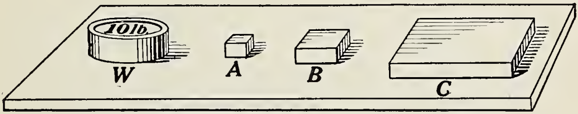



Fig. 209.—Illustrating the measurement of pressure.

Cut from a thin board three square blocks having edges $\frac{1}{2}$ inch, 1 inch and 3 inches, respectively, and lay them on a table ($A$, $B$, $C$, Fig. 209). These are so very light that we may neglect the forces exerted by them upon the table. Now place the 10-pound weight $W$ upon $B$; there is pressure thus produced upon the table and as the area of the surface acted upon is 1 square inch we say the pressure is 10 pounds per square inch.

Next, place the weight upon $C$. The area of the surface of the table now acted upon is 9 square inches, and we say the pressure on it is $1\frac{1}{9}$ pounds per square inch.

Finally place the weight on $A$. In this case the area acted upon is $\frac{1}{4}$ square inch; so the pressure is 10 pounds per $\frac{1}{4}$ square inch or 40 pounds per square inch.

In each case the total force exerted is the same but the pressure, or the force per square inch, differs, being in the three cases in the proportion 1 : 9 : 36.

**In specifying a pressure always give the force on unit area; as, pounds per square inch, grams per square centimetre, or tons per square yard.**

**172. Solids, Liquids, Gases.** The distinguishing properties of the three states of matter are:

Solids have definite volume and definite shape.

Liquids have definite volume but no definite shape.

Gases have neither definite volume nor definite shape.

A liquid offers no permanent resistance to forces tending to change its shape. It will yield to even the smallest force if continuously applied, but the rate of yielding varies with different liquids and it is this temporary resistance which constitutes **viscosity.**

The term **fluid** is applied to either a liquid or a gas.

**173. Pressure of a Fluid.** It is a matter of common experience that a fluid exerts a force upon the surface with which it is in contact. A wooden tank, such as we often see above buildings for fire-protection purposes, or beside the railway for supplying water to the locomotives, is bound with strong iron bands to prevent the water from pushing the staves outwards. Note, also, that the bands are closer together near the bottom than higher up, indicating that the pressure at the bottom is greater than near the top.

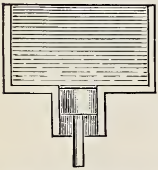


Fig. 210.—Pressure at the bottom of a vessel.

Consider a vessel like that in Fig. 210, having a piston inserted in the bottom. A force must be applied upwards on the piston to prevent the water from pushing the piston out. Let the force upwards required to balance the pressure of the water be 10 pounds, and the area of the piston be 5 square inches. Then the pressure of the water on the piston is 2 pounds per square inch.

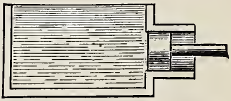


Fig. 211.—Pressure at the side of a vessel.

Next consider the case of a piston of the same size inserted in the side of the vessel (Fig. 211). As remarked above, the water exerts a force upon the piston.

If we adjust the depth of the water so that, as before, the force required to balance the pressure of the water is 10 pounds, then the **average** pressure of the water on the piston is 2 pounds per square inch. In this case it is necessary to say average pressure because of the fact of experience mentioned above, that the pressure depends upon the depth and so is not uniform over the surface of the piston. The manner in which the pressure varies with the depth will be taken up in the next chapter.

**174. Pressure at a Point.** We have just seen that pressure in a fluid varies from place to place, and we often use the phrase **pressure at a point.** Let us look into the precise meaning of the phrase.

When we say that the pressure at a point $B$ is 5 pounds per square inch we mean that if we had a square inch, against every point of which the fluid exerted the same thrust that it exerts at $B$, then the thrust against the square inch would be 5 pounds.

Consider a very small surface of area $A$, containing the point, so small indeed that the pressure upon it may be considered uniform all over it. Let $F$ be the total force exerted upon the area $A$. Then the force $P$ on unit area $= F/A$. This is the pressure at the point.

Suppose we press upon the hand with the flat end of a lead pencil with such a force that the pressure is 5 pounds per square inch. It is easily seen that the force applied is not equal to 5 pounds nor is the surface acted upon equal to a square inch. But if

$F$ = applied force in pounds,

and $A$ = area acted upon in square inches,

then pressure $= F/A = 5$ pounds per square inch.

**175. Transmission of Pressure by Fluids.** One of the most characteristic properties of matter is its power to transmit force. The harness connects the horse with its load; the piston

and connecting rods convey the pressure of the steam to the driving wheels of the locomotive. Solids transmit pressure only in the line of action of the force. Fluids act differently. If a globe and cylinder of the form shown in Fig. 212 is filled with water and a force exerted on the water by means of a piston, it will be seen that the pressure **is transmitted,** not simply in the direction in which the force is applied, but **in all directions**; because jets of water are thrown with velocities which are apparently equal from all the apertures. If the conditions are modified by connecting with the globe **U**-shaped tubes partially filled with mercury, as shown in Fig. 213, it will be found that when the piston is inserted, the change in level of the mercury, caused by the transmitted pressure, is the same in each tube. This would show that the pressure applied to the piston is transmitted **equally** in all directions by the water.

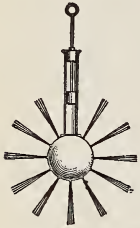


Fig. 212. — Pressure applied to the piston transmitted in all directions by the liquid within the globe.

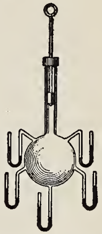


Fig. 213.—Transmission shown to be equal in all directions by pressure gauges.

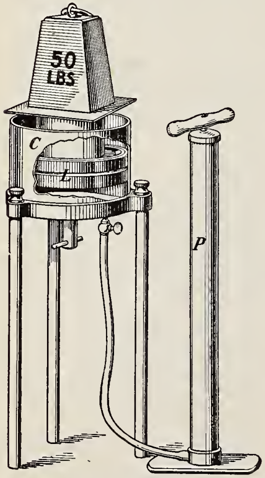



Fig. 214.—Transmission of pressure by a gas (air).

Next, let us use the apparatus shown in Fig. 214. The cylinder *C*, about 5 inches in diameter, is provided with a tightly-fitting piston *L*. On this a heavy weight (50 pounds) is placed. One end of a piece of heavy rubber tubing is attached to *C*

while the other end, by means of an ordinary bicycle tire valve, is joined to the bicycle pump *P*. On working the pump the weight is raised with very little effort. Careful experiments with similar apparatus show that, neglecting friction, if the area of *L* is 50 times that of the piston of the pump, only one pound force need be applied to the pump to raise the 50-pound weight,

Again, consider the vertical section of a closed vessel filled with some fluid, say, water, and fitted with pistons of equal area as shown in Fig. 215. Let a force *P* pounds be applied to the piston *A*. This will produce a force within the water which will be transmitted by the water to every surface with which it is in contact.

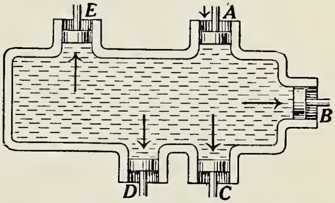


Fig. 215.—Diagram illustrating transmission of pressure.

The piston *A* will exert a thrust of *P* pounds upon the surface of the water in contact with it and this thrust will be transmitted not only to *C*, which is directly opposite *A*, but also to *D*, which is alongside *C*, to *B* which is at one end of the vessel, and to *E* which is at the top beside *A*. **The pressure is transmitted by the fluid in all directions and undiminished in intensity.**

If the pistons *C* and *D* were merged into a single one the thrust on it would be $2P$ pounds, or the thrust is proportional to the area of the surface.

If the area of the piston *A* is 5 square inches and the force applied to it be 10 pounds, there will be a pressure of 2 pounds on every square inch of the inner surface of the vessel.

**176. Pressure of a Fluid at Right Angles to the Surface.** From the fact that a fluid transmits pressure perfectly, that is, in every direction and without diminution, we must conclude that its particles are perfectly free to move about amongst

themselves, that the slightest force applied to a liquid can displace its particles.

**It must follow that when a fluid is at rest its pressure is at right angles to the surface upon which it acts.** This can be proved in the following way.

If it were possible, let the pressure at $A$ (Fig. 216) upon the side of the vessel be not at right angles to the surface, but in the direction $R$. Resolve the force into two components, $Q$ at right angles to the surface and $P$ parallel to the surface. The force $Q$ is balanced by the reaction of the side of the vessel, but $P$ is unopposed and it must cause a sliding of the particles of the liquid along the surface in the direction $AB$. But this is impossible as, by hypothesis, the fluid is at rest.

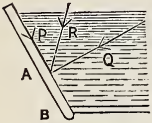


Fig. 216.—Pressure is at right angles to the surface.

We must, therefore, conclude that the pressure of a fluid is at right angles to the surface upon which it acts.

We are now in a position to state the following law:

**Pressure exerted anywhere upon a mass of fluid filling a closed vessel is transmitted undiminished in all directions, and acts with the same force on all equal surfaces and in a direction at right angles to them.**

This is known as ***Pascal's Law*** or ***Principle.***

**177. Mechanical Applications.** The transmission of pressure by a liquid equally in all directions is utilized in the hydraulic press and the hydraulic jack. Fig. 217 illustrates the general principle of action of these machines. $D$ and $E$ are two hollow cylinders connected by a tube $C$ and partly filled with water

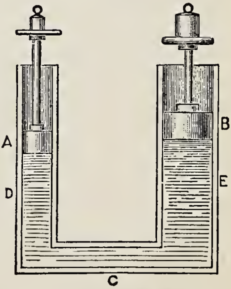


Fig. 217.—Illustrating the hydraulic press or the hydraulic jack.

or oil. *A* and *B* are two pistons, fitted into *D* and *E* respectively. Any force applied to *A* is transmitted by the liquid to *B*, and as the pressures on *A* and *B* are undiminished in intensity, the total forces exerted by the liquid upon *A* and *B* are proportional to their areas. Thus, if the area of *A* is 1 square inch and that of *B* is 10 square inches, then a weight of 1 pound on *A* will sustain a weight of 10 pounds on B.

For a description of Bramah's Press, see Sec. 237 and for a picture of the hydraulic jack see Fig. 219.

**178. Pressure at a Point in a Fluid.** Consider a vessel filled with fluid (Fig. 218), and let the area of the piston *A* be one unit, say, 1 square inch. Let *C* be any point within the mass of the fluid, and imagine it to be at the centre of a circular plane area *mn* (seen edgewise). Let the area of *mn* be 1 square inch.

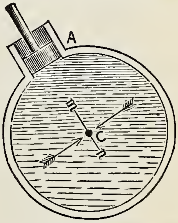


Fig. 218.—Pressure the same in all directions in a fluid.

If the piston is pushed inwards with a force of *P* pounds, the liquid will transmit a pressure of *P* pounds to every square inch of the inner surface of the vessel; also, each face of *mn* will be subjected to a force of *P* pounds.

Now the magnitude of this force does not depend on the direction in which the area *mn* is turned, that is, the pressure at *C* is the same in all directions.

**The pressure at a point in a fluid is the same in all directions.**

In this discussion no mention has been made of the pressure due to gravity acting upon the fluid. The law holds true, however, no matter what the source of the pressure may be.

## PROBLEMS AND EXERCISES

1. A fluid thrust of 1728 pounds is uniformly distributed over a surface whose area is 3 sq. ft. Find the measure of the pressure at a point in the surface (1) when the unit-area is 1 sq. in., (2) when it is 1 sq. ft., the unit of force in each case being 1 pound.

2. The pressure is uniform over the whole of a sq. yard of a plane area in contact with a fluid, and is 7776 pounds. Find the measure of the pressure at a point (1) when the unit of length is 1 in., (2) when it is 3 in., the unit of force in each case being the pound.

3. The thrust of a fluid against a circular plane, diameter 14 cm., is 770 kg.; if the pressure is uniform, find the measure of the pressure at a point (1) when the unit-area is 1 sq. mm., (2) when it is 1 sq. dm., the unit of force in each case being the gram.

4. A rectangular surface, length 50 cm. and width 4 cm., is subjected to a uniformly distributed fluid thrust of 4 kg. Find the measure of the pressure at a point (1) when the unit of length is 1 mm., (2) when the unit of length is 2 mm.; if the unit of force is the gram.

5. If the area of a piston inserted in a closed vessel is $3\frac{1}{4}$ sq. in., and if it is pressed with a force of 35 pounds, find the thrust which it will transmit to a surface of $7\frac{3}{7}$ sq. in.

6. A closed vessel is filled with liquid, and two circular pistons, whose diameters are respectively 2 cm. and 5 cm. inserted. If the thrust on the smaller piston is 50 grams, find the thrust on the larger piston when they balance each other.

7. A closed vessel is filled with fluid and two circular pistons whose diameters are respectively 3 in. and 7 in. inserted; if the thrust on the larger piston is $a$ pounds, find the thrust on the smaller.

8. The diameter of the large piston of a hydraulic press is 100 cm. and that of the smaller piston 5 cm. What force will be exerted by the press when a force of two kilograms is applied to the small piston?

9. The diameter of the piston of a hydraulic elevator is 14 in. Neglecting friction, what load, including the weight of the cage, can be lifted when the pressure of the water in the mains is 75 pd. per sq. inch?

10. The horizontal cross-section of the neck of a glass bottle, just capable of sustaining a pressure of 11 pounds to the sq. in., is $2\frac{3}{4}$ sq. in. It is filled with a fluid supposed weightless, and a piston is inserted into the neck. What is the least force that must be applied to the piston to break the bottle?

11. If the diameter of the small piston (Fig. 217) is 5 cm., and that of the larger one 2.5 metres, and if the small piston is pushed with a force of 8 gm., what force will it transmit to the large piston?

12. In the same machine the horizontal cross-section of the small piston is 3 sq. cm.; with what force must it be pushed that it may sustain a force of 7.25 kg. applied to a piston whose horizontal cross-section is 7 sq. dm.?

13. Find the mechanical advantage of the hydraulic jack shown in Fig. 219. The diameters of the large and small pistons are $\frac{7}{8}$ in. and $\frac{1}{2}$ in., respectively; the distances from the fulcrum to the small piston and to the end of the handle are 1 in. and 36 in., respectively. Find, also, the force which must be exerted on the end of the handle in order that the jack may raise a mass of 1 ton.

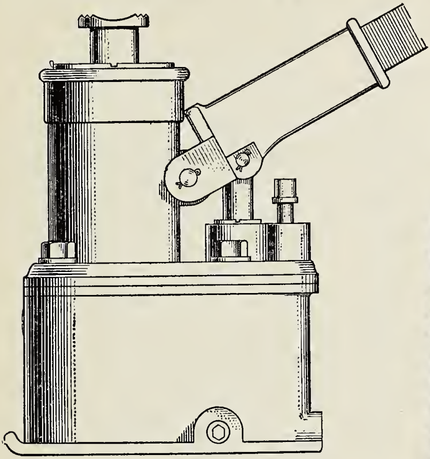


FIG. 219.—Hydraulic jack for an automobile.

14. Pour a small quantity of mercury into a tube of the form shown in the Fig. 220. Now pour some water into the larger branch.

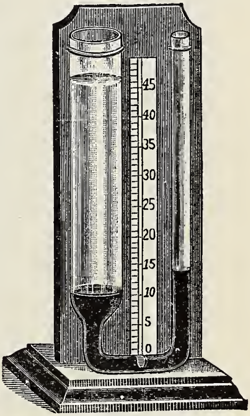



FIG. 220.—Experiment with two liquids.

(1) What changes take place in the levels of the mercury in the two branches? Why?

(2) How much water do you suppose must be put into the smaller branch to bring the mercury to the same level in each branch? Give reasons for your answer. Verify by pouring water into the smaller branch.

(3) How does the weight of the water in the large branch compare with that in the smaller one when the mercury is restored to the same level in each tube?

# CHAPTER XVIII

## Equilibrium of Fluids Under Gravity

**179. Liquids are Attracted Towards the Earth.** Anyone who has carried a pail of water need not be told that water is **heavy.** Every particle of it is attracted towards the earth and it is for this reason that liquids must be held in non-porous vessels, though these need not be covered as the liquid keeps to the bottom.

If bricks be laid one upon another there is a pressure upon the surface of any brick produced by those bricks above it. The one at the bottom has to bear the weight of all those above it. So it is in a vessel containing a fluid. The lower layers have to support all the fluid above them and we would expect them to be under pressure. Also there must be pressure upon the bottom of the vessel and, on account of the nature of the fluid, upon its sides as well. In the previous chapter we learned that within a fluid the pressure at a point is the same in all directions.

**180. Force Within the Liquid.** If we pierce a hole in the side or bottom of a vessel filled with water, the water rushes out and the farther the hole is below the surface the more quickly does the liquid escape. It is an old camper's experiment to obtain cold water from the bottom of a lake by lowering a bottle closed by a cork and so arranged that the water will force it into the bottle when it gets low enough down. These results show in a general way that the pressure depends upon the depth.

**181. Relation between Pressure and Depth.** To demonstrate that the pressure within a liquid increases with the depth, let us perform the following experiment:

Prepare a pressure gauge of the form shown in Fig. 221 by stretching a rubber membrane over a thistle-tube *A*, which is connected by means of a rubber tube with a **U**-shaped glass tube *F*, partially filled with coloured water. The action of the gauge is shown by pressing on the membrane. The pressure is transmitted to the surface of the water in *F* by the air in the tube and is measured by the difference in level of the water in the branches of the **U**-tube.

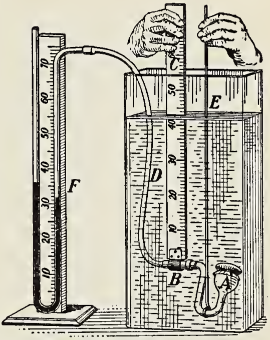



Fig. 221.—Apparatus to show that pressure is proportional to depth and is the same in all directions.

Now place *A* in a jar of water (which should be at the temperature of the room), and gradually push it downward (Fig. 221). The changes in the level of the water in the branches of the **U**-tube indicate an increase in pressure with the increase in depth.

Careful experiments show that **this pressure increases from the surface downward in direct proportion to the depth.**

Now, by means of the wire *E*, turn the thistle-tube *A* in different directions, the centre of the membrane being kept all the time at the same depth, and observe the levels in the **U**-tube. They remain steady. Evidently the upward, downward and lateral pressures are equal at the same depth.

We find therefore that **the pressure is equal in all directions at the same depth.**

**182. Pressure Independent of Shape of Containing Vessel.** In our proof of the law that the pressure within a liquid varies with the depth nothing was said about the shape of the containing vessel or the quantity of liquid present. It will be useful to investigate if this matter should be taken into consideration.

In the case of a vessel with vertical sides the pressure on the bottom is obviously the weight of the liquid. But what if the sides are not vertical? We can settle the question by means of the apparatus shown in Fig. 222. *A*, *B*, *C*, *D* are tubes of different shapes but made to fit into a common base. *E* is a detachable bottom held in position by a lever and weight.

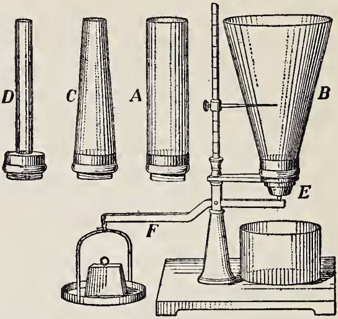



Fig. 222.—Pressures on the bottoms of vessels of different shapes and capacities.

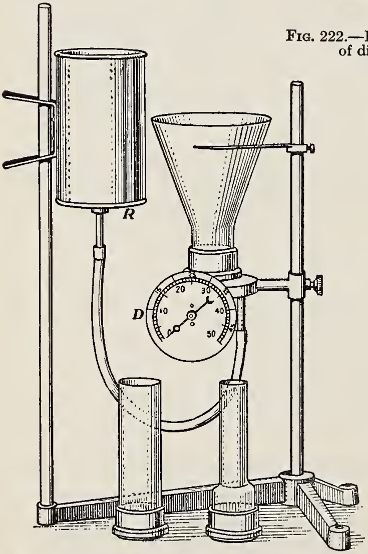



Fig. 223.—Alternative apparatus for showing that pressure does not depend on the shape of the vessel.

First, screw the cylindrical tube *A* in position and place a suitable weight on the scale-pan. Then let us pour water into the vessel until at last the pressure due to the water pushes the bottom down and allows the water to escape. With the pointer mark the height reached by the water when this happens. Now repeat the experiment using *B*, *C*, *D* in succession. We observe that the height of the water when the bottom drops down is the same for all the various vessels.

**Thus we see that the pressure on the bottom of a vessel produced by liquid in it depends only on the depth of the liquid, not at all upon the shape of the vessel or the amount of liquid in it.**

An alternative form of apparatus used in demonstrating this principle is illustrated in Fig. 223. The thrust of the water is shown on the dial *D*; the reservoir *R* may be raised or lowered in order to bring the water to the proper level.

**183. Pressure at Points in the Same Level.** Let *A* and *B* (Fig. 224) be two points in the same level in a liquid and let these points be the centres of the ends of a very small cylinder of the liquid.

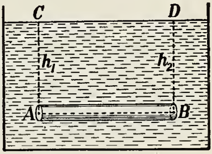



Fig. 224.—Pressures at points in the same level are equal.

Let us consider the forces acting upon this cylinder.

We have

(1) its weight acting vertically downward and consequently at right angles to *AB* the axis of the cylinder.

(2) the thrusts of the liquid (external to the cylinder) on the two ends of the cylinder, acting at right angles to the ends.

(3) the thrust of the liquid on the curved surface of the cylinder acting at right angles to it and consequently at right angles to the axis.

Now the only forces tending to make the cylinder move endwise are the thrusts on the ends, and since the cylinder is in equilibrium these thrusts must be equal. Hence the pressure at *A* must equal the pressure at *B*.

**The pressures at points in a liquid in the same horizontal plane are therefore equal.**

**184. Free Surface of Liquid a Horizontal Plane.** Let us investigate the relation of *C* and *D*, two points situated vertically above *A* and *B* (Fig. 224), on the surface of the liquid.

Let $AC = h_1$ units and let $BD = h_2$ units. Then since pressure is proportional to depth,

$$\begin{aligned} \text{pressure at } A &= kh_1, \\ \text{pressure at } B &= kh_2. \\ \text{But pressure at } A &= \text{pressure at } B. \\ \text{Hence} \quad kh_1 &= kh_2, \text{ or } h_1 = h_2. \end{aligned}$$

But $A$ and $B$ are in the same level and therefore $C$ and $D$ are in the same level.

Now $C$ and $D$ are any two points on the surface of the liquid (since $A$ and $B$ are any two points in the same horizontal plane in the liquid).

**It follows then that the free surface of a liquid at rest under the action of gravity is a horizontal plane.**

**185. Surface of Liquid in Connecting Vessels.** Pour water into a series of vessels, $A$, $B$, $C$, $D$, $E$ (Fig. 225) of different shapes, connected together so the liquid can pass freely from one to another. The water will rise in them so that all their surfaces will be in the same horizontal plane.

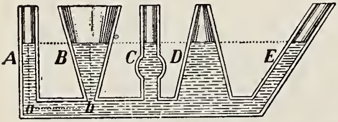


FIG. 225.—Surface of a liquid in connecting tubes in the same horizontal plane.

The reason for this can easily be given. Consider the vessels $A$ and $B$, and let $a$ and $b$ be two points in the liquid on the same horizontal plane. Now by hypothesis the liquid is at rest and therefore the pressure at $a$ toward $b$ is equal to that at $b$ towards $a$ since there is no motion from one to the other; but these pressures are equal only when $a$ and $b$ are at the same depth below the surface of the liquid. But the line $ab$ is horizontal, and hence **the surface of a liquid at rest is horizontal.**

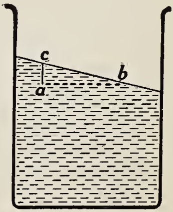


FIG. 226.—Diagram to show that the surface is horizontal.

It may be well, however, to consider this matter a little further. If it were

possible, suppose the surface of the liquid in a vessel not horizontal but inclined as in Fig. 226; and let *a* and *b* be on the same horizontal plane, *b* being on the surface and *a* below it. At *a* there is a downward pressure proportional to the depth *ac*, which pressure is transmitted in all directions. At *b* there is no fluid pressure; consequently the water particles will be forced toward *b* and the surface there will rise while that at *c* will sink until they all reach the same level.

**186. "Water Seeks its Own Level."** This is a familiar statement, and the cause of the water seeking its own level is the **force of gravity.** The water moves until its surface is at right angles to the direction of the force of gravity. Over a small area we consider the force of gravity to act in parallel vertical lines and the surface then would be a horizontal plane. But the force of gravity is directed towards the centre of the earth, and over an area of the earth's surface of considerable size, the radii of the sphere cannot be taken as parallel. As the surface of the water is at right angles to the directions of the force, that is, the earth's radii, the free surface of the liquid must be spherical. But the curvature is so slight that we do not notice it in the case of a pail of water, while when the body is a large one, like a great lake or the ocean, the curvature is evident enough.

**187. Calculation of Pressure: Examples.** 1. What is the pressure at a point, (*a*) 2 m. below, (*b*) 30 ft. below, the surface of water?

(*a*) Consider 1 sq. cm. of horizontal area at a depth of 2 m. Then the thrust upon this surface is equal to the weight of a vertical column of water standing upon it and reaching to the surface. Its volume = 200 c.c., and its weight = 200 gm. Hence the pressure = 200 gm. per sq. cm.

(*b*) Taking 1 sq. ft. of horizontal area at a depth of 30 ft., the volume of the vertical column upon this = 30 cu. ft., and its weight = 30 × 62.5 pd. = 1875 pd. Hence the pressure is 1875 pd. per sq. ft.

2. A tube is 10 m. long and 1 sq. cm. in cross-section. One end is screwed into the upper face of a cylindrical vessel of radius 7 cm. and height 2 cm. (Fig. 227). The tube and vessel are filled with water. Find

the weight of the water; also the pressure per sq. cm. as well as the whole thrust downwards upon the bottom.

FIG. 227.–Pressure, depth, and volume.

The volume of the cylinder $= \frac{22}{7} \times 7^2 \times 2 = 308$ c.c. The volume of the tube $= 1000$ c.c. Total volume $=$ 1308 c.c. and the weight of the water $= 1308$ gm. or 1.308 kg.

The bottom is 1002 cm. below the surface of the water. Consider a column standing on 1 sq. cm. and reaching upward 1002 cm.; its volume = 1002 c.c. and weight 1002 gm. Hence the pressure = 1002 gm. per sq. cm. The area of the base $= \frac{22}{7} \times 7^2 = 154$ sq. cm. Hence the whole thrust $= 154 \times 1002 = 154{,}308$ gm. or 154.308 kg.

We can solve the problem in a slightly different way. The thrust on the bottom will be equal to the weight of a column of water 154 sq. cm. in cross-section and 1002 cm. high or 154,308 c.c. Hence the whole thrust = 154,308 gm. The pressure $= 154{,}308 \div 154 = 1002$ gm. per sq. cm., as before.

**188. Hydrostatic Paradox.** The result just given illustrates the peculiar fact that by means of a small amount of a liquid we can obtain a very great pressure. In the case considered, if the liquid were conceived to become solid and to stand on the bottom of the vessel, the whole thrust downwards on the bottom would be its weight, or 1.308 kg.; but if the same matter is in the form of a liquid, the thrust downwards on the bottom is 154.308 kg. This result appears paradoxical, that is, seemingly absurd or contradictory.

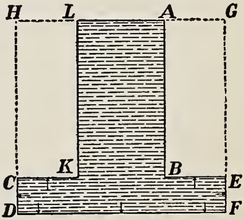



FIG. 228.—Explanation of the hydrostatic paradox.

Fig. 228 will help us to explain it. Consider a vessel of the shape shown, filled with water.

Let $AB = x$ cm. and $EF = y$ cm. and let the area of the bottom $= a$ sq. cm.

Then at the level $KB$ the pressure $= x$ gm. per sq. cm.

This pressure is transmitted, according to Pascal's Principle to every square centimetre of the bottom.*

But, in addition to this pressure on the bottom, we have a pressure of $y$ gm. per sq. cm. because of the water below the level $KB$.

Hence the pressure on the bottom is $(x + y)$ gm. per sq. cm. and the whole thrust is $(x + y)a$ gm.

**Consequently the whole thrust on the bottom is equal to the weight of liquid which would just fill the entire space *HDFG*, that is, the weight of a column of liquid standing on *DF* and reaching to the surface.**

## PROBLEMS

1. If the pressure of a liquid at a depth of 14 ft. 3 in. is 6 pd. to the sq. in., find the pressure at a depth of 21 ft. 8 in.

2. If the pressure at a depth of 5.6 metres is 2.8 gm. per sq. mm., what is the pressure at a depth of 7.5 cm.?

3. If the pressure on a sq. in. at a depth of 40 cm. is 10 pd., find the pressure 6 cm. lower down.

4. What is the pressure in gm. per sq. cm. at a depth of 100 m. in water? (Density of water, one gram per c.c.).

5. The area of the cross-section of the piston $P$ (Fig. 229), is 120 sq. cm. What weight must be on it to maintain equilibrium when the water in the pipe $B$ stands at a height of 3 m. above the height of the water in $A$?

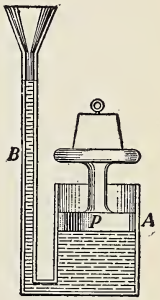


FIG. 229

6. The water pressure at a faucet in a house supplied with water by pipes connected with a distant reservoir is 80 pd. per sq. inch when the water in the system is at rest. What is the vertical height of the surface of the water in the reservoir above the faucet? (1 lb. water = 27.73 cu. in.)

7. Find the measure in pd. per sq. in. of the pressure at a point 72 ft. below the surface of a pool of water. (Density of water, $62\frac{1}{2}$ lbs. per cu. ft.)

*It may help the student, if he imagines a thin weightless piston across the vessel at the level $KB$.

8. A reservoir of water is 100 m. above the level of the ground-floor of a house. Find the pressure in gm. per sq. cm. of the water at a point in a water-pipe at a height of 10 m. above the ground-floor.

9. The pressure at a point within a body of water under the action of gravity is 100 pd. per sq. in. If the weight of a cu. ft. of water is 1000 oz., find the depth of the point below the surface.

10. The water in a canal lock rises to a height of 10 ft. against one side of a vertical flood-gate whose breadth is 12 ft. Find the thrust against it.

(In this case the pressure varies directly with the depth and hence the average pressure is equal to the pressure half-way down, that is, 5 ft. below the surface. The width is uniform and consequently the total force, or thrust, is equal to the area $\times$ average pressure.)

11. A rectangular box 2 cm. long, 1.5 cm. wide, and 8 mm. deep, is filled with water. Find the total force on (1) the bottom, (2) a side, (3) an end.

12. A rectangular vessel 80 cm. long, 20 cm. wide, and 60 cm. deep, supposed weightless, is placed on a horizontal table. Into its upper face is let perpendicularly a straight tube which rises to a height of 2 m. above this face, the internal cross-section of the tube being 1 sq. cm. The vessel and the tube are filled with water. Find the total force on (1) the bottom of the vessel, (2) a side, (3) an end, (4) the upper surface, (5) the table.

# CHAPTER XIX

## Buoyancy; Archimedes' Principle

**189. Buoyant Force of a Liquid.** We know very well that a liquid exerts an upward force upon a body which is either partially or completely immersed in it. A cork or a sea-gull bobs about on the surface of a lake, a heavy log floats on the river and is towed to the saw-mill, and even the great ship of ten thousand tons is supported by the water. In all these cases the object floats; its weight is entirely overcome by the upward force due to the water.

But an upward force is exerted also when the body is fully immersed. An expert swimmer can keep a drowning person from sinking, though out of the water he might not be able to lift the body at all. Sometimes in fishing, a heavy stone is attached to a rope and let down as an anchor. On pulling it up, to go to another place, comparatively little effort is needed as long as it is in the water, but it becomes decidedly heavier as soon as it comes to the surface.

Let us find out by experiment just how much of a body's weight is apparently lost when it is immersed in a liquid.

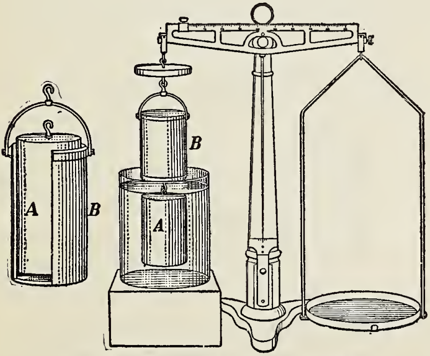


Fig. 230.—Determination of buoyant force.

**190. The Principle of Archimedes.** A suitable form of apparatus for the purpose is shown in Fig. 230. *A* is

a metal cylinder, closed at both ends, which fits exactly into a hollow socket *B*. Hook the cylinder to the bottom of the socket, suspend them from one end of the beam of a balance, and add weights to the other end to bring the balance to equilibrium. Next, surround *A* with water, as shown in the figure. The buoyancy of the water upon *A* destroys the equilibrium. Now carefully pour water in the socket *B*. It will be found that when *B* is just filled, equilibrium will be restored. The buoyant force is equal to the weight of the water displaced by the body.

This result has been obtained with water, but we might have used any other liquid, and it must also hold for a gas.

The apparatus just described is designed especially to demonstrate the law of buoyancy but we can easily dispense with it.

By means of a fine thread suspend a heavy body, such as a stone or a piece of iron, from one end of the balance and find its weight by placing weights on the other end. Let its weight be 158 grams. Then surround the body with water as in Fig. 231 and weigh again. Let the weight now be 137 grams. The buoyant force of the water is thus 158 − 137 = 21 grams. Next, lower the body into an overflow can (Fig. 232) and catch the overflow in a beaker or other vessel whose weight has been carefully determined. Weigh again and by subtraction find the weight of the water which has been displaced by the body. It will be found to be 21 grams.

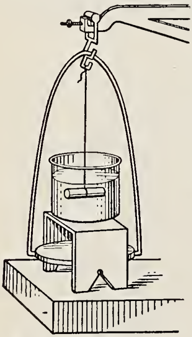


Fig. 231.—Finding the apparent loss in weight when a body is immersed in a liquid.

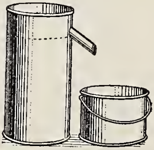


Fig. 232.—Overflow can.

If an overflow can is not available, lower the body into the water in a graduate and note the rise in the water. It will be found to be 21 c.c., the weight of which is 21 grams.

We therefore conclude:

**The buoyant force exerted by a fluid upon a body immersed in it is equal to the weight of the fluid displaced by the body;** or, in slightly different words,

**A body when weighed in a fluid loses in apparent weight an amount equal to the weight of the fluid which it displaces.**

This is known as the *Principle of Archimedes.*

It is stated that King Hiero of Syracuse, Sicily, suspected that his crown was not made of pure gold but contained some silver, and he asked the great scientist Archimedes (287-212 B.C.) to determine if such was the case. It is evident that if the volume of the crown were known, the weight, if of pure gold, could easily be calculated. If its actual weight was less, some substance lighter than gold must be combined with it. The solution of the problem was suggested to Archimedes by the buoyant action of the water when he was in a bath. According to tradition he leaped from the bath and rushed through the streets crying, "Eureka! Eureka!" (I have found it! I have found it!)*

**191. Theoretical Proof by Calculation.** Archimedes' principle is so important that a simple proof by calculation will be given. Consider a solid in the form of a cube to be immersed in water with its upper face horizontal (Fig. 233).

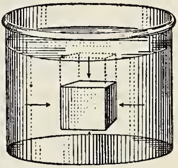


Fig. 233.—Buoyant force of a liquid on a solid.

Let the edge of the cube be 3 cm. in length and the upper face be 2 cm. below the surface.

Evidently the thrusts on the vertical sides balance, and the resultant vertical force due to the water

* See "A Short History of Science" by Sedgwick and Tyler (N.Y., 1918), page 113.

will be equal to the difference between the thrusts on the bottom and the top.

Now the thrust on the top is equal to the weight of a column of water standing on 9 sq. cm. and reaching to the surface, that is, having a height of 2 cm. The volume is 18 c.c. and the weight is 18 grams.

The thrust on the bottom (upwards) is equal to the weight of a column of water standing on 9 sq. cm. and reaching to the surface, that is, having a height of 5 cm. The volume is 45 c.c. and the weight is 45 grams.

Resultant thrust $= 45 - 18 = 27$ grams upwards.

But the volume of the cube is 27 c.c. and the weight of the water displaced by it $= 27$ grams.

**192. Principle of Flotation.** It is obvious that if the weight of a body **immersed** in a liquid is greater than the weight of the liquid displaced by it, the body will sink; but if less, the body will rise until it reaches the surface. Here it will come to rest when it has risen so much above the surface that the weight of the liquid then displaced is equal to the weight of the body.

**The weight of a floating body is equal to the weight of the liquid which it displaces when floating.**

It should also be observed that two parallel forces cannot be in equilibrium unless they act in the same straight line but in opposite directions. In the case of the floating body the weight acts downwards through the centre of gravity of the body, while the buoyant force acts upwards through the centre of buoyancy (that is, the centre of gravity of the fluid displaced). Consequently when floating the centre of gravity of the body must be in the same vertical line as the centre of buoyancy.

## PROBLEMS AND QUESTIONS

(Take 1 cu. ft. water = 62.5 lb.)

1. A cubic foot of marble which weighs 160 pounds is immersed in water. Find (1) the buoyant force of the water on it, (2) the weight of the marble in water.

2. Twelve cubic inches of a metal weigh 5 pounds in air. What is the weight when immersed in water?

3. If 3500 c.c. of a substance weigh 6 kg., what is the weight when immersed in water?

4. A piece of aluminium whose volume is 6.8 c.c. weighs 18.5 gm. Find the weight when immersed in a liquid twice as heavy as water.

5. A body whose volume is $2\frac{2}{5}$ cu. ft. weighs 420 pounds. Find its weight when $\frac{5}{6}$ of its volume is immersed in water.

6. A substance whose volume is $3\frac{1}{5}$ c.dm. weighs $7\frac{3}{5}$ kg. Find its weight when $\frac{3}{8}$ of its volume is immersed in a liquid one-half as heavy as water.

7. One c.dm. of wood floats with $\frac{3}{5}$ of its volume immersed in water. What is the weight of the cube?

8. A c.c. of cork weighs 250 mg. What part of its volume will be immersed if it is allowed to float in water?

9. A cu. in. of pine floats with $\frac{5}{7}$ of its volume in water. Find its weight.

10. A c.c. of poplar floats with $\frac{m}{n}$ of its volume out of water. Find its weight.

11. The weight of $2\frac{1}{4}$ cu. feet of elm is 124 pounds. What part of its volume will be immersed if it is allowed to float in water?

12. The weight of $6\frac{2}{3}$ c.dm. of cork is $1\frac{2}{3}$ kg. If it is allowed to float in water, how many c.dm. will remain above the surface?

13. A piece of wood whose mass is 100 gm. floats in water with $\frac{3}{4}$ of its volume immersed. What is its volume?

14. A piece of wood weighing 100 pounds floats in water with $\frac{2}{3}$ of its volume above the surface. Find its volume.

15. What is the least force which must be applied to a cu. ft. of larch which weighs 30 pounds that it may be wholly immersed in water?

16. A c.dm. of cork, weighing 480 gm., floats just immersed in water, when prevented from rising by a string attached to the bottom of the vessel containing the water. Find the tension of the string.

17. A cylindrical cup weighs 35 gm., its external radius being $1\frac{3}{4}$ cm., and its height 8 cm. If it be allowed to float in water with its axis vertical, what additional weight must be placed in it that it may sink?

18. A cylinder of wood, 8 in. long and weighing 15 pounds, floats vertically in water with 3 in. of its length above the surface. What is the tension of the string which will hold it just immersed in water?

19. The cross-section of a boat at the water-line is 150 sq. ft. What additional load will sink it 2 inches?

20. A scow with vertical sides is 25 ft. long and 12 ft. wide, and it sinks $2\frac{1}{2}$ in. when a team of horses walks on it. Find the weight of the team.

21. Why will an iron ship float on water, while a piece of the iron of which it is made sinks?

22. A vessel of water is on one scale-pan of a balance and counterpoised. Will the equilibrium be disturbed if a person dips his fingers into the water without touching the sides of the vessel? Explain.

23. A piece of coal is placed in one scale-pan of a balance and iron weights are placed in the other scale-pan to balance it. How would the equilibrium be affected if the balance, coal and weights were now placed under water? Why?

24. A block of wood 1 in. square and 6 in. long is tied at one end to the bottom of a tank on the inside. Mercury is poured into the tank until the block, when standing vertically, is just half immersed; then water is poured in until the block is entirely covered.

(*a*) Does the tension of the string that holds the block down change as the water is being poured in? Give reason for the answer.

(*b*) Would the tension of the string have been different had mercury been used instead of water? Why?

# CHAPTER XX

## Density and Specific Gravity

**193. Mass per Unit Volume.** Obtain cylindrical pieces of brass, iron and wood, having flat ends; also a cylindrical vessel such as a tin can.

Measure the diameters and lengths of the cylinders in cm. and calculate their volumes. Also measure the internal diameter of the can and its height up to a mark, in inches, and calculate its volume.

Weigh the cylinders in grams and calculate the mass per c.c. of each of the substances.

Weigh the vessel in pounds; then fill with water up to the mark and weigh again. Calculate the mass of the water per cu. in.

| | | | |
|---|---|---|---|
| **Examples.**—(*a*) Cylinder of iron. | Diam. | = | 2.34 cm. |
| | Length | = | 8.42 " |
| | By calculation, volume | = | 3.62 c.c. |
| | By weight, mass | = | 273.03 grams. |
| | Whence mass of 1 c.c. of iron | = | 7.54 grams. |
| (*b*) Tin vessel. | Diam. | = | 3.12 in. |
| | Height | = | 4.00 in. |
| | By calculation, volume | = | 30.58 c. in. |
| | By weight, mass | = | 1.10 lb. |
| | Whence mass of 1 c. in. of water | = | 0.036 " |
| | and " of 1 c. ft. " " | = | 62.2 " |

**The mass per unit volume of a substance is its density.** Thus, the density of the iron used is 7.54 grams per c.c.; that of water is 0.036 lb. per cu. in. or 62.2 lb. per cu. ft.*

**194. Density and Specific Gravity.** As we have just seen, the density of a body is its mass per unit volume.

---

* More accurately, 62.4 lb. per cu. ft. at 4° C. It is usual, however, to take 1 cu. ft. of water as 62.5 lb. or 1000 oz. For table of densities, see Appendix.

**The specific gravity of a substance is the ratio which the weight of a given volume of it bears to the weight of an equal volume of water.**

$$\text{Or, specific gravity} = \frac{\text{weight of body}}{\text{weight of equal volume of water}}.$$

As this is just a ratio it is expressed by a simple number, and is independent of any system of units; but it is related to density in the following way:

Let $W$ pd. = weight of a given volume (say 1 cu. ft.) of the substance

and $w$ pd. = weight of the same volume of water.

$$\text{Then sp. gr.} = \frac{W}{w},$$

$$= \frac{\text{density of substance}}{\text{density of water}}.$$

If now we use the C.G.S. system of units the density of water = 1 gm. per c.c.; and the number which expresses the specific gravity will also be the measure of the density. This, however, will not be the case if we use the F.P.S. units, as then the density will be the number of pounds per cu. ft., while the specific gravity will be the same as before.

**Example.**—Suppose the volume of a piece of cast-iron is 50 c.c. and that its weight is 361 gm. Find its specific gravity and its density.

The weight of 50 c.c. of water = 50 gm.

Therefore the sp. gr. of the iron $= \frac{361}{50} = 7.22$, which is the measure of the weight in grams of 1 c.c. of iron, or its density.

In the F.P.S. system the specific gravity is the same, but the density = $62.5 \times 7.22 = 451.25$ pounds per cu. ft.

## PROBLEMS

1. Find the mass of 140 c.c. of silver if its density is 10.5 gm. per c.c.

2. The specific gravity of sulphuric acid is 1.85. How many c.c. must one take to weigh 100 gm.?

3. A piece of granite weighs 83.7 gm. On dropping it into the water in a graduated vessel, the water rises from 130 c.c. to 161 c.c. (Fig. 234). Find the density of the granite.

4. A tank 50 cm. long, 20 cm. wide and 15 cm. deep is filled with alcohol of specific gravity 0.8. Find the weight of the alcohol.

5. A rectangular block of wood 5 x 10 x 20 cm. in dimensions weighs 770 gm. Find the density.

6. The specific gravity of pure milk is 1.086. What is the density of a mixture containing 500 c.c. of pure milk and 100 c.c. of water?

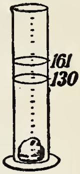


Fig. 234.

7. A body whose mass is 60 gm. is dropped into a graduated tube containing 150 c.c. of water. If the body sinks to the bottom and the water rises to the 200 c.c. mark, what is the density of the body?

**195. Specific Gravity of a Solid Heavier than Water.** The specific gravity of a solid body can easily be found by an application of Archimedes' principle.

Suspend the body whose specific gravity is to be determined from one end of a balance by means of a fine thread and find how much it weighs in air. Then bring the vessel containing water under the body and raise it until the body is fully immersed (Fig. 231, Sec. 190). Weigh again.

Let $m$ gm. = wt. of the body in air,

and $m_1$ gm. = wt. of the body in water.

Then $m - m_1$ gm. = loss of weight in water,

= wt. of an equal volume of water.

Hence, $$\text{sp. gr.} = \frac{m}{m - m_1},$$

= density in gm. per c.c.

**Example.**—A piece of iron weighed in air 263.5 gm., and in water 226.4 gm. Find its specific gravity.

Here loss of weight in water = 37.1 gm.,

= wt. of equal vol. of water.

Hence, sp. gr. $= \frac{263.5}{37.1} = 7.10$,

and the density of the iron = 7.10 gm. per c.c.

**196. Specific Gravity of a Solid Lighter than Water.** Select a heavy body which will cause the light body to sink in the water when attached to it, and proceed as follows:

1st. Weigh the body in air. Let the weight = $m$ grams.

2nd. Attach the sinker to hang below the body. Weigh both, with the sinker only in the water. Let this weight = $m_1$ grams.

3rd. Weigh them when both are in the water. Let the weight = $m_2$ grams.

Now the only difference between the 2nd and 3rd operations is that in the former the body is weighed in air, in the latter in the water. The sinker is in the water in both cases.

Hence, $m_1 - m_2$ = buoyancy of the water on the body,
= wt. of the water displaced by the body,

and the sp. gr. $= \dfrac{m}{m_1 - m_2}$.

**197. The Specific Gravity Bottle.** The specific gravity bottle, one form of which is shown in Fig. 235, is specially adapted for finding the specific gravity of liquids. The procedure is as follows:

1st. Weigh the bottle empty = $m$ grams.
2nd. Weigh it filled with water = $m_1$ "
3rd. Weigh it filled with the liquid = $m_2$ "

Then the water which fills the bottle weighs $m_1 - m$ grams,
and the liquid which fills it weighs $m_2 - m$ "

Then the sp. gr. of the liquid $= \dfrac{m_2 - m}{m_1 - m}$.

FIG. 235.—Specific gravity bottle.

**Example.**—A bottle empty weighed 21.10 gm.; when filled with water, 71.22 gm.; when filled with alcohol, 61.73 gm. Find the sp. gr. of the alcohol.

Weight of water filling bottle = 50.12 gm.
Weight of alcohol filling bottle = 40.63 "

Hence, sp. gr. $= \dfrac{40.63}{50.12} = 0.81.$

**198. Specific Gravity of a Liquid by Archimedes' Principle.** In finding the specific gravity of a liquid by Archimedes' principle take a heavy body (say, a glass stopper) and weigh it in air, when immersed in water and when immersed in the liquid.

Let weight of sinker in air $= m$ grams
" " " " water $= m_1$ "
" " " " liquid $= m_2$ "

Then weight of water displaced by sinker $= m - m_1$ grams
and " " liquid " " " $= m - m_2$ "

Then the sp. gr. of liquid $= \dfrac{m - m_2}{m - m_1}$.

This also expresses the density of the liquid in grams per c.c.

**Example.**—A glass stopper weighed 100 gm. in air, 60 gm. in water and 70 gm. in gasoline. Find the sp. gr. of the gasoline.

Wt. of water displaced by stopper = 40 gm.
" " gasoline " " " = 30 "
Hence, sp. gr. of gasoline $= \dfrac{30}{40} = 0.75$.

**199. The Hydrometer.** The approximate specific gravity of a liquid is a quantity which it is often necessary to determine quickly and for this purpose an instrument known as a **hydrometer** has been devised.

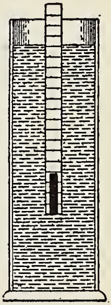


Fig. 236.—Illustration of the principle of the hydrometer.

The principle underlying its action may be illustrated as follows. Take a straight rod of wood, of cross-section 1 sq. cm. and (say) 25 cm. long, and bore a hole in one end. After inserting enough shot to make the rod float upright in water, plug up the hole. After this, mark on one of the long faces a centimetre scale, and dip the rod in hot paraffin to render it impervious to water. Now place the rod in water (Fig. 236) and suppose it to sink to a depth of 16 cm. Then the weight of the rod = weight of water displaced = 16 grams.

Again place it in a liquid whose density is to be determined, and suppose it to sink 20 cm.

FIG. 237.—The hydrometer.

Then the volume of the liquid displaced = 20 c.c., and this = wt. of the rod = 16 grams.

Then the specific gravity of the liquid = $\frac{16}{20}$ = 0.80.

It is evident, also, that the rod could be marked so as to indicate the specific gravity directly. Thus,

| | |
|---|---|
| for readings | 12, 16, 20 cms. |
| the sp. gr. is | 1.25, 1.00, 0.80. |

For commercial purposes the hydrometer is usually constructed in the form shown in Fig. 237. At the end of a slender stem *B* is a float *A*, and a little chamber *C* which contains mercury and makes the instrument take an upright position when in a liquid. The graduations are either on the outside of the stem or on a paper within it. The weight and volume are so adjusted that the instrument sinks to the mark at the lower end of the stem when in the densest liquid to be tested, and to the mark at the upper end when in the least dense liquid. The scale is marked so as to indicate directly any density between the limits chosen. By making the float *A* much larger than the stem the instrument is rendered more sensitive.

As the range of an instrument of this class is necessarily limited, special instruments are constructed for use with different liquids. Thus one instrument is used for testing the density of milk, another for the acid in a storage battery, and so on.

FIG. 238.—Storage battery hydrometer.

That for testing a storage battery is illustrated in Fig. 238. The lower end is thrust into the battery and, by pressing the rubber bulb and letting it go, enough acid is drawn into the tube to float the hydrometer. The depth to which it sinks shows the general condition of the liquid in the battery.

## PROBLEMS

1. A body whose mass is 6 gm. has a sinker attached to it and the two together weigh 16 gm. in water. The sinker alone weighs 24 gm. in water. What is the density of the body?

2. A body whose mass is 12 gm. has a sinker attached to it and the two together displace when submerged 60 c.c. of water. The sinker alone displaces 12 c.c. What is the density of the body?

3. A uniform wooden rod 5 cm. square and 30 cm. long is loaded so that it floats upright in water with 20 cm. below the surface. If the rod were placed in alcohol (s.g. 0.8) what length of it would be below the surface?

4. If a body when floating in water displaces 12 c.c., what is the density of a liquid in which when floating it displaces 18 c.c.?

5. A piece of metal whose mass is 120 gm. weighs 100 gm. in water and 104 gm. in alcohol. Find the volume and density of the metal, and the density of the alcohol.

6. A hydrometer floats with $\frac{2}{3}$ of its volume submerged when floating in water, and $\frac{3}{4}$ of its volume submerged when floating in another liquid. What is the density of the other liquid?

7. A cylinder of wood 8 in. long floats vertically in water with 5 in. submerged. (*a*) What is the specific gravity of the wood? (*b*) What is the specific gravity of the liquid in which it will float with 6 in. submerged? (*c*) To what depth will it sink in alcohol whose density is 0.8?

8. A mass of lead is suspected of being hollow. It weighs 2486 gm. in air and 2246 gm. in water. What is the volume of the cavity? (s.g. of lead = 11.3.)

9. How much silver is contained in a gold and silver crown whose mass is 407.44 gm., if it weighs 385.44 gm. in water? (Density of gold 19.32 and of silver 10.52 gm. per c.c.).

10. The mass of a piece of limestone (sp. gr. = 2.637) is 256.34 gm. What is its apparent weight in water?

11. The apparent weight of a mineral when weighed in water is 195.46 gm. If its specific gravity is 2.678, what is its mass?

12. Find the apparent weight of 5 c.c. of gold (sp. gr. = 19.3) in mercury (sp. gr. = 13.6).

13. What is the least weight that must be placed upon a cu. ft. of cork (sp. gr. = 0.25) that it may float totally immersed in a liquid whose specific gravity is 0.9?

14. What is the least weight that must be placed upon a piece of wood weighing 20 pounds and floating with $\frac{3}{5}$ of its volume immersed in a liquid whose specific gravity is 1.5 that it may be totally immersed?

15. A cylinder of cork weighs 10 gm. and its specific gravity is 0.25. Find the least force that will immerse it (1) in water, (2) in a liquid whose specific gravity is 0.75.

16. A body (sp. gr. = 0.5) floats on water. If the weight of the body is 1 kg., find the number of c.c. of it above the surface of the water.

17. A body floats in a fluid (sp. gr. = 0.9) with as much of its volume out of the fluid as would be immersed if it floated in a fluid (sp. gr. 1.2). Find the specific gravity of the body.

18. A cubical block of wood (sp. gr. = 0.6) whose edge is 1 ft. floats, with two faces horizontal, down a fresh water river out to sea, where a fall of snow takes place, causing the block to sink to the same depth as in the river. If the specific gravity of the sea water is 1.025, find the weight of the snow on the block.

19. A ship, of mass 1000 tons, goes from fresh water to salt water. If the area of the section of the ship at the water-line is 15,000 sq. ft., and her sides vertical where they cut the water, find how much she will rise, taking the specific gravity of sea water as 1.026.

20. A beaker partly full of water is balanced accurately on the scales; then a piece of lead (sp. gr. = 11) weighing 66 gm., held by the hand at the end of a fine thread, is lowered into the water without touching the glass. What weight must be added to the opposite side to restore equilibrium?

21. *A*, *B*, and *C* are three beakers filled to the top with water:

(1) A block of wood weighing 30 gm. (sp. gr. = 0.4) is placed in *A*.

(2) A piece of lead measuring 3 c.c. (sp. gr. = 11) rests at the bottom of *B*.

(3) The lead and wood are fastened together and placed in *C*.

Find the change (if any) that has taken place in the weight of each beaker, giving full explanation in each case.

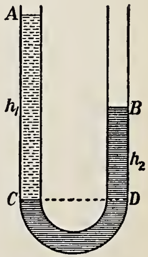


FIG. 239.—Liquids of unequal density in a bent tube.

**200. Liquids in a Bent Tube.** It is possible to find the density of one liquid with respect to another by means of a bent tube, provided the liquids do not mix.

Pour the liquids in and allow them to come to equilibrium (Fig. 239). Let *A* and *B* be their free surfaces and *C* their common surface.

It is evident that the liquid $AC$ is not so dense as the other.

Let $d_1$, $d_2$ (gm. per c.c.) be the densities, and $h_1$, $h_2$ (cm.) be the heights of the free surfaces above the horizontal plane $CD$ drawn through their common surface.

Since the liquids are in equilibrium, it is evident that the pressure at $C$ = the pressure at $D$.

The pressure per sq. cm. at $C$ = the weight of a column of liquid of density $d_1$ gm. per c.c., having a cross-section 1 sq. cm. and height $h_1$ cm.

$$= d_1 \times h_1 \text{ gm. per sq. cm.}$$

Similarly the pressure per sq. cm. at $D$

$$= d_2 \times h_2 \text{ gm. per sq. cm.}$$

Hence, $$d_1 \times h_1 = d_2 \times h_2,$$

or $$d_1/d_2 = h_2/h_1.$$

Hence, when the liquids are in equilibrium their densities are inversely as the heights of their free surfaces above their common surface.

**Example.**—Let the column $AC$ be oil and the rest water. By measurement $$h_1 = 20 \text{ cm.}, h_2 = 18 \text{ cm.}$$

Hence, density of oil, $d_1 = \frac{18}{20}$ density of water,

$$= 0.9 \text{ gm. per c.c.}$$

## PROBLEMS

1. Two liquids which do not mix are contained in a bent tube. If their specific gravities are 1.2 and 1.8 respectively, and the height of the first above their common surface is 15 in., find the height of the other.

2. In a bent tube a column of mercury (sp. gr. 13.6) is balanced by a column of alcohol (sp. gr. 0.8). If the height of the former is (1) 4 cm., (2) 10 cm., (3) 15 cm., what in each case is the height of the latter?

3. Two tanks are connected by a pipe. Into one tank is poured salt water (sp. gr. 1.03), and into the other a very light oil (sp. gr. 0.5). The oil is found to be 5 ft. above their common surface. Find the height of the water.

4. Mercury and ether are poured into a bent tube. The mercury stands 5.25 cm. when the ether stands 100 cm. above their common surface. If the density of the ether is 0.715 gm. per c.c., what is the density of the mercury?

5. Two liquids that do not mix are contained in a bent tube. The difference of their levels is 40 cm. and the height of the denser above their common surface is 70 cm. Compare their densities.

6. If water and a denser liquid which does not mix with it are placed in a U-tube, the internal cross-section of which is 1 sq. cm., the difference of their levels is found to be 4 cm., and the height of the liquid above their common surface is 10 cm. What is the specific gravity of the liquid?

## REVIEW QUESTIONS

1. State Pascal's Principle. How would you demonstrate the truth of the Principle?

2. State three laws governing the pressure in a liquid at rest under the action of gravity.

3. What is meant by the "hydrostatic paradox"? Explain it.

4. State Archimedes' Principle and tell how you would demonstrate it experimentally.

5. State the principle of flotation. How would you verify it?

6. Distinguish between density and specific gravity.

7. How would you use Archimedes' Principle to find:

   (*a*) the s.g. of a substance heavier than water;
   (*b*) the s.g. of a substance lighter than water;
   (*c*) the s.g. of a liquid?

8. How could you find the specific gravity of mercury by using a specific gravity bottle, if the mercury at your disposal filled only part of the bottle?

# CHAPTER XXI

## Pressure of the Air—The Barometer

**201. Air has Weight.** Though we cannot see the air, we are fully convinced that it is a substance which actually exists and is quite as real as the solid soil or the water of the ocean. The air offers a resistance to the rapidly moving automobile or railway train; and were it not a real substance the aeroplane could not soar upon it or drive itself forward by its propeller. Sometimes great trees are blown over or mountainous waves are raised upon the sea. These disturbances are not due to some imaginary force but are caused by real masses of matter sweeping forward over the surface of the earth.

However, the air is so thin and fluid that we might almost expect it to escape the laws of weight. We speak of a thing being as "light as air"; but it is not difficult to demonstrate that air has weight and that its weight is not so small as many people seem to think.

From an ordinary (not gas-filled) electric light bulb the air has been carefully removed and the space within is almost a perfect vacuum. Take one of these bulbs (one with a broken filament) and, having heated the butt-end in a flame, remove the brass plug. Then, by means of a delicate balance, weigh the glass portion which is left. Next make a scratch on the sharp glass tip with a file and then by a smart tap break off the tip. This will make a hole in the bulb and the air will rush in and fill it. Now weigh the bulb, including the tip, again. The weight will be distinctly greater than before.

**Example.**—The following measurements were made:

| | |
|---|---|
| Weight of bulb at first........ | 24.572 gm. |
| Weight of bulb + air......... | 24.755 " |
| Increase in weight........ | 0.183 " |

An attempt was also made to measure the capacity of the bulb and then to calculate the weight of 1 litre of air.

On forcing the bulb down into water in a graduate the entire volume was 160 c.c. Now the density of glass is about 2.5 grams per c.c. and the mass of the bulb is 24.572 grams; hence, the volume occupied by the glass = 24.572 ÷ 2.5 = 10 c.c. (approx.).

Consequently the capacity of the bulb = 160 − 10 = 150 c.c.

Hence, 150 c.c. of air weighed 0.183 grams.

The temperature was 19.5° C. and the barometer read 73 cm.

Using the laws of expansion of a gas, we find that the weight of 1 litre at 0° C. and 76 cm.

$$= \frac{1000 \times 0.183}{150} \times \frac{292.5}{273} \times \frac{76}{73} = 1.36 \text{ grams.}$$

The experiment to determine the weight of a given volume of air can be performed more satisfactorily with the gas flask shown in Fig. 240. It can be connected to an air-pump for removing the air from the globe, or to one for forcing air into it; and a stop-cock allows the vessel to be made air-tight.

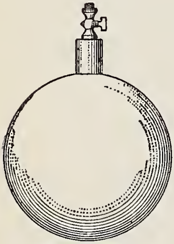


Fig. 240.—Globe for weighing air.

First, let the air be removed from the vessel and the stop-cock closed, and then let it be weighed. Then admit air and weigh again. Finally, cautiously force air into it and weigh a third time. The third weight will be greater than the second and the second greater than the first. The capacity of the flask can be determined by filling it with water and weighing it.

By observing the temperature of the gas, the pressure to which it is subjected and the barometric pressure, the weight of the gas at standard temperature and pressure can be determined. Careful experiments show that

**1 litre of air at 0° C. and 76 cm. pressure = 1.293 grams.**

From this we find that 1 cu. ft. = 1.28 ounces or 12 cu. ft. = 1 lb. (approximately). Consequently the air in a room 20 × 24 × 15 ft. weighs 600 lb. It is not so light after all.

**202. Pressure of the Atmosphere.** The reason why the air is heavy is because it is attracted to the earth by the force of gravity, and just as liquids exert pressure upon all surfaces with which they are in contact, so must the air do the same. The bed of the ocean is subjected to enormous pressure by the water above it, and in the same way the surface of the earth must sustain a pressure from the aerial ocean which rests upon it. As we have seen, the pressure in the water is directly proportional to its depth; in the atmosphere the pressure becomes less the higher above the earth one goes. Thus the pressure at sea-level at Halifax, N.S., or Victoria, B.C., is greater than in the Rocky Mountains.

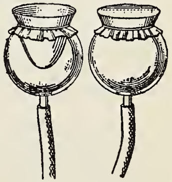


FIG. 241. — Rubber membrane forced inwards by the pressure of the air.

That the atmosphere exerts pressure can be demonstrated by many simple experiments. Tie a piece of thin sheet rubber over the mouth of a thistle-tube (Fig. 241) and exhaust the air from the bulb by suction or by means of an air-pump. As the air is exhausted the rubber is pushed inward by the pressure of the outside air. Again, fill a bottle with water, and place a sheet of writing paper over its mouth. Then, holding the paper in position with the palm of the hand, invert the bottle (Fig. 242). The pressure of the atmosphere against the paper prevents the water from running out. Numerous other experiments can be performed to show the same effect.

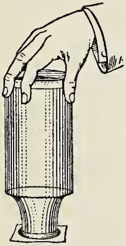


FIG. 242.—Demonstrating atmosphereic pressure.

If one end of a tube is thrust into water and the air is withdrawn from it by suction, the water rises in the tube. This phenomenon was known for ages and was accounted for by the simple statement that **nature abhors a vacuum.** In 1640 the Grank Duke of Tuscany dug a deep

well, but found that the water could not be raised more than 32 feet above its level in the well. He applied to the aged scientist Galileo for an explanation, and though the latter had proved that air had weight he did not connect that fact with the problem. He simply inferred that the horror felt by nature had its limitations. After his death the problem was solved by his pupil Torricelli, who was made, by the Grand Duke, professor of mathematics in the Academy of Florence, in succession to Galileo. Torricelli showed definitely that the reason why water rose in the pump only to a height of 32 feet was because the pressure of the atmosphere was not able to push it up any higher.

## PROBLEMS AND EXERCISES

1. Fill a tumbler and hold it inverted in a dish of water as shown in Fig. 243. Why does the water not run out of the tumbler into the dish?

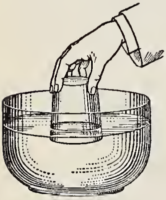


FIG. 243.

2. Take a bent glass tube of the form shown in Fig. 244. The upper end of it is closed, the lower open. Fill the tube with water. Why does the water not run out when it is held in a vertical position?

3. Why must an opening be made in the upper part of a vessel filled with a liquid to secure a proper flow at a faucet inserted at the bottom? Can the water be emptied from a flexible rubber bag if the bag has a single small opening in it?

FIG. 244.

4. Fill a narrow-necked bottle with water and hold it mouth downward. Explain the action of the water.

5. On the tin top upon a pot of jam is sometimes seen the instruction:—"To open, puncture and push up at edge." Give the reason for this.

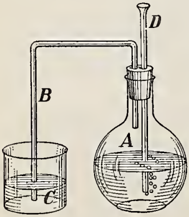



FIG. 245.

6. Boil water in a flask $A$ arranged as in Fig. 245, conducting the steam through the tube $B$ into cold water $C$. Remove the heat. Observe

and explain what happens. Next remove the tube *D*, plugging the hole in the cork through which it passes. Repeat the experiment and explain what happens.

7. Explain the action of a fountain-pen filler or medicine dropper (Fig. 296).

8. A new half-gallon tin can had half an inch of water placed in it. This water was boiled vigorously for two minutes or longer, the can was removed from the heater, and the cap screwed on tightly while the steam was still escaping. The action which took place when the can cooled is shown in Fig. 246. Explain.

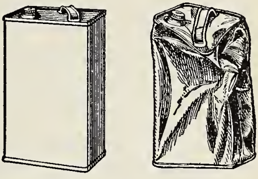

(*a*) (*b*)

FIG. 246.— (*a*) Can before cooling; (*b*) after cooling.

9. A flask weighs 280.60 gm. when empty, 284.19 gm. when filled with air and 3060.60 gm. when filled with water. Find the weight of 1 litre of air.

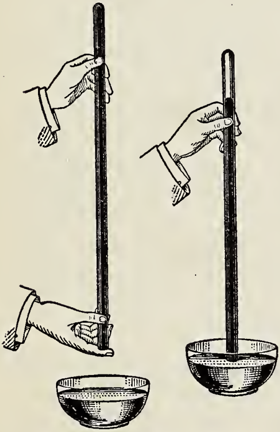


FIG. 247.—Mercury column sustained by the pressure of the air.

**203. Torricelli's Experiment.** Torricelli reasoned that since a water column rises to a height of 32 ft., and since mercury is about 14 times as heavy as water, the atmosphere would be able to support a mercury column only about $\frac{1}{14}$ of 32 ft., or approximately 28 in. in height. Under his direction Vincenzo Viviani, one of his pupils, performed an experiment similar to the following:

Take a glass tube about 1 metre (39 inches) long (Fig. 247), closed at one end, and fill it with mercury. Then, stopping the open end with the finger, invert it and place it in a vertical position, with the open end under the surface of the mercury in a bowl. Remove the finger. The

mercury will fall in the tube, and, after oscillating up and down, will come to rest with the surface of the mercury in the tube between 28 and 30 inches (71 and 76 cm.) above the surface of the mercury in the bowl.

The experiment resulted precisely as Torricelli expected, and conclusively showed that the column of mercury was sustained by the pressure of the atmosphere upon the surface of the mercury in the bowl. The empty space above the mercury is called a Torricellian vacuum.

When a report of this experiment reached France it created a sensation among the scientists there, but it was not repeated by them until 1646, as no suitable tubes were available before that date. In that year the experiment was performed by Pierre Petit, of Rouen, in conjunction with the great Pascal, who concluded "that the vacuum is not impossible in nature, and that she does not shun it with so great a horror as many imagine." Pascal reasoned that if the mercury column is simply held up by the pressure of the air the column should be shorter at a higher altitude. He asked his brother-in-law, Périer, who resided at Clermont, in the south of France, to test it on the Puy-de-Dôme, a near-by mountain over 1000 yards high. Using a tube about 4 ft. long, which had been filled with mercury and then inverted in a vessel containing mercury, Périer found that the column fell over 3 inches (8 cm.) on going to the summit. Later Pascal tried the experiment at the base and the top of the tower of Saint-Jacques-de-la-Boucherie, in Paris, which is about 150 feet high. There was a difference of 2 lines (about 0.5 cm.).

**204. The Barometer.** In his experiments Torricelli says he aimed "not simply to produce a vacuum, but to make an instrument which shows the mutations of the air, now heavier and dense, now lighter and thin"; and the modern mercury barometer, which is designed to measure the pressure of the atmosphere, is similar in principle to that constructed by Torricelli. Two forms of the instrument are in common use.

**205. The Cistern Barometer.** This is simply a convenient arrangement of the original Torricellian experiment. The bowl, or cistern, and the tube are permanently mounted on a board, and a scale, engraved on the metal case protecting the glass tube, shows the height of the mercury in the tube above the surface of the mercury in the cistern.


Fig. 248.—The cistern barometer.

A common form of the instrument is shown in Figs. 248, 249. The cistern has a flexible leather bottom which can be raised or lowered by a screw *C*, in order to adjust the level of the mercury. Before taking the reading, the screw is turned until the tip of the pointer *P* (which is the zero of the scale on the case) just touches the mercury. To do this, the level is slowly changed until the image of the tip just reaches the tip itself. The height of the column is then read directly from the scale on the case, the reading being made with accuracy by the assistance of a vernier.

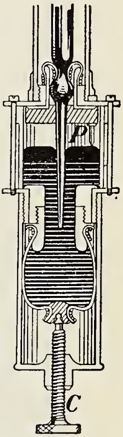



Fig. 249.—Section of the cistern.

In constructing a barometer of this kind the mercury must be very pure, since impure mercury has a different specific gravity and besides it adheres to the glass. Also, all bubbles of air and of moisture must be carefully removed. In order to do this the mercury is boiled. First, the tube is filled about one-third full of mercury which is then boiled over a charcoal fire or a large gas flame. Then more is added and the boiling continued, until at last the whole is thoroughly boiled. The temperature of boiling is so high (357° C.) that all the air and the moisture are completely removed. The

operation is sometimes shortened and made easier by a suitable arrangement whereby an air-pump removes much of the air and the moisture, and also causes the mercury to boil at a lower temperature.

**206. The Siphon Barometer.** This barometer consists of a tube of sufficient length, sealed at one end and bent into **U**-shape at the other (Fig. 250). When filled with mercury and held in an upright position the mercury in the long closed tube falls until the atmospheric pressure on the open end is just sufficient to balance a column of mercury extending from the level in the open tube to the level in the closed tube. A scale is attached to or engraved upon each branch. The upper scale gives the height of the mercury in the closed branch above a fixed point and the lower scale gives the distance of the mercury in the open branch below the same point. The sum of the two readings is the height of the barometric column.

Fig. 250.—Siphon barometer.

**207. The Aneroid* Barometer.** In this barometer no liquid is used. The air presses upon the flexible corrugated cover of

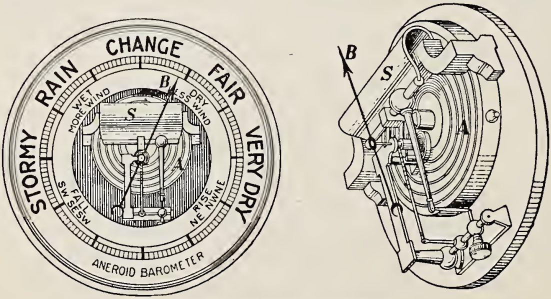



Fig. 251.—Aneroid barometer.

*Aneroid, from Greek *a* = not, *neros* = wet.

a circular, air-tight metal box *A* (Figs. 251, 252) from which the air has been partially exhausted. The cover, which is usually supported by a spring *S*, responds to the pressure of the atmosphere, being forced in when the pressure is increased and coming outwards when it is decreased. The movement of the cover is very small but it is multiplied and transmitted to an index hand *B* by a system of delicate levers and a chain, or by gear wheels. The circular scale is graduated by comparison with a mercury barometer.

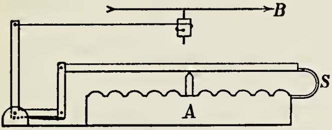


Fig. 252.—Simplified diagram of aneroid barometer.

The aneroid is not so accurate as the mercury barometer, but it is very portable and very sensitive and is in very common use. It is specially serviceable in determining differences of level. A good aneroid will indicate a fall in pressure in going from the cellar to the attic of a house.

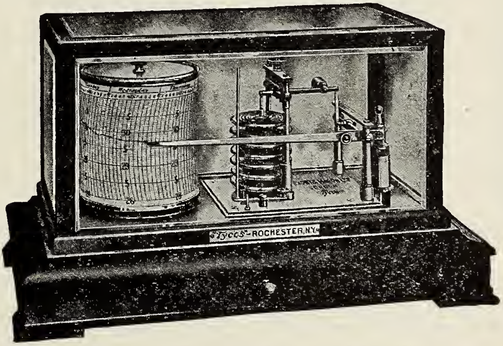


Fig. 253.—Barograph or self-recording barometer.

On the face of the aneroid barometer is often seen the words, "stormy, rain, change, fair, very dry." They have little meaning, and the barometer by itself cannot indicate

with certainty the nature of the coming weather. However, there are some laws which have been found to hold. If the barometer falls rapidly we may expect strong winds; and if it is low, rain or snow is likely to fall. If it is rising, fine weather is probably coming, and if it stays high and steady, fine weather is likely to continue. The barometer is highest in calm, clear, cold winter weather. Barographs, or self-registering barometers, have been devised on the principle of the aneroid. In these an index carries a pen which makes a continuous record upon a strip of paper on a revolving drum. (Fig. 253).

**208. Calculation of Atmospheric Pressure.** If we know the barometric height at a given place we can calculate the pressure of the atmosphere there. For example, suppose the barometer stands at 76 cm. (Fig. 254). Then the pressure of the atmosphere at $A$ is equal to the pressure of the mercury at $B$. Consequently to find the atmospheric pressure in grams per sq. cm. we have only to find the weight of a column of mercury 1 sq. cm. in section and 76 cm. in height, that is, the weight of 76 c.c. of mercury.

Fig. 254.

Now mercury expands as its temperature rises, and consequently the weight of 1 c.c. of mercury depends on the temperature. The following table gives the values for three temperatures:

| Temperature. | | Wt. of 1 c.c. | Wt. of 76 c.c. |
|---|---|---|---|
| −2° C. = | 28°.4 F. | 13.600 grams. | 1033.600 grams. |
| 0 | 32 | 13.596 " | 1033.296 " |
| 25 | 72 | 13.534 " | 1028.584 " |

In a barometer like that illustrated in Fig. 248 the scale is engraved on the brass case. Now this case increases in length as the temperature rises, and if we desire to determine accurately the pressure of the atmosphere we must make allowance for this too. It is usual to read the height of the mercury and also the temperature indicated by the thermometer attached to the case, and then to reduce the reading

to zero, that is, to determine what the reading would be if the temperature fell to zero. If, further, it is desired to compare the atmospheric pressures at various places, as is done in the Meteorological Service, it is usual also to reduce the readings to sea-level, that is, to determine what the readings would be if the barometers were lowered down deep holes until they reached the level of the sea. In order to facilitate these reductions, tables have been prepared from which the corrections to be applied for various temperatures and altitudes may be found without much labour.

To illustrate the amount of these corrections the following example is given:

Temperature 72° F. = 25° C.
Altitude 1000 ft. = 304.8 metres.

| | in. | mm. |
|---|---|---|
| Reading of barometer | 28.900 | 735.6 |
| Correction for temperature | −.113 | −3.0 |
| | 28.787 | 732.6 |
| Correction for altitude | +1.02 | +25.8 |
| Reading at freezing point and sea-level........... | 29.807 | 758.4 |

## PROBLEMS

(In the following questions the density of mercury is to be taken as 13.6 gm. per c.c. or as 7.858 oz. per. cu. in. = 848 lb. per cu. ft.)

1. Find the atmospheric pressure per sq. in. when the mercury barometer stands at 30 in.

2. Find the pressure of the atmosphere on a square centimetre when the mercury barometer stands at 76 cm.

3. Three barometers are constructed to use liquids whose specific gravities are respectively 7.2, 2.9, and 11.8. Find the atmospheric pressure on a sq. inch (1) when the first barometer stands at 4.8 ft., (2) when the second stands at 11.52 ft., (3) when the third stands at 87.63 cm.

4. Three barometers are constructed to use liquids whose specific gravities are respectively 13.6, 5.17, and 2.06. Find the atmospheric pressure on 1 sq. cm., (1) when the first barometer stands at 70 cm., (2) when the second stands at 2 m., (3) when the third stands at 5 m.

5. If in ascending a mountain the barometer falls from 30 in. to 20 in., find the decrease in the total force exerted by the atmosphere on an area of 10 sq. ft.

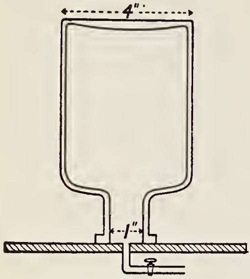



Fig. 254*a*—Vertical section of a bottle on the plate of an air-pump.

6. The density of mercury being 13.6 gm. per c.c., find the pressure of the atmosphere in dynes per sq. cm. when the barometer stands at 75 cm.

7. The outer diameter of the mouth of the bottle shown in section in Fig. 254*a* is $1\frac{1}{2}$ in. and the inner diameter is 1 in. The bottom of the bottle is 4 in. in diameter. The mouth, which is ground smooth, is placed upon the plate of an air pump and the air is removed from within. If the bottle weighs 1 pd. and the barometer stands at 30 in.,

(*a*) What force will be required to lift the bottle off?

(*b*) What is the pressure exerted by the bottle on the plate?

**209. Determination of Heights by the Barometer.** Since the pressure of the atmosphere decreases with the elevation above sea-level it is evident that the barometer may be used to measure the difference between the altitudes of two places. Aneroid barometers are actually used to determine heights in reconnaissance surveying, and the altimeter of an

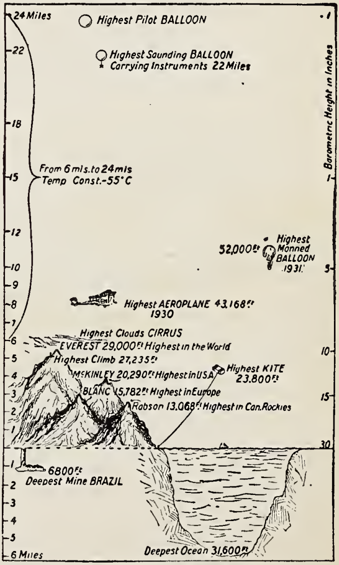



Fig. 255.—Atmospheric conditions at different heights.

aeroplane consists of an aneroid barometer calibrated in feet of elevation.

If the density of the air was uniform, its pressure, like that of liquids, would vary directly with its depth; but the air is very compressible and the lower layers are much denser than those above them. As a consequence the relation between barometric height and altitude is somewhat complicated.

**It has been found that for small elevations a fall of 1 inch in the mercury column corresponds to a rise of 900 ft. in elevation.**

For heights less than 1000 metres (3280 ft.) the following formulas have been found to hold:

Let $H$, $h$ be the barometric heights at the lower and upper stations, and $T$, $t$ be the temperatures at the lower and upper stations.
Then, if $T$ is in degrees Fahr.,

$$\text{Difference in height} = 52{,}494 \left(\frac{H-h}{H+h}\right)\left(1+\frac{T+t-64}{900}\right) \text{ feet.}$$

If $T$ is in degrees centigrade,

$$\text{Difference in height} = 16{,}000 \left(\frac{H-h}{H+h}\right)\left(1+\frac{2(T+t)}{1000}\right) \text{ metres.}$$

Fig. 255 shows roughly the conditions of atmospheric pressure at heights up to 24 miles.

**210. Buoyancy of Gases.** It is evident that Archimedes' principle applies to gases as well as to liquids. A simple experiment to demonstrate the buoyant force of air is illustrated in Fig. 256. A hollow metal or glass globe $A$ is suspended from one end of a short balance beam and is counterpoised by a small weight $B$. If the air exerts a buoyant force, as in a liquid, the force upward on $A$ must be greater than that on $B$, and if the air be removed from about the balance the globe $A$ should sink. On putting the apparatus under the receiver of an air-pump and exhausting the air, the globe at once sinks.

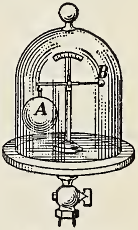


FIG. 256.—Buoyancy of air.

**A gas exerts upon a body immersed in it a buoyant force which is equal to the weight of the gas displaced by the body;** and, of course, if this buoyant force is greater than the weight of the body, the body will rise.

**211. Balloons and Air-Ships.** The use of air-ships or balloons is made possible by the buoyancy of the air. A balloon is a large, light, gas-tight bag filled with some gas lighter than air, usually hydrogen or illuminating gas. Helium is the ideal gas for the purpose as it will not take fire, but up to the present comparatively little of this gas has been available. In Fig. 257 is illustrated the great British air-ship

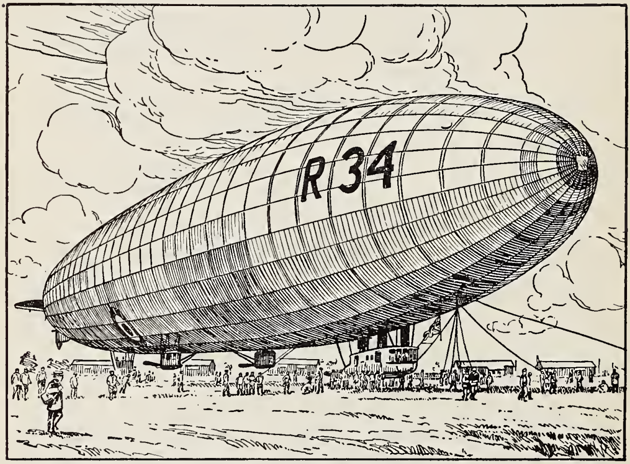



Fig. 257.—The British air-ship R-34, the first to cross the ocean. It left East Fortune, Scotland, July 2, and reached Long Island, N.Y., July 6, 1919. Time of flight, 108 hours. The return was made in 75 hours. Length, 672 ft.; diameter, 79 ft.; volume 2,000,000 cu. ft.; crew and passengers, 30.

R-34, which, during the summer of 1919, made the journey from Great Britain to the United States and back. By means of propellers these air-ships can be driven in any desired direction.

A balloon will continue to rise so long as its weight is less than the weight of the air which it displaces, and when there is a balance between the two forces, it simply floats at a constant height. In the case of an ordinary balloon the aeronaut maintains his position by adjusting the weight of the balloon to the buoyancy of the air. When he desires to ascend, he throws out ballast. To descend he allows gas to escape and thus decreases the buoyancy. The power airships can be made to rise or sink or turn to the right or left by means of suitable elevators and rudders.

It will be interesting to compare the lifting powers of a balloon filled with different gases. Let its capacity be 80,000 cu. ft. (1699 cu. metres). If it were spherical the diameter would be 48.6 ft. Balloons were originally nearly spherical in shape, though most nowadays are in the form of a 'sausage.'

The weight of 1 cu. m. of hydrogen = 0.09 kg.; of helium, 0.18 kg.; of illuminating gas, 0.75 kg.; of air, 1.29 kg. (at standard pressure and temperature).

Hence the weight of 1699 cu. m. of hydrogen = 152.9 kg.; of helium, 305.8 kg.; of illuminating gas, 1274.3 kg.; and the same volume of air weighs 2191.7 kg., which is the buoyant force of the air (neglecting the volume of the material of the balloon and its basket).

The lifting force, therefore, if the balloon is filled with

| | | | | | | |
|---|---|---|---|---|---|---|
| Hydrogen | = 2191.7 | − | 152.9 | = | 1938.8 | kg. |
| Helium | = 2191.7 | − | 305.8 | = | 1885.9 | " |
| Illuminating gas | = 2191.7 | − | 1274.3 | = | 917.4 | " |

Thus the lifting power of helium is about $\frac{11}{12}$, while that of illuminating gas is $\frac{4}{7}$ that of hydrogen.

**212. Height of the Atmosphere.** There are several ways of obtaining an estimate of the height of the atmosphere, but no means of determining that height accurately. From twilight effects a height of about 40 miles has been calculated. It would seem that above this height the air ceases to reflect light, but other evidence shows that it extends far beyond. Meteors, or shooting stars, which consist of small masses of matter made incandescent by the heat produced as they rush

through the atmosphere, have been observed at heights of over 100 miles. The aurora borealis, or northern lights, is probably a phenomenon in our atmosphere, and measurements of brilliant displays seen in the north of Norway show that it usually attains a height of 110 km. (70 mi.) and sometimes as much as 600 miles.

## QUESTIONS AND PROBLEMS

1. Why should the gas-bag be subject to an increased strain from the pressure of the gas within as the balloon ascends?

2. Aeronauts report that balloons have greater buoyancy during the day when the sun in shining upon them than at night when it is cold. Account for this fact.

3. If the volume of a balloon remains constant, where should its buoyancy be the greater, near the earth's surface or in the upper strata of the air? Give reasons for your answer.

4. An aluminium block is placed on one pan of a balance and a lead weight on the other and they are in equilibrium? The whole is put in a vessel and the air removed from it. Describe what happens and explain why. What would happen if the balance were lowered into water?

5. The volume of a balloon is 2,500 cu. m. and the weight of the gas-bag and car is 100 kg.; find its lifting power when filled with hydrogen gas, the density of which is 0.0000899 gm. per c.c. while that of air is 0.001293 gm. per c.c.

6. Find the lifting power of the same balloon when filled with helium, which is twice as dense as hydrogen.

7. If the balloon were filled with illuminating gas, which is 8 times as dense as hydrogen, would it rise? If so, find the lifting power.

8. A balloon had a capacity of 80,000 cu. ft. The gas-bag, net about it, and the basket together weighed 985 pounds. How great a load could it carry when filled with hydrogen? (1 cu. ft. air = 0.08 lb.; of hydrogen = 0.0056 lb.)

9. The ordinary balloons used during the Siege of Paris in 1870 had a capacity of about 70,000 cu. ft. and the weight of the balloon and car was about 1000 pd. Find the lifting power when filled with coal gas whose density is 0.4 that of air.

# CHAPTER XXII

## Boyle's Law and The Kinetic Theory of Gases

**213. Compressibility and Expansibility of Gases.** We have already referred to the fact that gases can be compressed and that they will expand if allowed to do so. Indeed this is the distinguishing feature of a gas. A solid has both a definite volume and a definite shape; a liquid has a definite volume but no definite shape,—it will take the shape of the vessel which holds it; but a gas has neither definite volume nor definite shape (Sec. 172). If a quantity of gas is introduced into a closed vessel it will spread out and go into every corner of it, no matter what the shape may be. If the stopper be removed from a bottle containing ammonia we soon smell the pungent odour of ammonia gas everywhere in the room; or if hydrogen sulphide be introduced into a building (for instance, in natural gas) it before long reveals its undesired presence in all parts of the house.

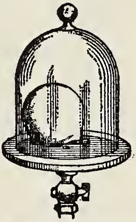


Fig. 258 —When the air is removed from the receiver the toy balloon expands.

In its efforts to escape, the gas exerts a pressure against the walls of the vessel enclosing it. This can be illustrated in the following way. Place a toy balloon or a half-inflated rubber from a football under the receiver of an air-pump and operate the pump. (Fig. 258.) As the air about the bag is continually removed, the bag expands; and when the air is admitted again the bag resumes its former volume.

To account for this we imagine the bag to be the seat of two contending factions,—the troops of molecules within endeavouring to keep back the invading hosts of molecules without. Incessantly they rush back and forth, perpetually

striking against the surface of the bag. As the enemies are withdrawn by the action of the pump, the defenders within gain the advantage and, pushing forward, enlarge their boundary which at last, however, becomes so great that the outsiders can again hold it in check.

**The never-ceasing impact of the molecules of the gas against a surface produces the pressure exerted by the gas. This view of a gas is known as the *Kinetic Theory of Gases*.**

### QUESTIONS AND EXERCISES

1. Arrange apparatus as shown in Fig. 259. By suction remove a portion of the air from the flask, and, keeping the rubber tube closed by pressure, place the open end in a dish of water. Now open the tube. Explain the action of the water.

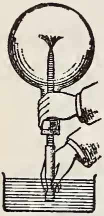


Fig. 259.

Fig. 260.

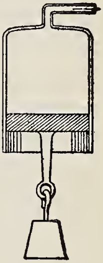


Fig. 261.

2. Guericke took a pair of hemispherical cups (Fig. 260) about 1.2 ft. in diameter, so constructed that they formed a hollow air-tight sphere when their lips were placed in contact; and at a test at Regensburg before the Emperor Ferdinand III and the Reichstag, in 1654, showed that it required sixteen horses (four pairs on each hemisphere), to pull the hemispheres apart when the air was exhausted by his air-pump. Account for this.

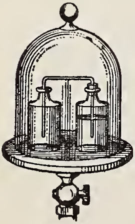


Fig. 262.

3. If an air-tight piston is inserted into a cylindrical vessel and the air exhausted through the tube (Fig. 261), a heavy weight may be lifted as the piston rises. Explain this action.

4. A rubber tube with thin walls collapses when used for connecting an air-pump with a vessel from which the air is being withdrawn. Explain.

5. A bottle partly filled with water is closed with a perforated cork and connected by a bent tube with an uncorked bottle as shown in Fig. 262. Explain what happens when this arrangement is placed under the receiver of an air-pump and the air is exhausted.

**214. Effect of a Rise in Temperature.** If we place the rubber bag used in the last experiment in an oven, it expands, showing that the pressure upon the inner surface of the bag has increased. Now there are the very same molecules within—no increase in the number—and we must conclude that **a rise in temperature causes the molecules to move with greater speeds, and this produces the increased pressure.**

A very good analogy is the action of a number of bees placed in a closed vessel provided with a glass window through which their movements can be observed. At low temperatures the bees are quite dormant but as the vessel is gradually heated they become very active indeed.

**215. Relation Between Volume and Pressure of a Gas—Boyle's Law.** It is a matter of importance to know the change produced in the volume of a given mass of gas when it is subjected to different pressures. This relation was first determined experimentally in 1660 by the distinguished Irish chemist, Robert Boyle (1627-1691).

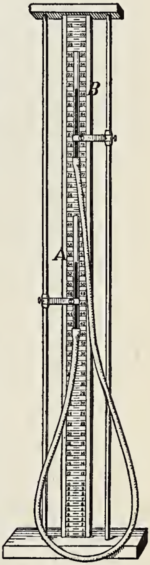


FIG. 263.—Boyle's Law apparatus. *A*, closed tube; *B*, open tube.

The apparatus shown in Fig. 263 is suitable for the investigation of this relation.

Two glass tubes, *A* and *B*, are supported in such a way that either may be raised or lowered. The upper end of *A* is closed, that of *B* is open, and their lower ends are joined by a heavy rubber tube. The rubber tube and part of *A* and *B* are filled with mercury. The tube *A* is of uniform bore and the volume of the air may be taken proportional to the length of the tube occupied by it, this being obtained from the scale against which it is placed.

First, adjust the amount of mercury so that when it is at the same level in both glass tubes $A$ is about half-full of air, which should be dry.

Read the barometer and record its height in cm. of mercury. When the mercury is at the same level in both tubes the air is under the pressure of one atmosphere, *i.e.*, the pressure shown by the barometer.

Now lower $B$ as far as it will go. Do this rather slowly. The temperature of the gas should remain the same, and a rapid change in volume produces a change in the temperature. Then read the levels of the mercury in the two tubes. The level of the mercury in the open tube is below the level of that in the closed tube and the pressure to which the imprisoned gas is subjected is now 1 atmosphere *minus* the difference in the levels of the mercury. Raise $B$ a few centimetres and take the readings again. Continue this until $B$ is as high as it can go. When the level of $B$ is above that of $A$ the pressure on the imprisoned air is 1 atmosphere *plus* the difference in level.

The tube $A$ should not be handled for fear of raising the temperature of the inclosed air; and, as has already been remarked, the air should not be compressed or expanded quickly.

The following results were obtained with such an apparatus:

| Level of Mercury in | | Difference between the Levels | Height of Barometer | Total Pressure in cm. of Mercury $= P$ | Length of Air in Tube $= V$ | Product $P \times V$ |
|---|---|---|---|---|---|---|
| Closed Tube | Open Tube | | | | | |
| 46.2 | 7.6 | −38.6 | 74.6 | 36.0 | 30.1 | 1083.6 |
| 51.6 | 21.0 | −30.6 | " | 44.0 | 24.7 | 1086.8 |
| 55.3 | 32.3 | −23.0 | " | 51.6 | 21.0 | 1085.7 |
| 59.2 | 48.1 | −11.1 | " | 63.5 | 17.1 | 1085.8 |
| 61.7 | 61.7 | 0.0 | " | 74.6 | 14.6 | 1089.2 |
| 63.8 | 76.3 | +12.5 | " | 87.1 | 12.5 | 1088.7 |
| 65.5 | 91.9 | +26.4 | " | 101.0 | 10.8 | 1090.8 |
| 66.9 | 107.7 | +40.8 | " | 115.4 | 9.4 | 1084.7 |
| 68.0 | 124.8 | +56.8 | " | 131.4 | 8.3 | 1090.6 |
| 68.5 | 132.7 | +64.2 | " | 138.8 | 7.8 | 1082.6 |

Reading of top of closed tube, 76.3 cm.

**From this experiment we learn that the pressure and volume vary in such a way that the product $P \times V$ is constant.** If the pressure is doubled the volume becomes half as great, if the pressure is multiplied threefold, the volume becomes one-third, etc. In other words,

**If the temperature is kept constant, the volume of a given mass of air varies inversely as the pressure to which it is subjected.**

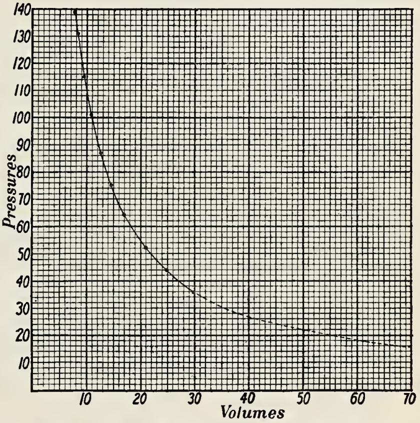



**Fig. 264.**—Graph showing how volume of a gas changes with the pressure.

This relation is generally known as *Boyle's Law*. In France it is called Mariotte's Law, because it was independently discovered by a French physicist named Mariotte (1620-1684), fourteen years after Boyle's publication of it in England.

Fig. 264 shows the graph obtained by plotting the results given in the above table. This curve, whose equation is $PV =$ a constant, is a rectangular hyperbola.

**216. Alternative Boyle's Law Experiment.** Fig. 265 illustrates another form of apparatus suitable for demonstrating Boyle's Law.

It consists of the large reservoir $A$ to which are connected the glass tube $B$, which is closed at the top and the metal tube $C$ on which is mounted the pressure gauge $D$. The large cylinder is partly filled with a light oil which rises in $B$ and $C$ when air is pumped into $A$ through the valve $E$. The volume of the air in $B$ is given by the scale mounted alongside $B$ and the corresponding pressure is read on the pressure gauge. Pressures less

than atmospheric may be obtained by exhausting air from the space above the oil in $A$.

This piece of apparatus is easily manipulated, the use of mercury is avoided and it is not necessary to read the barometer. If a good pressure gauge is used the results compare very favourably with those obtained by the method previously described.

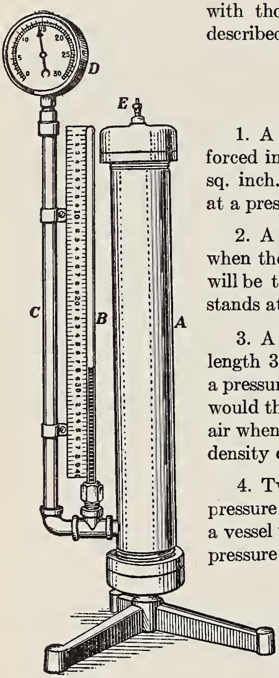


Fig. 265.—Ahrens' apparatus for Boyle's Law.

## PROBLEMS

1. A tank whose capacity is 2 cu. ft. has gas forced into it until the pressure is 250 pd. per sq. inch. What volume would the gas occupy at a pressure of 75 pd. per sq. inch?

2. A gas-holder contains 22.4 litres of gas when the barometer stands at 760 mm. What will be the volume of the gas when the barometer stands at 745 mm.?

3. A cylinder whose internal dimensions are: length 36 in., diameter 14 in., is filled with gas at a pressure of 200 pd. per sq. inch. What volume would the gas occupy if allowed to escape into the air when the barometer stands at 30 in. (Take density of mercury as 848 lb. per cu. ft.).

4. Twenty-five cu. ft. of gas, measured at a pressure of 29 in. of mercury, is compressed into a vessel whose capacity is $1\frac{1}{2}$ cu. ft. What is the pressure of the gas?

5. A mass of air whose volume is 150 c.c. when the barometer stands at 750 mm. has a volume of 200 c.c. when carried up to a certain height in a balloon. What is the reading of the barometer at that height?

6. A piston is inserted into a cylindrical vessel 12 in. long, and forced down within 2 in. of the bottom. What is the pressure of the inclosed air if the barometer stands at 29 in.?

7. The density of the air in a gas-bag is 0.001293 gm. per c.c. when the barometer stands at 760 mm.; find its density when the barometric height is 740 mm.

8. An open vessel contains 100 gm. of air when the barometer stands at 745 mm. What mass of air does it contain when the barometer stands at 755 mm.?

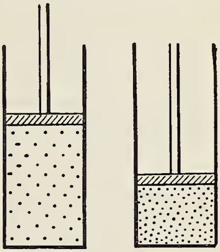
9. Oxygen and other gases, used for welding and other purposes, are stored in steel tanks. The volume of a tank is 6 cu. ft., and the pressure of the gas at first was 15 atmospheres. After some had been used the pressure was 5 atmospheres. If the gas is sold at 6 cents a cu. ft., measured at atmospheric pressure, what should be charged for the amount consumed?

10. In one form of sounding apparatus a slender glass tube closed at one end is lowered, open end down, to the bottom of the ocean, and an ingenious arrangement allows one to see to what height the water has risen in the tube. Suppose that the tube is 45 cm. long and the water rises to within 1.5 cm. of the closed end.

(*a*) What pressure (in atmospheres) has the inclosed air been subjected to?

(*b*) Taking the barometric height to be 76 cm.; the sp. gr. of mercury to be 13.6 and that of sea-water to be 1.026, find the depth of the water.

**217. Explanation of Boyle's Law.** This law naturally follows from the kinetic theory of gases. Suppose a certain quantity of a gas is in a cylinder closed by a piston, and let the gas at first occupy any definite volume (Fig. 266*a*). The molecules dart about in all directions and maintain a pressure upon the inner surface, which exactly balances the downward push of the piston. Next, let the piston be thrust down until the volume is one-half as great (Fig. 266*b*). Then the number of molecules within this space is twice as great as before and the blows delivered against its sides are twice as numerous, and consequently the pressure exerted by the gas is twice as great. In the same way, if the volume is reduced to $\frac{1}{n}$th part, the pressure exerted will be $n$ times as great.



(*a*) (*b*)
Fig. 266.—Pressure in *B* is twice that in *A*.

Let successive volumes be $v, \frac{1}{2}v, \frac{1}{3}v, \ldots\ldots \frac{1}{n}v.$

Then corresponding pressures are $p, 2p, 3p, \ldots\ldots np.$

Now $$p \times v = 2p \times \tfrac{1}{2}v = 3p \times \tfrac{1}{3}v = \ldots\ldots np \times \tfrac{1}{n}v,$$

that is, the pressure $\times$ the volume is constant $= k$ (say).

Then $$p = k\frac{1}{v}, \text{ or } p \text{ varies inversely as } v.$$

Now if the volume of a gas is reduced to $\frac{1}{2}$, its density becomes 2 times as great; if to $\frac{1}{3}$, its density is 3 times as great; if to $\frac{1}{n}$th, its density is $n$ times as great. From this we see that the density varies inversely as the volume. Consequently we say that **the pressure exerted by a gas is directly proportional to its density.**

This is simply another statement of Boyle's Law.

nit **218. The Speed of the Molecules.** The average speed of the molecules of a gas at a given temperature may be calculated in the following way. Consider a cubical vessel (Fig. 267), 1 cm. to the edge, containing a certain quantity of gas which, of course, exerts equal pressures on all the surfaces. Though the motions of the particles are indiscriminately in all directions, striking one surface and rebounding from it to strike another or perhaps to collide with another molecule; yet it seems reasonable to assume that the pressures on the six sides of the cube would be maintained if the molecules were divided into three equal sets; one set moving continually back and forth parallel to $AB$ and keeping up a bombardment against the two sides perpendicular to $AB$; the second moving parallel to $AC$ and bombarding the two sides perpendicular to $AC$; the third moving parallel to $AD$ and bombarding the two sides perpendicular to $AD$; and the molecules all moving with a speed which is the average of all the speeds.

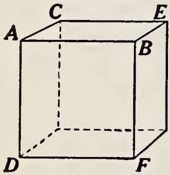


Fig. 267.—Speed of molecules of gas in a cubical vessel.

Let us consider the first set, namely those moving parallel to $AB$, and taking $n$ to be the total number of molecules in the cube, the number of the first set will be $\frac{1}{3}n$.

Let the speed of the molecules be $V$ cm. per sec., and the mass of each be $m$ grams. Each moving molecule will possess a momentum $mV$.

Suppose a molecule to strike against the side $CD$ with speed $V$; we assume that it rebounds with the same speed. In this way a momentum

$mV$ in one direction on impact is changed into one of equal amount in the opposite direction, or the change of momentum at one impact $= 2mV$. Now the speed is $V$ cm. per sec., and the molecule will travel across the cube and back, a distance of 2 cm., in the $\frac{2}{V}$th part of a sec. In 1 sec. it will do this $\frac{V}{2}$ times, that is, each molecule will make $\frac{V}{2}$ impacts against a side every second, and as in each impact there is a change of momentum of $2mV$, it is clear that in 1 sec. upon each side 1 sq. cm. in area there will be produced a change of momentum,

$$\tfrac{1}{3}n \times \frac{V}{2} \times 2mV = \tfrac{1}{3}nmV^2.$$

This gives rise to a pressure $p$ (say) upon the side.

According to Newton's Second Law,

$$\text{Force} = \text{Rate of change of momentum.}$$

In the present case, force $= p$ (dynes per sq. cm.),

time $= 1$ sec.

Change of momentum $= \frac{1}{3}nmV^2$ units.

Hence, $p = \frac{1}{3}nmV^2$ dynes per sq. cm.

Now $nm =$ entire mass of the molecules in 1 c.c.,

$= \rho$, the density of the gas.

Hence, $p = \frac{1}{3}\rho V^2$, or $V = \sqrt{\frac{3p}{\rho}}$.

This velocity is not strictly the average of all the velocities but is the square root of the mean square velocity.

We see then that $p$ varies as $\rho$ if $V$ is constant, that is if the temperature is constant. This is Boyle's Law.

Let us now calculate the velocity for a gas,—for example, hydrogen, under standard pressure and temperature.

For hydrogen, $\rho = 0.0000899$ gm. per c.c.

Also $p = 1033.296 \times 980$ dynes per sq. cm. (Sec. 208),

and hence $V = \sqrt{\frac{3 \times 1033\ 296 \times 980}{0.0000899}} = 183{,}820$ cm. per sec.

In the same way the velocity for any other gas may be calculated.

TABLE OF MOLECULAR VELOCITIES

| Gas | Velocity |
|---|---|
| Hydrogen................ | 1838 m. = 6032 ft. per sec. |
| Nitrogen................. | 493 " = 1618 " " " |
| Oxygen ................ | 461 " = 1514 " " " |
| Carbon Dioxide........... | 393 " = 1291 " " " |

For a fuller treatment of the kinetic theory of gases see Maxwell's "Theory of Heat," Chapter XXII, or Edser's "Heat for Advanced Students," Chapter XIII.

## PROBLEMS

1. 22.4 litres of nitrogen, oxygen and carbon dioxide weigh 28, 32 and 44 gm., respectively. Find the molecular velocities for each of these gases and compare with the values given in the above table.

2. Find the velocity of a molecule of helium taking the density of helium as twice that of hydrogen.

# CHAPTER XXIII

## Air-Pumps and Air Appliances.

**219. Air-Pump.** In Fig. 268 is shown the construction of a common type of pump used for removing the air from a vessel.

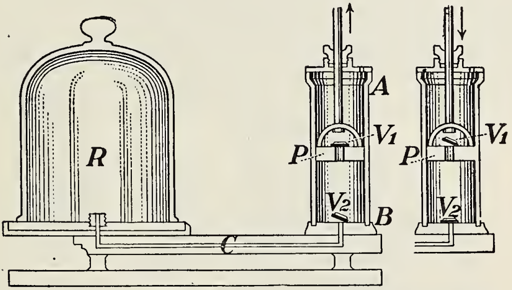



Fig. 268—Common form of air-pump. *AB*, cylindrical barrel of pump; *R*, receiver from which air is to be exhausted; *C*, pipe connecting barrel with receiver; *P*, piston of pump; $V_1$ and $V_2$, valves opening upwards.

Its operation is as follows:—When the piston $P$ is raised, the valve $V_1$, in it, remains closed, due to its own weight and the pressure of the air above it. The air in $R$ expands, and some of it passes by way of the pipe $C$, into the lower portion of the barrel, lifting the valve $V_2$ in doing so. When the piston descends, the valve $V_2$ remains closed, and the air in the barrel passes up through the valve $V_1$ and escapes outside. Thus at each up-and-down stroke a fraction of the air is removed from the receiver, $R$. The pump will continue to act until the air on expanding from $R$ is no longer able to lift the valve $V_2$, or when the pressure of the air below the piston is insufficient to raise the valve $V_1$. It is evident, therefore, that only a partial vacuum can be obtained with a pump of this kind. To secure more complete exhaustion, pumps have been constructed in which the valves are opened and closed automatically as the

piston moves, but even with these the air cannot all be removed from the receiver, since at each double-stroke the air in it is reduced only by a fraction of itself.

**220. The Geryk or Oil Air-Pump.** This pump is much more efficient than that just described. Its action is as follows:—The piston *J* (Fig. 270), made air-tight by the leather washer *C* and by being covered with oil, moves up and down in the cylinder. The tube *A*, opening into the chamber *B* surrounding the cylinder, is connected to the vessel from which the air is to be removed. On rising, the piston pushes before it the air in the cylinder, and on reaching the top it pushes up *G* about ¼ inch, thus allowing the imprisoned air to escape through the oil into the upper part of the cylinder, from which it passes out by the tube *D*.

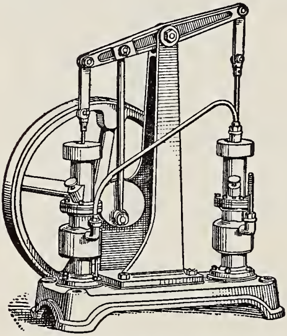


Fig. 269.—An oil air-pump with two cylinders.

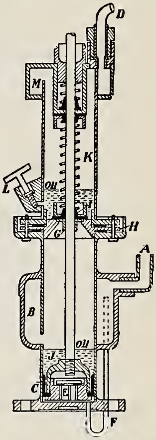



Fig. 270.—Vertical section of a cylinder of an oil air-pump.

When the piston descends the spring *K*, acting upon the packing *I*, closes the upper part of the cylinder, and the piston on reaching the bottom drives whatever oil or air is beneath out through the tube *F*, or allows it to go up through the valve *E*, into the space above the piston.

Oil is introduced into the cylinder at $L$. When the pump has two cylinders they are connected as shown in Fig. 269. With one cylinder the pressure of the air can be reduced to $\frac{1}{4}$ mm. of mercury, while with two a reduction to $\frac{1}{500}$ mm., it is claimed, can be quickly obtained.

**221. Rotary Air-pump.** This new type of pump (Fig. 271) will exhaust air to a pressure of 0.001 mm. It is of light weight, and is reliable and quiet in operation. It is also much more rapid in its action than the ordinary air-pumps.

Within the outer case of the pump is a fixed hollow cylinder provided with an inlet tube $E$ and an outlet $F$, which is fitted with a valve $L$.

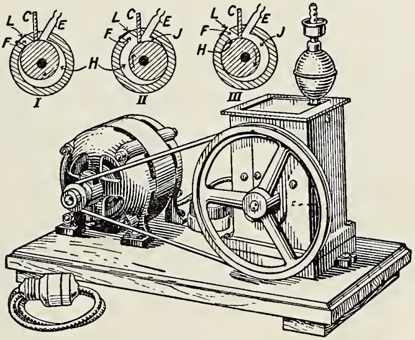



FIG. 271.—A rotary air-pump. In $I$, $II$, $III$ is shown a vertical section of the cylinder at different stages of a rotation.

Inside this cylinder is a second cylinder mounted eccentrically on an axle which is driven by a large pulley. As the inner cylinder rotates, it is always in contact with a portion of the outer cylinder. A metal plate $C$ works up and down through a slot cut in the outer cylinder, always resting on the rotating cylinder. The pump case is filled with oil so that only the inlet tube $E$ projects.

In position $I$ the space $H$ is in communication with the inlet tube $E$, which is connected to the vessel from which the air is being removed. As the cylinder rotates in the direction of the arrow to position $II$, the air in $H$ is cut off from that in the vessel and is compressed, while air from the vessel expands into the space $J$. In position $III$ most of the air in $H$ has been driven out through the valve $L$ while the space $J$ is nearing its

maximum size. As the rotation continues, position $I$ is reached again and the cycle repeats.

**222. The Condensation Vacuum Pump.** The wonderful uses made of highly-exhausted glass bulbs in investigations into the nature of matter, in the production of X-rays, in the wireless telegraph, in the radio, and for other purposes, has led to the invention of several kinds of high-vacuum pumps. A recent and rapid type, known as Langmuir's condensation pump, is constructed on a new principle, which may be explained with the help of Fig. 272.

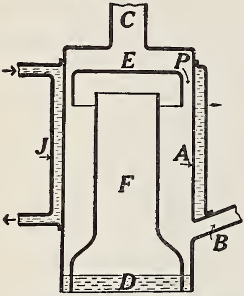



Fig. 272.—Diagram to explain Langmuir's condensation pump.

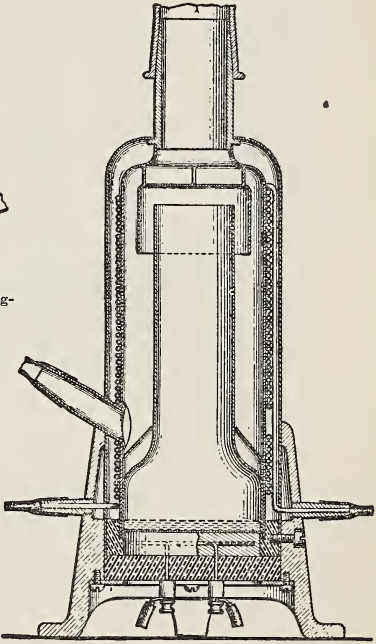


Fig. 273.—Showing actual construction of the condensation pump.

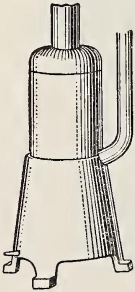


Fig. 274. — Outer view of the condensation pump.

A metal cylinder $A$ is provided with two openings, $B$ and $C$. The latter is connected to the vessel to be exhausted and the former is joined to an auxiliary pump which, itself, must be able to produce a vacuum of

about 0.1 mm. Within the cylinder is a funnel-shaped tube $F$ which rests on the bottom of $A$. Suspended from the top of $A$ is a cup $E$, inverted over the upper end of $F$. A water-jacket $J$ surrounds the wall of $A$ from the level of $B$ to a height somewhat above the lower edge of the cup $E$. Mercury is placed in the cylinder, as indicated at $D$.

By applying heat to the bottom of the cylinder the mercury is caused to evaporate. The vapour passes up through $F$, and, being deflected by $E$, is directed downwards and outwards against the water-cooled wall of $A$. The gas as it comes from the vessel which is being exhausted enters the pump at $C$, passes down between $E$ and $A$ and at $P$ meets the stream of mercury vapour which forces it down along the wall of $A$ and out of the tube $B$ where the auxiliary pump takes it and removes it. The mercury which condenses on the water-cooled wall falls downwards and returns to $D$, ready to be vaporized again.

A detailed drawing of one form of the pump is given in Fig. 273. In the base is a simple electric heater which produces the mercury vapour. Around the wall of the pump and just inside the outer casing is a coiled tube through which water is kept running. The ends of this tube are seen projecting outwards near the base. The tube leading to the auxiliary pump is higher up on the left, while the tube from the vessel which is being exhausted is at the top. The outer appearance of the pump is shown in Fig. 274.

The pump as just described is constructed of metal but it is often made of glass, in a quite different shape. It is very rapid in its action and there is no lower limit to which the pressure may be reduced. Pressures lower than $\frac{1}{1000000}$ of a dyne per sq. cm., or 0.000,000,007,5 mm. of mercury have been produced.

**223. Air-Compressors.** The simplest compression pump is that used for inflating rubber tires. Its construction is seen in Fig. 275. When the piston $P$ is pushed down, the air in the cylinder is forced through the valve $v$ into the inner tube of the tire $T$, the valve immediately closing to check the air from going back. On lifting the piston a partial vacuum is produced in the

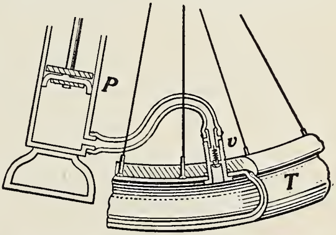



FIG. 275.—Air-compressor for pneumatic tire.

cylinder and the air from outside enters, going past the soft cup-shaped leather forming a part of the piston. When the piston is moving downward this leather is pressed against the inside of the cylinder, thus preventing the air from escaping. Each downward stroke forces more air into the tire until at last it becomes sufficiently hard. In a bicycle tire the pressure seldom exceeds 45 pounds per square inch, while in automobile tires the pressures run from 30 to 90 pounds.

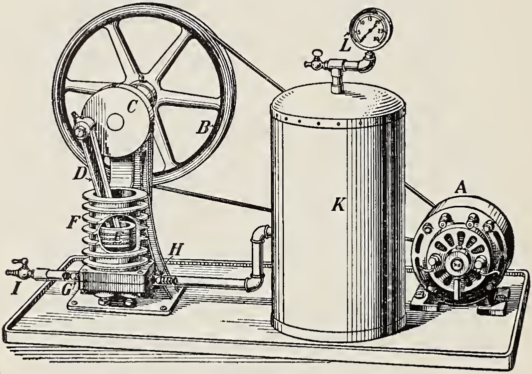



Fig. 276.—A convenient form of compressor.

Another style of compressor is illustrated in Fig. 276. A motor $A$ drives the wheel $B$ on the opposite end of whose axis is the crank disc $C$ to which the connecting rod $D$ is attached. This causes the solid piston $E$ to move up and down in the cylinder $F$. As the piston rises, air is drawn in through the intake $I$ and goes past the valve $G$ into the cylinder. As it descends, this air is forced through the valve $H$ into the tank $K$. The air is rapidly driven into $K$ and the pressure, which is measured by the gauge $L$, quickly rises. If a vessel is connected with $I$, air will be removed from it, but this pump is not suited for producing a high vacuum.

**224. Examples.** 1. The capacity of the receiver of an air pump is $R$ c.c. and that of the cylinder or barrel $C$ c.c. Assuming that there are no leaks

and neglecting the force needed to lift the valves, find the pressure in the receiver after $n$ strokes, if the pressure at first is atmospheric.

At the end of the first up-stroke the $R$ c.c. of air in the receiver has expanded to fill a volume of $(R + C)$ c.c.

Let the new pressure $= P_1$.

Then by Boyle's Law,

$$P_1 \times (R + C) = A \times R, \text{ where } A \text{ is atmospheric pressure,}$$

or
$$P_1 = \frac{R}{R + C} A.$$

We have now in the receiver $R$ c.c. at pressure $P_1$. At the end of the second up-stroke the $R$ c.c. of air in the receiver again expands to fill a volume of $(R + C)$ c.c.

Let the new pressure $= P_2$.

Then
$$P_2 \times (R + C) = P_1 \times R,$$

or
$$P_2 = \left(\frac{R}{R + C}\right) P_1 = \left(\frac{R}{R + C}\right)^2 A.$$

Similarly the pressure at the end of $n$ strokes is given by

$$P_n = \left(\frac{R}{R + C}\right)^n A.$$

2. The capacity of the receiver of an air-compressor is $R$ c.c. and that of the cylinder $C$ c.c. Find the theoretical pressure after $n$ strokes, taking the original pressure in the receiver as one atmosphere.

At the end of the first compression stroke we have compressed into $R$ c.c. a mass of air which formerly occupied $(R + C)$ c.c. at atmospheric pressure.

Let the new pressure $= P_1$.

Then by Boyle's Law,

$$P_1 \times R = A \times (R + C),$$

or
$$P_1 = \frac{R + C}{R} A.$$

At the end of the second compression stroke we have compressed into $R$ c.c. a mass of air which originally occupied $(R + 2C)$ c.c. at its original atmospheric pressure.

Let the new pressure $= P_2$.

Then
$$P_2 \times R = A \times (R + 2C),$$

or
$$P_2 = \frac{R + 2C}{R} A.$$

Similarly the pressure at the end of $n$ strokes is given by

$$P_n = \frac{R + nC}{R} A.$$

## PROBLEMS

1. The capacity of the receiver of an air-pump is twice that of the barrel; what fractional part of the original air will be left in the receiver after (*a*) the first stroke, (*b*) the third stroke?

2. The capacity of the barrel of an air-pump is one-fourth that of the receiver; compare the density of the air in the receiver after the first stroke with the density at first.

3. The capacity of the receiver of an air-compressor is ten times that of the barrel; compare the density of the air in the receiver after the fifth stroke with its density at first.

4. The capacity of the barrel of an air-pump used to exhaust a litre flask is 250 c.c.; compare the density of the air in the flask after the second stroke with its original density.

5. The capacity of the barrel of an air-compressor used to force air into a tank, whose capacity is one litre, is 200 c.c.; compare the density of the air in the tank after the fifth stroke with its density at first.

**225. Pressure Gauge.** The construction of this useful device is shown in Fig. 277. The action depends on the fact that if air or water is forced into a bent tube the tube tends to straighten out. The gauge is attached to a tank or a pipe by the nipple *G*. Within the case is a bent metal tube *AA* (the middle portion not being visible in the diagram). When air or water is forced into it under pressure, it tries to straighten out. Now the lower end is rigidly fixed, but the upper end is free to move, and the higher the pressure in the tube the greater is the motion of this end. By means of a metal strip *B* this end is joined to the short arm of the lever *C* which turns about the pin *D*. On the other end of *C* are teeth which mesh with the small pinion *E*, and the hand *F* is on the end of the axis of the pinion. Hence as the free end *A* moves, its motion is multiplied and transmitted to the hand which moves around the dial. The spring *H* takes up any loose motion in the mechanism.

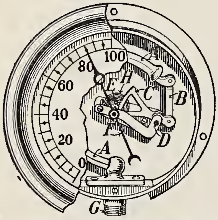


Fig. 277.—Pressure gauge.

**226. Air-Brakes.** One of the many uses of compressed air is to set the brakes on railway trains. Fig. 278 illustrates the principal working parts of the Westinghouse air-brakes in common use in this country. A steam-driven air-compressor pump *A* and a tank *B* for compressed air are attached to the locomotive. The equipment on each car consists of (i) a cylinder *C* in which works a piston, directly connected, by a piston-rod *D* and a system of levers, with the brake-shoe *G*,

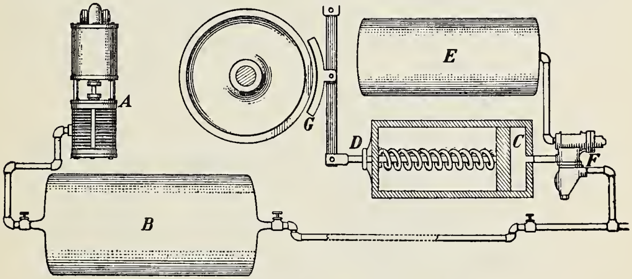


FIG. 278.—Air-brakes in use on railway trains.

(ii) a secondary tank *E*, and (iii) a system of connecting pipes and a special "triple-valve" *F* which automatically connects *B* with *E* when the air from *B* is admitted to the pipes, but which connects *E* with the cylinder *C* when the pressure of the air is removed.

When the train is running, pressure is maintained in the pipes and in the tank *E*, and the brakes are free; but when the pressure is decreased, either by the engineer or by the accidental breaking of a connection, the inrush of air from *E* to *C* forces the piston forward and sets the brakes against the wheels. To take off the brakes, the air is again turned into the pipes, the valve *F* then connects *B* with *E* and the air in *C* is allowed to escape, while the piston is forced into its original position by a spring.

The principle of the action of the triple-valve is illustrated in Fig. 279.

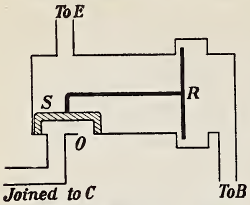



FIG. 279.—Diagram to explain the principle of "triple valve."

When the air pressure is in the pipes the piston *R* and the slide valve *S* are pushed to the left. The cylinder *C* is now connected to the atmosphere through the opening *O*, while air passes around *R* into the tank *E*. When the pressure in the pipes is removed *R* and *S* move to the right and *E* and *C* are connected.

**227. Diving-Bell.** This is made of steel or iron, heavy enough to sink in the water when the open side is downwards,

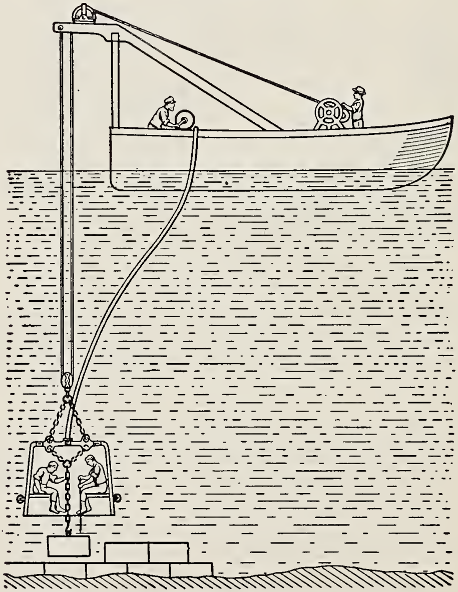


FIG. 280.—Laying a stone foundation with a diving-bell.

and large enough to accommodate two or more workmen. In Fig. 280 is shown how blocks of stone or cement are placed

when a pier of a bridge is being constructed. The bell is let down from a boat or from a wooden staging built over the water. From an air-compressor on the boat air is forced into the bell, thus preventing the water from entering it and also supplying the men with fresh air to breathe. Surplus air escapes at the lower edge of the bell.

**228. Pneumatic Caisson.** The pneumatic caissons used in laying the foundations of bridges, piers, elevators, etc., are operated on the same principle as the diving bell. A section of a typical caisson is shown in Fig. 281. The sides of the caisson are extended upward and are strongly braced to keep back the water. Masonry, or concrete, $C$, $D$, placed on top of the caisson, presses it down upon the bottom, while compressed air, forced through a pipe $P$ drives the water from the working chamber and also sustains the men. To leave the caisson the workman climbs up and passes through the open door $B$ into the air-lock $L$. The door $B$ is then closed and the air is allowed to escape from $L$ until it is at atmospheric pressure. Then door $A$ is opened and the workman climbs out. In order to enter, this procedure is reversed. Material is hoisted out in the same way or is sucked out by a mud-pump. As the earth is removed, the caisson sinks. until at last the solid rock is reached. The entire

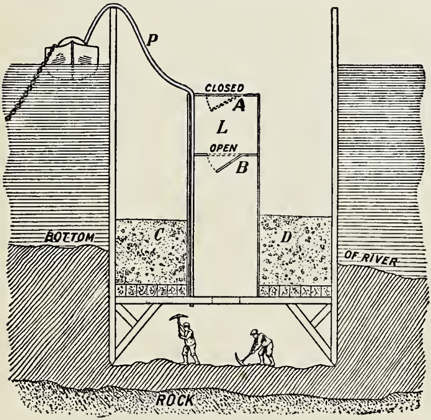


Fig. 281.—Section of a pneumatic caisson.

caisson is then filled with solid concrete, and a permanent foundation for a dock or bridge is thus obtained.

**229. Diving Suit.** The modern diver is incased in an air-tight weighted suit. (Fig. 282). He is supplied with air from above, through pipes or from a compressed-air reservoir attached to his suit. The air escapes through a valve into the water.

Fig. 282.—Diver's suit.

Manifestly the pressure of the air used by a diver or a workman in a caisson must balance the pressure of the outside air, and the pressure of the water at his depth. The deeper he descends, therefore, the greater the pressure to which he is subjected. The ordinary limit of safety is about 80 feet; but divers have gone much deeper than this. In March, 1915, a United States submarine sank in the harbour of Honolulu. A diver went 288 feet under water and walked along the top of the ship, and in the course of salvaging it he made five descents to a depth of 306 feet.

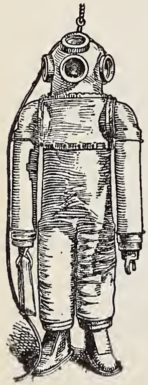


Fig. 283.—Flexible armoured diving suit.

Fig. 283 shows a recently designed flexible armoured suit, which carries its own air supply. A telephone instrument is mounted in the helmet so that the diver is in constant communication with his assistants on the surface. The sleeves terminate in a pair of iron claws which are operated from the inside. A record depth of 361 feet has been obtained by using this suit.

The armour makes it possible to supply air at atmospheric pressure since the pressure of the water is carried by the suit. The diver is able to remain longer at great depths and can be brought to the surface more quickly, without injury, than when wearing the ordinary suit.

**230. Some Other Uses of Compressed Air.** Another useful application is the pneumatic drill, used chiefly for boring holes in rock for blasting. In it the steel drill is held in the end of a cylinder within which a piston is made to move back and forth by allowing compressed air to act alternately on its two end faces. Each time the piston moves forward it delivers a vigorous blow upon the end of the drill, and as it does this several times per second the drill enters the rock quite rapidly. The pneumatic hammer, which is similar in principle, is used for riveting and in general foundry work. Steam could be used in place of air, but the pipes conveying it would be hot, and water would be formed from it.

By means of a blast of sand, projected by a jet of air, castings and also discoloured stone and brick walls are cleaned. Figures on glass are engraved in the same way. Tubes for transmitting letters or telegrams, or for carrying cash in our large retail stores, are operated by compressed air. It is used also for spraying trees, for spray-painting and for many other purposes which cannot be mentioned here.

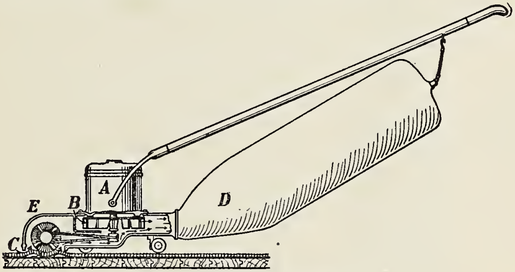



FIG. 284.—The vacuum cleaner.

**231. Vacuum Appliances.** The vacuum cleaner (Fig. 284) is an extremely useful practical application of air currents. The motor *A* drives the fan *B* which creates a current of air in through the opening *C*. If the opening is placed against a carpet the air rushing in through the carpet carries with it

dust and other dirt. This dirt is trapped in the bag *D* while the air passes out through its cloth walls.

The Westinghouse vacuum brake is used on heavy motor trucks and busses. Its construction is somewhat similar to the compressed air brake (Fig. 278) but the piston is actuated by producing a vacuum on one side of the piston. The normal air pressure on the other side then moves the piston and sets the brakes.

D-mt

# CHAPTER XXIV

## Water Pumps and the Siphon

**232. Water Pumps.** From very early times pumps have been employed for raising water from reservoirs, or for forcing it through tubes. It is certain that the suction pump was in use in the time of Aristotle (born 384 B.C.). The force-pump was probably the invention of Ctesibius, a mechanician who flourished in Alexandria in the second century B.C. To Ctesibius is also attributed the ancient fire-engine, which consisted of two connected force-pumps, spraying alternately.

**233. Suction or Lift-Pump.** The construction of the common suction-pump is shown in Fig. 285. During the first strokes the suction-pump acts as an air-pump, withdrawing the air from the suction pipe *BC*. As the air below the piston is removed its pressure is lessened, and the pressure of the air on the surface of the water outside forces the water up the suction pipe, and through the valve $V_1$ into the barrel. On the down-stroke the water held in the barrel by the valve $V_1$ passes up through the valve $V_2$, and on the next upstroke it is lifted up and discharged through the spout *G*, while more water is forced up through the valve $V_1$ into the barrel by the external pressure of the atmosphere.

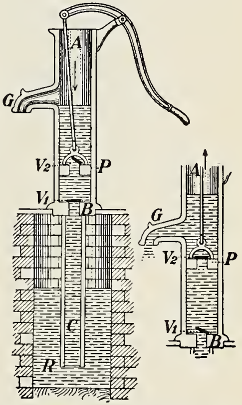



Fig. 285. — Suction-pump, *AB*, cylindrical barrel; *BC*, suction-pipe; *P*, piston; $V_1$ and $V_2$, valves opening upwards; *R*, reservoir from which water is to be lifted.

It is evident that the maximum height to which water, under perfect conditions, is raised by the pressure of the atmosphere cannot be greater than the height of the water column which the air will support. Taking the relative density of mercury as 13.6 and the height of the mercury barometer as 30 inches, this height would be $\frac{30}{12} \times 13.6 = 34$ feet. To this height, then, above the level of the water in the well, the atmosphere can raise the water, and, of course, for the pump to lift the water higher its piston must be immersed in this water column. Consequently the pump rod must extend downwards within 34 feet of the level of the water in the well. As a matter of fact, on account of the air within the water and the vapour from the water, the piston should be within 25 feet of the surface of the water in the well.

**234. Force Required to Operate the Pump.** Let us investigate the force which is needed to operate the pump when the water in the cylinder stands, say, 2 feet above the piston and the water in the well is 22 feet below the piston.

Let the area of the piston be 12 sq. in. and let the atmospheric pressure be $A$ pd. per sq. in.

Then the down-thrust on the piston

$$= 12\ (A + \text{pressure due to 2 ft. of water}).$$

But the up-thrust on the piston

$$= 12\ (A - \text{pressure due to 22 ft. of water}).$$

Hence, the resultant down-thrust on the piston

$$= 12\ (\text{pressure due to 24 ft. of water})$$

$$= \frac{12 \times 24 \times 62\frac{1}{2}}{144} = 125 \text{ pd.}$$

The handle of the suction pump is usually a lever of the first class and the force which must be applied to the end of the handle is consequently much less than the tension in the piston rod which we have calculated.

The tension in the piston rod is of course greater than 125 pd. by the amount of the friction and the weight of the piston and piston rod.

**235. Force-Pump.** When it is necessary to raise water to a considerable height, or to drive it with force through a nozzle, as for extinguishing fire, a force-pump is used. Fig. 286 shows the most common form of its construction. On the up-

stroke a partial vacuum is formed in the barrel, and the air in the suction tube expands and passes up through the valve $V_1$. As the plunger is pushed down, the air is forced out through the valve $V_2$. The pump, therefore, during the first strokes acts as an air-pump. As in the suction-pump, the water is forced up into the suction pipe by the pressure of the air on the surface of the water in the reservoir. When it enters the barrel it is forced by the plunger at each down-stroke through the valve $V_2$ into the discharge pipe. The flow will obviously be intermittent, as the outflow takes place only as the plunger is descending. To produce a continuous stream, and to lessen the shock on the pipe, an air chamber $F$ is often inserted in the discharge pipe. When the water enters this chamber it rises above the outlet $G$, which is somewhat smaller than the inlet, and compresses the air in the chamber. As the plunger is ascending, the pressure of the inclosed air forces the water out of the chamber in a continuous stream.

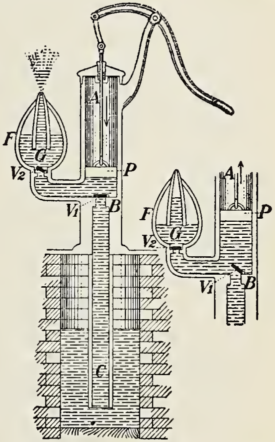



Fig. 286.—Force-pump. $AB$, cylindrical barrel; $BC$, suction-pipe; $P$, piston; $F$, air chamber; $V_1$, valve in suction-pipe; $V_2$, valve in outlet pipe; $G$, discharge pipe; $R$, reservoir from which water is taken.

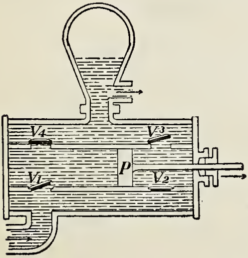



Fig. 287.—Double-action force-pump. $P$, piston; $V_1$, $V_2$, inlet valves; $V_3$, $V_4$, outlet valves.

**236. Double-Action Force-Pump.** In Fig. 287 is shown the construction of the double-action force-pump. When the piston is moved forward in the direction of the arrow, water is drawn into the

back of the cylinder through the valve $V_1$, while the water in front of the piston is forced out through the valve $V_3$. On the backward stroke water is drawn in through the valve $V_2$ and is forced out through the valve $V_4$. Pumps of this type are used as fire engines, or for any purposes for which a large continuous stream of water is required. They are usually worked by steam or other motive power.

**237. Hydraulic Press.** This machine is ordinarily used whenever great force is to be exerted through short distances, as in pressing goods into bales, extracting oils from seeds, making dies, testing the strength of materials, etc. Its construction is shown in Fig. 288. $A$ and $B$ are two cylinders connected with each other and with a water cistern by pipes closed by valves $V_1$ and $V_2$. In these cylinders pistons $P_1$ and $P_2$ work through water-tight collars, $P_1$ being moved by a lever. The bodies to be pressed are held between plates $C$ and $D$. When $P_1$ is raised by the lever, water flows up from the cistern through the valve $V_1$ and fills the cylinder $A$. On the down-stroke the valve $V_1$ is closed and the water is forced through the valve $V_2$ into the cylinder $B$, thus exerting a force on the piston $P_2$, which will be as many times that applied to $P_1$ as the area of the cross-section of $P_2$ is that of the cross-section of $P_1$. It is evident that by decreasing the size of $P_1$, and increasing that of $P_2$, an immense force may be developed by the machine.

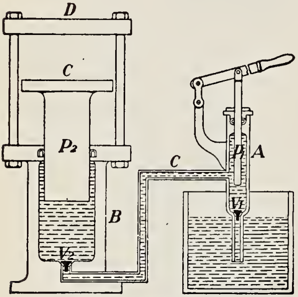


FIG. 288.—Bramah's hydraulic press.

### PROBLEMS AND EXERCISES

1. What is the greatest height to which water can be raised by a common pump when the mercury barometer stands at 76 cm., the sp. gr. of mercury being 13.6?

2. How high can sulphuric acid be raised by a common pump when the mercury barometer stands at 27 in., the sp. gr. of sulphuric acid being 1.8 and that of mercury being 13.6?

3. How high can alcohol be raised by a lift-pump when the mercury barometer stands at 760 mm. if the relative densities of alcohol and mercury are 0.8 and 13.6 respectively?

4. Neglecting friction and the weight of the moving parts, find the force which must be applied to the piston rod in a common pump (Fig. 285) to raise the piston when the water stands 1 ft. above the piston and the water in the well is 17 ft. below the piston. The diameter of the piston is $3\frac{1}{2}$ in.

If the handle is straight and the distances from the piston rod and the end of the handle to the fulcrum are 5 in. and 30 in., respectively, what force must be applied at the end of the handle to raise the piston?

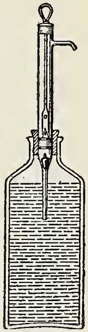


Fig. 289

5. Connect a glass model pump with a flask, as shown in Fig. 289. Fill the flask (*a*) full, (*b*) partially full of water, and endeavour to pump the water. Account for the result in each case.

6. The area of the piston of the force pump shown in Fig. 286 is 12 sq. in. The water in the well is 20 feet below the level of the piston and the

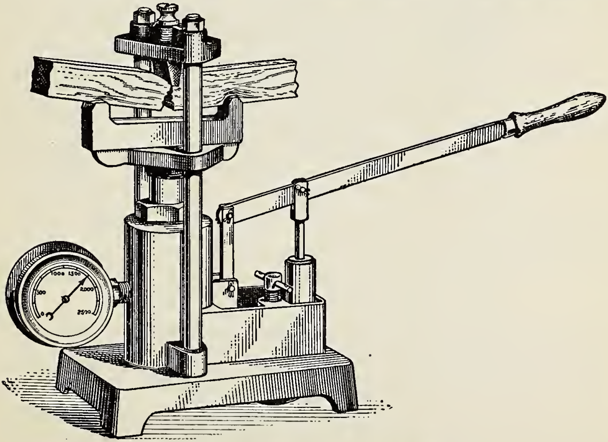



Fig. 290.—Experimental hydraulic press.

pump is being used to deliver water to a tank 30 ft. above the level of the piston. Find the force which must be applied to the piston, (1) on the up-stroke of the piston, (2) on the down-stroke of the piston.

7. In the experimental hydraulic press shown in Fig. 290, the distances from the fulcrum to the small piston and to the end of the handle are as 1 to 6.25. The diameters of the pistons are as 1 to 4. Find the mechanical advantage of the press.

8. If the area of the small piston (Fig. 290) is 0.1 sq. in., what force must be applied at the end of the handle in order that the pressure gauge may read 2500 pd. per sq. in.?

9. What will be the up-thrust on the large piston under the conditions given in problem 8?

**238. Siphon.** If a bent tube is filled with water, and placed in a vessel of water and the ends unstopped, the water will flow freely from the tube, so long as there is a difference in level in the water in the two vessels. A bent tube of this kind, used to transfer a liquid from one vessel to another at a lower level, is called a siphon.

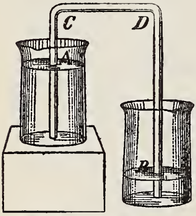


FIG. 291.—The siphon.

To understand the cause of the flow consider Fig. 291.

The pressure at $A$ tending to move the water in the siphon in the direction $AC$

= the atmospheric pressure − the pressure due to the weight of the water in $AC$;

and the pressure at $B$ tending to move the water in the siphon in the direction $BD$

= the atmospheric pressure − the pressure due to the weight of the water in $BD$.

But since the atmospheric pressure is the same in both cases, and the pressure due to the weight of the water in $AC$ is less than that due to the weight of the water in $BD$, the force tending to move the water in the direction $AC$ is greater than the force tending to move it in the direction $BD$; consequently

a flow takes place in the direction $ACDB$. This will continue until the vessel from which the water flows is empty or until the water comes to the same level in each vessel.

**239. The Aspirating Siphon.** When the liquid to be transferred is dangerous to handle, as in the case of some acids, an aspirating siphon is used. This consists of an ordinary siphon to which is attached an offset tube and stopcock, as shown in Fig. 292, to facilitate the process of filling. The end $B$ is closed by the stopcock and the liquid is drawn into the siphon by suction at the mouth-piece $A$. The stopcock is then opened and the flow begins.

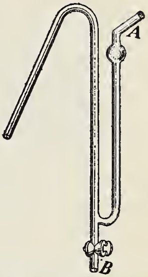



FIG. 292.—The aspirating siphon.

## PROBLEMS AND QUESTIONS

1. Upon what does the limit of the height to which a liquid can be raised in a siphon depend?

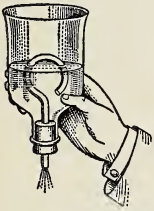


FIG. 293.—Intermittent siphon or Tantalus cup.

2. Over what height can (*a*) mercury, (*b*) water, be made to flow in a siphon?

3. How high can sulphuric acid be raised in a siphon when the mercury barometer stands at 29 in., taking the relative densities of sulphuric acid and mercury as 1.8 and 13.6, respectively?

4. Upon what does the rapidity of flow in the siphon depend?

5. Arrange apparatus as shown in Fig. 293. Let water from a tap run slowly into the bottle. What takes place? Explain.

6. Natural reservoirs are sometimes found in the earth, from which the water can run by natural siphons faster than it flows into them from above (Fig. 294). Explain why the discharge through the siphon is intermittent.*

*Such intermittent springs exist near Atkins, in the mountain region of southwestern Virginia; near Giggleswick, in Yorkshire, England; and in Germany.

7. Arrange apparatus as shown in Fig. 295. Fill the flask $A$ partly full of water, insert the cork, and then invert, placing the short tube in water. Explain the cause of the phenomenon observed.

Fig. 294.—A intermittent spring.

8. A boat on the beach is full of water. How could you empty it with the help of a suitable length of rubber hose? Could you use the same method to get the bilge water out of a boat floating on water? Explain.

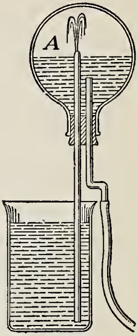



Fig. 295.

9. Find the greatest height over which a liquid of density $\rho_1$ can be carried by a siphon when the height of the barometer is $h$, the density of the liquid used in the barometer being $\rho$.

10. What would be the effect when the siphon is working of making a hole in it (Fig. 291), (1) at $C$, (2) between $A$ and $C$, (3) at $D$, (4) between $C$ and $D$, (5) between $D$ and $B$?

Omit

# CHAPTER XXV

## SURFACE TENSION

**240. Surface Tension Met With Everywhere.** In studying the behaviour of liquids of all sorts, whether contained in ordinary vessels, or in the body of an animal or a plant, whether at a low or a high temperature, we continually meet with a peculiar phenomenon which has been found to be of great importance in the various processes of nature. It is especially prominent when the quantity of liquid is small.

Numerous experiments, many of them easily performed, illustrate the effect and we shall examine some of them.

**241. Formation of a Drop.** On slowly forcing water from a medicine dropper, it gradually gathers at the end, becoming more and more globular, and at last breaks off and falls. (Fig. 296.) We can see that the drop is approximately spherical. When mercury falls on the floor it breaks up into a multitude of shining globules which retain their shape indefinitely. Why do they not flatten out?

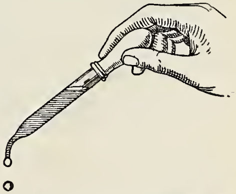


FIG. 296.—A drop of water assumes the globular form.

If melted lead is poured through a sieve at the top of a tower it forms into drops which harden on the way down and which finally appear as solid spheres of shot.

While in some cases we can see the drops growing, the final separation from the mass of liquid is ordinarily so sudden and the subsequent motion is so rapid, that it is impossible to trace the successive stages. The drops, also, are usually quite small. In the following experiment, however, the process of formation

is so slow and the drop is so large that the effect of surface tension can be conveniently observed.

Aniline is an oily liquid which at ordinary temperatures is denser than water. When poured into water it does not mix with it, but falls to the bottom, and the colour assumed by the aniline renders the surface between the water and the aniline clearly visible at a considerable distance. However when heated above 80° C. it rises to the surface of the water.

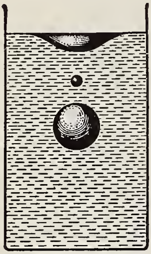


Fig. 297.—Aniline oil drop forming in water.

Into a beaker about 9 inches high and $4\frac{1}{2}$ inches in diameter pour water to the depth of about 7 inches. Then add about 80 c.c. of aniline. Place the beaker above a burner and heat gently until a temperature of about 80° is reached.

The hot aniline now rises to the surface, spreads out, and, coming in contact with the air, is cooled and collects in the form of a drop, an inch or more in diameter, hanging down from the mass at the surface. As the drop grows in size a neck is formed, which, after a while, gets thinner at two places; and when it breaks away the large drop is followed by a small one which is known as Plateau's spherule (Fig. 297). If the temperature is maintained at about 80° the drops will continue to be formed.

Observe the oscillations in the form of the drop as it descends.

If time is not available to perform this experiment the aniline oil may be placed in a small separating funnel, the tube of which is just below the surface of water in a large test tube. When the tap is opened slightly a fairly large drop of aniline forms slowly at the mouth of the tube and finally breaks away. Similar results may be obtained by placing water coloured with a little fluorescin in the funnel and mineral oil or coal oil in the tube (Fig. 298).

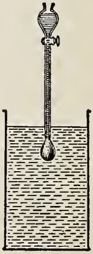


Fig. 298.—Water drop forming in mineral oil.

Various stages in the development of a drop of water are illustrated in Fig. 299.

FIG. 299.—Stages in the development of a drop of water. (*From photographs by Boys.*)

Another beautiful experiment, due to the Belgian physicist Plateau, referred to just above, is as follows:

Put water in a beaker and then carefully pour alcohol on top of it. About 40 per cent. of water to 60 per cent. of alcohol is best, but there may be considerable variation from this proportion. Now introduce olive oil into it by means of a pipette* (Fig. 300). If it is of the same density it will neither sink nor rise on account of gravity. **It assumes a spherical form as though an enveloping skin was trying to compress the oil into a smaller space.** For a given volume, a sphere has less surface area than a body of any other form.

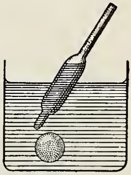


FIG. 300.—A sphere of olive oil in a mixture of water and alcohol.

One of these olive oil spheres may be kept in a stoppered bottle for years. It will probably rise to the top or sink in time but the addition of a little alcohol or water to the mixture, as the case may demand, will cause the sphere to float midway between top and bottom as before.

**242. A 'Skin' on the Surface.** Many other experiments strongly suggest that liquids are enclosed in a thin skin or membrane, which continually tends to contract.

(1) Fill a wine-glass or a small tumbler brimful of water, and then carefully drop into it coins, buttons or other bits of metal. The water

*For useful hints see "Soap Bubbles" by C. V. Boys.

slowly rises above the top of the glass, appearing to be restrained within a skin which clings at its edges to the glass. The surface becomes more and more convex until at last the skin breaks and the water runs over the edge.

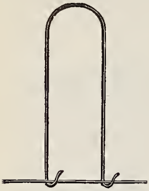


Fig. 301.—Stirrup for placing a needle on the surface of water.

(2) Place a clean, dry sewing needle on the surface of water by lowering it so that both ends will touch the surface at once. In doing this use a fine wire bent in the form shown in Fig. 301. With a little care it can be done. The surface is made concave (Fig. 302) by laying the needle on it, and in the endeavour to contract and smooth out the hollow, sufficient force is exerted to support the needle, though its density is $7\frac{1}{2}$ times that of water. When once the water has wet the needle the water rises against the metal and now the tendency of the surface to flatten out will draw the needle downwards.

Fig. 302.—Needle on the surface of water kept up by surface tension.

If the needle is magnetized, it will act when floating like a comnass needle, showing the north and south direction.

This experiment may be varied by using a safety razor blade in place of the needle.

(3) A wire sieve is wet by water, but if it is covered with paraffin wax, the water will not cling to it. Make a dish out of copper gauze having about twenty wires to the inch; let its diameter be about six inches and height one inch. Bind it with wire to strengthen it. Dip it in melted paraffin wax, and while still hot knock it on the table so as to shake the wax out of the holes. An ordinary pin will still pass through the holes, and there will be over 10,000 of them. On the bottom of the dish lay a small piece of paper and pour water on it. Fully half a tumblerful of water can be poured into the vessel and yet it will not leak. The water has a skin over it, which will suffer considerable stretching before it breaks. Give the vessel a jolt, the skin breaks and the water at once runs out. A vessel constructed as described will also float on the surface of water.

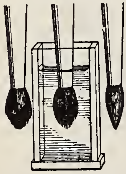



Fig. 303.—*a*, dry; *b*, in water; *c*, wet. Surface tension holds the hairs of the brush together.

(4) When a brush is dry, the hairs spread out as in Fig. 303*a*, but on wetting it they cling together (Fig. 303*c*). This is due to the surface film which contracts and draws the hairs together. That it is not due simply to being wet is seen from Fig. 303*b*, which shows the brush in the water but with the hairs spread out.

A bit of aluminium-leaf or gold-leaf rests quietly on the surface of water, though the former is $2\frac{1}{2}$, and the latter 19, times as dense as the water. In both cases they are not heavy enough to break through the skin on the surface. Remember, however, that **this surface layer is not a skin in the ordinary meaning of that term.** It is made of liquid, though it is reasonable to suppose that the constitution of the surface layer is somewhat different from that of the rest of the liquid.

**243. Surface Tension in Soap Films.** The surface tension of water is beautifully shown by soap bubbles and films. In these there is very little matter, and the force of gravity does not interfere with our experimenting. It is to be observed, too, that in the bubbles and films there is an outside and an inside surface, each under tension.

In an inflated toy balloon the rubber is under tension. This is shown by pricking it with a pin or untying the mouth-piece. At once the air is forced out and the balloon becomes flat. A similar effect is obtained with a soap bubble. Let it be blown on a funnel, and the small end be held to a candle flame (Fig. 304). The outrushing air at once blows the flame aside, which shows that the bubble behaves like an elastic bag.

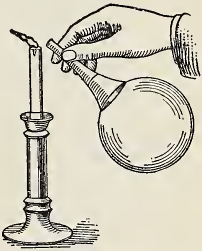


FIG. 304.—Soap-bubble blowing a candle flame aside.

There is a difference, however, between the balloon and the bubble. The former will shrink only to a certain size; the latter first shrinks to a film across the mouth of the funnel and then runs up the funnel ever trying to reach a smaller area.

Again, take a ring of wire about 2 inches in diameter, with a handle on it (Fig. 305). To two points on the ring tie

a fine thread with a loop in it. Dip the ring in a soap solution,* and obtain a film across it with the loop resting on the film. This film is a thin layer of water bounded by two surfaces, the soap making it more permanent. Now puncture the film within the loop: The film which is left contracts, becomes as small as possible and thus draws the loop into a circle, since the area of a circle is greater than that of any other surface having an equal perimeter.

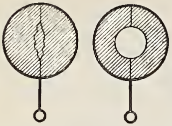


Fig. 305.—A loop of thread on a soap-film.

As in the bubble, the surface acts like a stretched sheet of india-rubber, but there is a further difference between them. The tension in the sheet of rubber depends on the amount of stretching, and may be greater in one direction than in another; whereas **the tension in the soap film remains the same however much the film is extended, and the tension at any point is the same in all directions along the film.**

**244. The Cause of Surface Tension.** Surface tension effects are due to cohesion. A little consideration would lead us to expect the molecules at the surface to act in a manner somewhat different from those in the interior of a liquid. Let *a* be a molecule well within the liquid (Fig. 306). The molecule is attracted on all sides by the molecules very close to it, within its sphere of action, which is extremely small, and as the attraction is in all directions it will remain at rest. Next consider a molecule *b* which is just on the surface. In this case there will be no attraction on *b* from above except by air molecules, which may be neglected, but the neighbouring molecules within the liquid will pull it downwards. Thus there are forces pulling the surface molecules into the

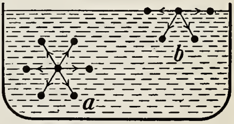



Fig. 306.—Behaviour of molecules within the liquid and at its surface.

*See method of preparation, page 347.

liquid, bringing them all as close together as possible, so that the area of the surface will be as small as possible. It is for this reason that the water forms in spherical drops, since, as has been remarked, for a given volume, the sphere has the smallest surface.

It should be noted that surface tension effects are not limited to surfaces of separation between liquids and air; they exist wherever there is a surface of separation, whether between liquid and gas, liquid and liquid, liquid and solid, or gas and solid.

### QUESTIONS

1. It is not easy to pour water from a tumbler into a bottle without spilling it, but by holding a glass rod as in Fig. 307, the water runs down into the bottle and none is lost. The glass rod may be inclined, and the water still follows it. Explain the action.

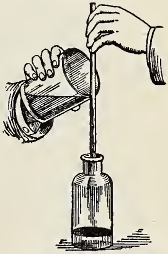


Fig. 307.—How to utilize surface tension in pouring a liquid.

2. Water may be led from the end of an eaves-trough into a barrel by means of a pole almost as well as by a metal tube. Why is this?

3. Some insects are able to run about on the surface of water, often quite rapidly. Explain.

4. Why does the end of a stick of sealing-wax or of a rod of glass assume a rounded form when heated in a flame?

5. Explain why a tent sheds rain in spite of the many small openings between the threads of the fabric. Why will rubbing the hand over the inner surface of the tent while it is raining cause a leak?

**245. Units by which Surface Tension is Measured.** Let us consider next the units by which we shall measure this force called surface tension.

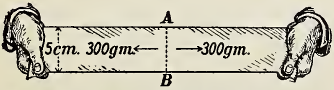



Fig. 308.—Diagram to illustrate how the surface tension of a liquid is designated.

In Fig. 308 is shown a strip of paper, 5 cm. wide, stretched lengthwise with a force of 300 gm. Across a section

*AB* of the strip—like a tug-of-war tending to separate the strip into two pieces—is a tension of 300 gm., which is 60 gm. per linear cm.

A stretched surface may be considered to be composed of strips like this and **the Surface Tension is measured in units of force per unit of length, usually dynes per cm.**

**246. Surface Energy.** To inflate a rubber balloon or a bicycle tire, or to blow a soap-bubble requires an expenditure of work; and when these bodies contract they exert a force and thus can do work.

The fact that a soap-film will contract and exert a force can be well shown as follows: Bend a wire into a rectangular shape (Fig. 309) and dip it into a soap solution. On taking it out it is covered with a film. Hold it horizontal and across it lay a thin straight wire *mn*; then puncture the side *Q* of the film. The two surfaces (the upper and the lower) of the film *P* which is left exert a pull on the wire in the direction shown by the arrow, and draw the wire over to the end *ab*, thus reducing the area of the film to as small dimensions as possible.

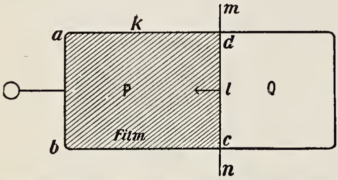


Fig. 309.—The wire *mn* is drawn to the left by the tension of the film.

Take hold of the ends of the wire and pull the wire out and release it again and again.

By an experiment somewhat similar to this the magnitude of the surface tension may be determined (see Sec. 250).

It is evident that the greater the width *cd* of the rectangle, the greater will be the entire force drawing the wire in the direction of the arrow, *i.e.*, at right angles to the axis of the wire. (Two parallel strips of stretched sheet rubber, each one centimetre wide, will exert twice the force which one of them exerts.)

Let the width *cd* of the rectangle be $l$ cm., and let the pull exerted by each surface of the film, on each cm. of the movable wire, be $T$ dynes. **This force per centimetre which a single liquid surface exerts is called the Surface Tension of the**

**liquid.** Then the entire force exerted upon the wire by the two surfaces of the film = $2Tl$ dynes. If the length *ad* of the film is $k$ cm., the work which the film $P$ can do in contracting = $2Tlk$ ergs.

Just as we say that a bent bow or a stretched sheet of rubber possesses potential energy, so we can say that the film possesses potential energy, and its amount is equal to the work which it can do in contracting, that is, $2Tlk$ ergs. But its area = $2lk$ sq. cm. Hence the potential energy per sq. cm. = $2Tlk \div 2lk = T$ ergs.

Again, if one takes hold of the wire and moves it to the right (Fig. 309) a distance $x$ cm., thus increasing the area of the film by $2lx$ sq. cm., the work which one does is $2Tlx$ ergs, and the work done per sq. cm. = $T$ ergs.

Hence we have the relation: **The measure of the surface tension of a liquid is equal to the measure of its potential energy per sq. cm. of the surface; or it is equal to the measure of the work done in enlarging the surface of the liquid one unit of area.**

Or, in more particular terms: **If the surface tension is $T$ dynes per linear cm., then the surface energy is $T$ ergs per sq. cm. and the work done in enlarging the surface by one sq. cm. is $T$ ergs.**

The question of surface tension arose chiefly through the consideration of the rise of liquids in capillary tubes, *i.e.*, tubes so fine as to admit only a hair (Latin, *capillus*, a hair); but the subject of surface tension is a very broad one with numerous applications. Hence, it is better to use the name *surface tension* than the name *capillarity*, by which it is sometimes known.

**Example.**—If *ad* (Fig. 309) measures 10 cm. and *dc*, 6 cm., find the force exerted on the wire *mn* by a soap film whose surface tension is 28 dynes per cm. Find also the potential energy of the whole surface of the film due to surface tension.

Force on wire $= 2 \times 28 \times 6 = 336$ dynes.
Potential energy $= 28$ ergs per sq. cm.,
$= 2 \times 28 \times 6 \times 10 = 3360$ ergs.

### PROBLEMS

(Take $T$ for a soap solution as 28 dynes per cm.).

1. If $ab$ (Fig. 309) measures 5 cm., what force is needed to pull the wire $mn$ to the right? How much work is done in pulling it a distance of 6 cm.?

2. Calculate the work done in blowing a soap bubble 10 cm. in diameter.

3. Find the work done in expanding a bubble from 3 cm. to 10 cm. in diameter.

**247. Angle of Contact or Capillary Angle.** If a plate of glass is held vertically in water, the liquid in the surface,


(*a*) Water. (*b*) Mercury.

Fig. 310.—Angle of contact of water and mercury.

where it touches the glass, is drawn up above the level of the general surface (*a*, Fig. 310). If the glass be lifted from the water some water will cling to it. The water is said to wet the glass. If the glass be held in mercury the liquid surface in contact with the glass is depressed (*b*, Fig. 310), and if the glass be removed from the mercury none of the mercury will adhere to it. Mercury does not wet glass.

**The angle which the tangent to the liquid surface where it meets the surface of the solid makes with the common surface of the liquid and the solid is called the angle of contact or the capillary angle ($\theta$, Fig. 310).**

The size of this angle depends on the third medium, above the liquid. Thus if oil is used instead of air the angle is much altered. It also depends very materially on the condition of the surfaces. The slightest contamination on the surface of water or on the solid will alter the angle considerably. Figure 310 *a* illustrates the usual condition for water and glass.

With perfectly clean water and glass the angle of contact *BAC* is very small, probably zero, but with slight contamination it may reach 90°, *i.e.*, it does not rise on the surface of the glass at all. Figure 310 *b* illustrates the effect with mercury and glass. Here the angle of contact is obtuse, varying from 129° to 143°.

**248. Level of Liquids in Fine Tubes.** If a small glass tube is held upright in water the liquid rises within the tube and,

Water. Mercury.

FIG. 311.—Level of liquid in a fine tube.

both inside and outside, the surface curves upwards where it touches the glass (Fig. 311). This effect can be observed more easily if a little colouring matter (fluorescin, for example) is added to the water. If mercury is used instead of water,

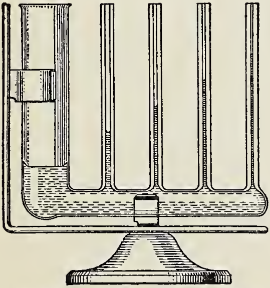


FIG. 312.—Showing the elevation of water in capillary tubes.

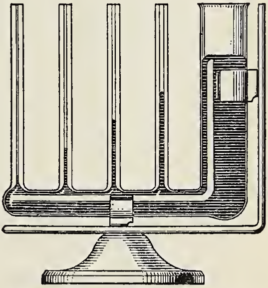


FIG. 313.—Showing the depression of mercury in capillary tubes.

the liquid within the tube takes a lower level than that outside, and the surface at the glass curves downwards instead

of upwards. In these experiments the glass should be perfectly clean.

It is interesting to observe the effect with tubes of various sizes.

Fig. 312 shows capillary tubes having different internal diameters connected to a tube of large diameter. It will be seen that in each of the capillary tubes the level is above that of the water in the large tube and that **the finer the tube the higher is the level of the water.** With alcohol the liquid rises, though not so much, but with mercury the liquid is depressed. The behaviour of mercury is shown in Fig. 313. In this case **the finer the tube, the greater is the depression of the mercury.**

**249. Calculation of the Rise of Liquid in a Tube.** Consider a tube held vertically in a liquid which wets it. The liquid rises on the outside slightly, but on the inside to a considerable height (Fig. 314).

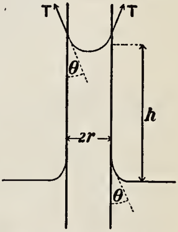


FIG. 314.—Surface tension supports the column in the tube.

The phenomenon is "explained" by stating that the attraction of the molecules of the liquid for those of the glass is greater than the attraction of the molecules of the liquid for each other. The surface of the liquid meets the glass along an inner circumference of the tube, and the attraction exerted, across this line, between the surface molecules of the liquid and those of the glass, is sufficient to support the raised column.

The action may be likened to the weight of the ink in the small rubber bag of a self-filling fountain pen, being supported, when the pen is held point upward, by the cement which fastens the bag to the nib-holder.

Let $T$ denote the surface tension in dynes per cm.

$r$ " " radius of tube in cm.

$h$ " " mean height of column in cm.

$\rho$ " " density of the liquid in gm. per c.c.

$\theta$ " " angle of contact.

The force $T$ acts in a direction making an angle $\theta$ with the vertical; hence its component in the vertical direction is equal to $T \cos \theta$.

The surface of the liquid pulls the inner surface of the tube inwards and downwards, acting in the direction of the tangent to the liquid surface where it touches the tube, and the reaction of the tube lifts the liquid upwards.

The length of the line of contact of the liquid and inner surface of the tube $= 2\pi r$ cm., and hence the total force upwards in the direction of the axis of the tube

$$= 2\pi r T \cos \theta \text{ dynes.}$$

This balances the weight of the raised column of liquid, which $= \pi r^2 h \rho g$ dynes.

Equating the total force upwards to the total force downwards, we have

$$\pi r^2 h \rho g = 2\pi r T \cos \theta,$$

$$\text{and } T = \frac{h \rho r g}{2 \cos \theta}, \text{ or } h = \frac{2T \cos \theta}{\rho r g}.$$

**From this we see that $h \propto \frac{1}{r}$, or the height to which the liquid is drawn up is inversely proportional to the radius of the tube.** In the case of a liquid, such as mercury, which is depressed, the depression is inversely proportional to the radius of the tube.

In a glass tube of radius 1 mm. the water rises about 1.4 cm. Hence in one of radius $\frac{1}{1000}$ mm. the rise would be 14 metres. It has been surmised that the distribution of sap in plants is partially due to capillary action, but this will not account for the *rate* at which water rises in trees.

The tube of a barometer should be large, otherwise a correction for capillarity is necessary. If the tube has a diameter of 2 mm. the mercury is depressed 4.6 mm., but if it is 2 cm. (about 0.8 inch) or greater, the correction for depression is so small that it may be neglected.

**250. Measurement of Surface Tension.** One of the simplest methods of finding the surface tension of a liquid is shown in Fig. 315.

Construct the fork-shaped arrangement *a* from a piece of wire, making the distance between the parallel prongs about 6 cm.

Measure the distance between the prongs of the fork carefully and then suspend it from one arm of a balance. Place a beaker of soap solution under the fork so that the cross-bar of the fork is about one-half centimetre above the liquid when the beam is horizontal (Fig. 315).

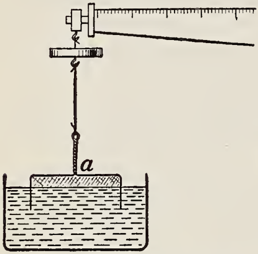


FIG. 315.—Measuring the surface tension of a soap solution.

Raise the beaker in order to wet the fork; then lower it, break the film and weigh carefully,

Repeat the operation without breaking the film and weigh again.

Subtract these two weights to find the pull exerted by the film on the cross-bar. Reduce this to dynes and divide the force in dynes by twice the distance between the prongs (because of the two surfaces of the film). This gives the surface tension in dynes per centimetre length of film.

Repeat the experiment with different sized forks and also with distilled and tap water. When using water the distance between the cross-bar and the surface of the water must be less and it is more difficult to get the film to last long enough to make the weighing, but a little patience will produce very good results. Compare the results with those given in Sec. 252.

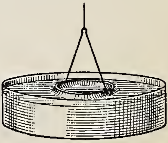


FIG. 316.—Finding surface tension by using a ring of platinum wire.

For water, alcohol and other liquids whose films do not last long, the circular ring of platinum wire (Fig. 316) gives more

satisfactory results. In this case the force in dynes is divided by twice the circumference of the ring.

**251. Capillary Tube Method.** A common method of measuring the surface tension of a liquid is to observe the rise (or fall) of the liquid in a capillary tube and then use the formula

$$T = \frac{h\rho r g}{2 \cos \theta}. \quad \text{(Sec. 249.)}$$

This requires the angle of contact to be known. For water and some other pure liquids it may be taken as zero, in which case $T = \frac{1}{2} h\rho r g$.

The radius of the tube may be found by weighing the tube before and after a column of mercury has been placed in it. Since the density of mercury is known, the volume of the mercury is easily calculated and by measuring the length of the column, its area of cross-section and radius follow.

If we assume the surface tension of distilled water to be known, the surface tensions of other liquids may be found easily without calculating $r$.

Place the lower end of the capillary tube in distilled water in a beaker and measure the distance to which the water rises in the tube above the level of the water in the beaker (Fig. 317). In doing this apply the mouth to the upper end of the tube and draw the water up until it rises nearly to the top and then allow it to settle to its final position.

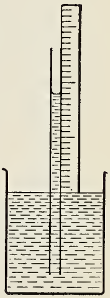


FIG. 317. — Comparing the surface tensions of different liquids.

Wash the tube out well with the next liquid to be used and repeat the measurement.

Then if $h_1$ is the height for water and $\rho_1$ its density and $h_2$ is the height for the second liquid and $\rho_2$ its density, the surface tension of the

$$\text{liquid} = \frac{h_2\,\rho_2}{h_1\,\rho_1} \times \text{the surface tension of water.}$$

The surface tension of distilled water is about 73 dynes per centimetre length.

Assuming this, find the surface tensions of a soap solution and tap water and alcohol.

**252. Table of Surface Tensions.** The value of the surface tensions of various liquids when in contact with air, water or mercury are given in the following table:

Table of Surface Tensions at 20° C. (In Dynes per cm.)

| Liquid | Density (gm. per c.c.) | Tension of Surface Separating the Liquid from | | |
|---|---|---|---|---|
| | | Air | Water | Mercury |
| Water | 1 | 73 | .. | 392 |
| Mercury | 13.6 | 520 | 392 | ... |
| Carbon Bisulphide | 1.27 | 31 | 42 | 387 |
| Chloroform | 1.49 | 28 | 27 | 415 |
| Alcohol, Ethyl | 0.79 | 24 | .. | 364 |
| Olive Oil | 0.91 | 35 | 19 | 317 |
| Turpentine | 0.89 | 29 | 12 | 241 |
| Petroleum | 0.80 | 30 | 29 | 271 |

A soap solution made according to the directions on page 347 has a surface tension of approximately 28 dynes per cm. when in contact with air. The value for tap water (Toronto) is not materially different from that of distilled water.

## PROBLEMS

1. If the distance between the prongs of the fork (Fig. 315) is 5 cm. and if the pull exerted by the film is 0.286 gm., find the surface tension of the soap solution.

2. If the circumference of the platinum ring (Fig. 316) is 8 cm. and if the additional weight due to the pull of the film when distilled water is used, is 1.192 gm., find the surface tension of the water.

3. Calculate the heights to which pure water, alcohol and turpentine will rise in capillary glass tubes (*a*) 1 mm., (*b*) 0.2 mm., in diameter.

4. Calculate the depression of mercury in the tubes, taking $\theta = 140°$.

5. If distilled water rises to a height of 10 cm. in a capillary tube, how high will chloroform rise in the same tube?

6. Two parallel plates, separated by a space $d$, stand vertically in a liquid, having density $\rho$, surface tension $T$ and angle of contact $\theta$. Show that the height $h$ to which the liquid will rise is

$$h = \frac{2T \cos \theta}{g\rho d}.$$

Compare this with the height in a cylindrical tube whose diameter is equal to the distance between the plates.

(Consider the equilibrium of a portion of the liquid between the plates 1 cm. in length.)

7. If two lantern slide cover glasses are bound together, slightly separated at one edge and placed in a shallow vessel containing coloured water, the water climbs between the plates as shown in Fig. 318. Explain the action.

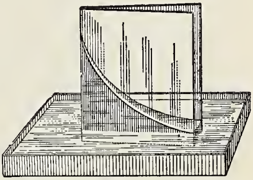


FIG. 318.—Water rises between the two plates of glass which touch along one edge.

**253. Attraction and Repulsion between Bodies on the Surface of Water.** It has often been noticed that bubbles, small sticks and straws floating on still water appear to attract each other. They gather in groups or become attached to the edge of the containing vessel. This effect can be easily illustrated by means of two discs sliced off a cork, placed on the surface of the water. When they get within a certain distance (about 1 cm.) they run together. If the water does not wet either body they will still attract each other; but when two bodies, one of which is wet and the other is not, are brought near together they will appear to repel each other.

These actions can be explained in the following way. Let $P$ and $Q$ be two plates suspended by threads near together in a liquid.

First, let the liquid wet both plates (Fig. 319). Let $a$, $a$ be points on the surface of the liquid at its ordinary level, away from the plates, and $c$ be a point on the same level in the liquid between the plates. As the liquid is in hydrostatic equilibrium the pressures exerted by the liquid at these three

points must be equal, each being equal to that of the atmosphere. If one ascends from *c* towards *b* the pressure diminishes, while if one descends below *c* the pressure increases. Consequently the pressure of the liquid between the plates is less than that of the atmosphere which presses on the outer surface of the plates, and the plates will be pushed together, as indicated by the arrows.

Next, take two plates which are not wet by the liquid (Fig. 320). These may be plates of glass, or aluminium, covered with paraffin. Here the pressures at *a*, *a*, as also at *c* between the plates, are all equal, each being that of one atmosphere. Hence the pressures at *b*, *b* in the outer liquid

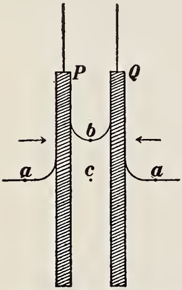


Fig. 319.—If both of the plates are wet they are attracted.

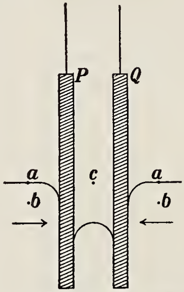


Fig. 320.—If both plates are not wet they are attracted.

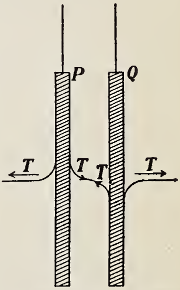


Fig. 321.—If one plate is wet and the other is not they are repelled.

are greater than the pressure on the same level between the plates, and the plates will consequently be pushed together as before.

This may be demonstrated also by means of two short iron rods floating on the surface of mercury.

**It should be noted that no change in pressure occurs in crossing the level surface of a liquid but there is an abrupt change of pressure in crossing the concave or convex surface between the plates.**

Finally, let the liquid wet one plate but not the other.

When the plates come sufficiently near together the surface of the liquid between the plates assumes the form shown in Fig. 321. It then has no level portion.

The tension of the surface on the outside pulls the plates with the force $T$ in the horizontal plane and although the surface curves upwards to meet the plate the horizontal pull is the same as though the horizontal surface continued right up to the plate. This tends to draw the plates apart. The tension of the surface between the plates exerts an equal force, but in a direction making, let us say, the angle $a$ with the horizontal. Resolving this in the horizontal plane, the force drawing the two plates together is $T \cos a$, and as this is smaller than $T$ acting in the opposite direction, the plates will be drawn apart.

The above results can be neatly illustrated in the following way:

Obtain two hollow glass balls about 2 cm. in diameter and cover one with paraffin. Attach a weight to each (with wax or otherwise) so that they may float rather more than half immersed. They will appear to repel each other. If both are clean glass or both paraffined they will attract each other. If the glass balls are not available, flat corks may be used instead.

**254. Small and Large Bubbles.** The following experiment illustrates the fact that the pressure within a small bubble is greater than that within a larger one.

*AB* (Fig. 322) is a **U**-shaped glass tube with a side tube attached to its centre and provided with taps at $C$, $D$ and $E$. The tubes have an internal diameter of about one-quarter inch and are flared out to about one-half inch diameter at $A$ and $B$. (Short pieces of glass may be joined by rubber tubing and pinch-cocks may be used instead of taps).

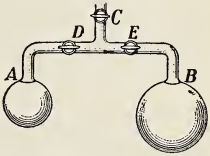



Fig. 322.—Apparatus for showing that the pressure in a small bubble is greater than that in a larger one.

The tap $E$ is closed and a small bubble is blown at $A$ with $C$ and $D$ open. Then $D$ is closed and a larger bubble is blown at $B$ with $E$ open. $C$ is then closed and $D$ opened, providing communication between the two bubbles. The large bubble at $B$ becomes still larger and the small one shrinks to a flat film across the tube.

A similar action takes place with large and small drops of water. This may be shown as follows:

Take a capillary tube bent in the form of a **U**, and fill it by drawing water through it. Then put a large drop of water on one end of the tube and a small drop on the other. Note which drop gains in size. What conclusion can be drawn regarding the pressures in the two drops?

**255. Calculation of Pressure in a Bubble or Drop.** Consider the soap bubble represented in Fig. 323. There is a state of equilibrium between the internal pressure tending to make it expand and the surface tension effect tending to make it contract.

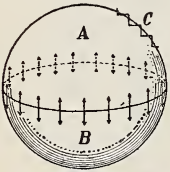


FIG. 323.—Calculation of pressure in a bubble.

Let the internal pressure be $P$ dynes per sq. cm. and the surface tension $T$ dynes per cm. Let the radius of the bubble be $R$ cm. Since the film is very thin $R$ may be taken as both the internal and external radius.

The two halves of the bubble $A$ and $B$ are similar to the two halves of a child's hollow rubber ball cemented together around the equatorial circle. In the case of the bubble the "cement" consists of the force of surface tension, $T$ dynes per cm. length of the equatorial circle. Since there is an inner and an outer surface the total force tending to hold one half of the bubble to the other half

$$= 2 \times 2\pi R \times T \text{ dynes.}$$

The internal pressure, $P$ dynes per sq. cm., acts everywhere at right angles to the surface but we can consider the surface as the limiting condition of the "stair-step" arrangement shown at $C$, when the number of steps becomes infinite.

The thrust tending to push one of the hemispheres away from the other is obviously equal to the sum of all the thrusts on the horizontal parts of

the steps in the surface of the hemisphere

$$= P \times \text{(sum of all the areas of horizontal parts)}$$
$$= P \times \text{(area of equatorial circle)}$$
$$= P \times \pi R^2.$$

Hence, $P \times \pi R^2 = 2 \times 2\pi R \times T,$

or $$P = \frac{4T}{R}.$$

In the case of a water drop the same argument holds except that we have only one surface and consequently

$$P \times \pi R^2 = 2\pi R \times T,$$

or $$P = \frac{2T}{R}.$$

**For both bubbles and drops, then, the pressure is inversely proportional to the radius of the sphere.**

This pressure due to a curved surface accounts for the sudden change of pressure as we cross the surface of separation in a capillary tube or in a liquid between parallel plates (Sec. 253).

The pressure calculated is, of course, pressure in excess of ordinary atmospheric pressure.

## QUESTIONS AND PROBLEMS

(For surface tensions see Sec. 252.)

1. When a soap bubble bursts the water from it is thrown in every direction. Account for this.

2. Why are small drops of mercury resting on a horizontal surface more nearly spherical than larger ones?

3. Two drops of mercury 1 mm. and 2 mm. in diameter, respectively, coalesce. Compare the pressure within the liquid due to surface tension in the two original drops and in the one formed by their union.

4. One soap bubble, 8 cm. in diameter, is on one end of a U-tube, and another, 3 cm. in diameter, is on the other end. Find the pressure in each bubble. If there is a free passage from one to the other, which one will increase in size?

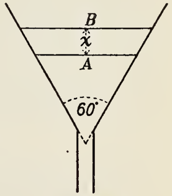


FIG. 324.

5. A soap film in position $A$ across a 60° funnel (Fig. 324) is moved to position $B$, $x$ cm. from $A$, by blowing air through the stem of the funnel. Find the work done against surface tension if the radius at $A$ is $r$ cm.

**256. Experimental Illustrations of Surface Tension.** (1) If small fragments of camphor are placed upon the surface of clean water they at once move about almost as if alive. The camphor dissolves slowly in the water, and the surface tension of a solution of camphor is smaller than that of pure water. Consequently if the camphor dissolves more rapidly at one side of the fragment than at the other, the surface tension on the first side will be diminished and the greater surface tension on the other side of the fragment will draw the fragment away.

This can be easily shown by rinsing a glass at the tap, filling it with water, and then scraping with a pen-knife small fragments of camphor which are allowed to fall upon the surface. They dart about, but if the surface of the water be touched with the finger the movements will likely cease, being arrested by the grease from the finger communicated to the water. Very little grease is required. Lord Rayleigh found that 0.8 milligram of olive oil on a circular surface 84 cm. in diameter was sufficient. From this he calculated that an oily film 2 millionths of a millimetre in thickness is sufficient to arrest the camphor movements.

(2) The surface tension of alcohol is much smaller than that of water (see Table p. 332). Scatter lycopodium powder over the surface of a thin layer of water, and then place a drop of alcohol on the surface. At the place where the alcohol is, the tension is immediately reduced, equilibrium is destroyed and the superficial film of the liquid is set in motion. This will be shown by the lycopodium powder. If the water is very shallow this motion will drag the water away from the place where the alcohol is, and will lay bare the bottom of the vessel.

(3) Rinse a glass under the tap and fill it with water, and scatter lycopodium powder as in the last experiment. Now touch the middle of the surface with a finger which has been rubbed against the hair. Enough grease will come off the finger to contaminate the water, and reduce its surface tension, and the surface layer will be drawn away from the place where the finger touched the surface. A patch will be entirely free from the powder.

(4) Hold a drop of ether close to the surface of water. The vapour of the ether condenses on the surface, reduces the surface tension and causes an outward motion, producing a dimple on the surface.

(5) Pour clean water on a level board so as to form a shallow pool 2 inches wide and 2 or 3 feet long. Near its middle lay a scrap of paper and on one end place a cake of soap (Fig. 325).

The paper is soon seen to move along the surface away from the soap. Here the soap in dissolving weakens the surface film, and the tension in the other portion draws the surface layer away from the soap.

FIG. 325.—Soap reduces the surface tension.

(6) Cut a piece of paper into the shape of a fish (Fig. 326). On its tail put a drop of amyl alcohol (or of fusel oil) and place it on the surface of clean water. The fish swims about in a very interesting way. Why does it stop at last?

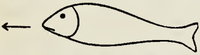


FIG. 326.—The paper fish moves to the left.

Cut the shape of an **S** from paper (Fig. 327), and put a drop of amyl alcohol on each end of it. It spins about like a pin-wheel.

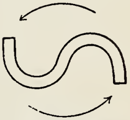


FIG. 327.—The paper spins about.

By using a shallow dish and a vertical attachment these motions can be projected on the screen.

Where the amyl alcohol is placed, the surface film is weakened, and the tension in the other parts of the surface draws the surface film away from these places, causing the motion of the pieces of paper.

(7) *A* is a glass bulb, with a small one beneath it, on the end of a small glass tube (Fig. 328). Mercury in the lower bulb makes the tube float upright in water. At *d* is a piece of wire gauze attached (by wax) to the small tube.

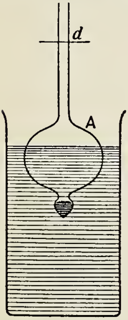


FIG. 328.—Surface tension upon *d* can hold the bulb down.

When floating in water a considerable part of the large bulb *A* is above the surface, and it requires quite a force to push it down.

Now press it down until the gauze touches the surface. The water wets it and clings to each wire. This tension will be sufficient to hold the tube down in the water.

While down put a few drops of ether (or alcohol) on the surface of the water. At once the gauze breaks away and rises as shown in the figure. Adding the ether weakens the surface tension.

The size of the piece of gauze will depend on the size of the bulb and the amount of mercury holding it down; but it will be easy to find suitable dimensions.

A simpler form of the apparatus is shown in Fig. 329. It consists of a hat-pin through a cork, with a piece of lead to keep it upright. In place of the wire gauze a cardboard disc *d* may be used. This can be pared down until it is just able to hold the cork down.


FIG. 329. — Simpler form of of apparatus, but not so satisfactory.

In this case the disc is held by the tension exerted only around its edge, while with the gauze the surface clings to each wire and so the total tension is greater.

(8) Two simple methods of removing grease from cloth are based on surface tension. The fatty oils have a greater surface tension than benzine. Hence if one side of a grease-spot on a piece of cloth is wetted with benzine the tension is greatest on the side of the grease. Consequently the portions consisting of a mixture of grease and benzine will be drawn towards the grease and away from the benzine.

In order to cleanse the grease-spot, first apply the benzine in a ring all round the spot, and gradually bring it nearer to the centre of the spot. The grease will be chased to the middle of the spot and if a fibrous substance such as blotting-paper is placed in contact with the cloth, the grease will escape into it. If the benzine had been applied to the centre of the spot the grease would have been spread out into the cloth.

The second method is to apply a hot iron to one side of the cloth and blotting-paper to the other. The surface tension diminishes as the temperature rises. Hence the grease draws away from the hot iron and escapes into the blotting-paper.

Try these two methods.

Omit

# CHAPTER XXVI

## Some Applications of Surface Tension

### I.—*In Agriculture*

**257. Action of the Water in the Soil.** Surface tension undoubtedly plays an important part in supplying moisture to the soil. If a lump of loaf-sugar is placed with one corner in water, the liquid gradually rises and spreads until it completely permeates the lump. The soil behaves similarly. Like the sugar, it is composed of small particles with spaces between them. If water falls upon it, some will pass down through it and run away, but a considerable amount will cling to the surfaces of the particles and gather in the spaces between them. If water is supplied at the side or underneath, as sometimes in irrigation, the water spreads upwards and throughout the mass and much of it remains there owing to surface tension.

**258. Evaporation at the Surface.** The water at the upper surface evaporates, and its place is supplied, as far as possible, by water drawn up by surface tension. The depth from which water can be raised by capillary action differs in different soils and for different conditions of the soil. The finer the texture is, the higher the possible rise.

Experiment has shown that capillary movement can take place through a column 5 feet in height. In this case the soil must be moist to begin with. On the other hand, if the soil is well dried the capillary rise may be less than 1 foot.

It has been shown, also, that evaporation from soil takes place entirely from the layers very near the surface.

**259. Retaining the Moisture in the Soil.** The problem of preventing the rise of the water to the surface and its loss by evaporation is a very important one, especially in those countries where there is no rainfall for months in succession or where the entire yearly rainfall is small, not more than ten of fifteen inches.

It has been found that if a soil after a rain is exposed to very arid conditions, with a high surface temperature and a hot dry wind, the soil at the surface will lose water much faster than it can be brought up from below by capillary action, and a layer of dry soil may be formed on the surface which will be so dry that it will act as a protecting covering.

One of the most effective means of conserving soil moisture, however, is by "mulching," *i.e.*, by covering the surface of the soil with some loosely packed material, such as straw, leaves or stable manure. The spaces between the parts of such substances are too large to admit of capillary action, and hence the water conveyed to the surface of the soil is prevented from passing upwards any further, except by slow evaporation through the mulching layer. A loose layer of earth spread over the surface of the soil acts in the same way, and the same effect may be attained by hoeing the soil or stirring it to the depth of one or two inches with harrows or other implements.

In the semi-arid regions of the United States, Argentina, the Canadian West and other countries, in which the average rainfall lies between 10 and 20 inches, good crops of selected grain can be grown by proper cultivation.

In some cases only one crop can be grown in alternate years, the year of no crop being used to preserve the moisture in the soil. In our Canadian West during a dry season it is found that land which was "summer fallowed" the year before produces the heaviest crop.

## II.—*In Dyeing*

**260. The Process of Dyeing.** There is great variety both in the materials to be dyed and in the colouring matter to be applied to them, and we are not surprised to find that the phenomena observed in the process of dyeing are very complicated. No single hypothesis as to the nature of the action taking place will account for all the results obtained.

In some cases chemical action undoubtedly takes place; in others the process is probably physical, and there is evidence that capillary action or surface tension is of great importance.

The experiments which follow suggest ways in which the dye is transferred to the fabric:

(1) Into vessel *A* (Fig. 330) pour clean water, and into vessel *B* a weak solution of saponine (1 gram of saponine to 500 c.c. of water).

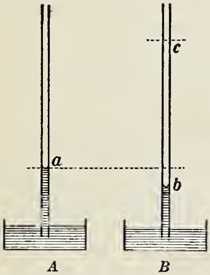


FIG. 330.—Showing concentration in the surface layer.

Hold a capillary tube in *A*. The water rises to a level *a*. Then remove the tube and hold it in *B*. The liquid now rises only to level *b*, considerably below level *a*.

This shows that the saponine solution has a smaller surface tension than clean water has.

Now draw the solution in *B* up to the level *c* and let it go suddenly. The column rapidly falls to level *a* and then settles less rapidly down to *b*.

While falling from *c* to *a* the liquid at the surface is being renewed constantly, and so the constitution of the surface layer is very approximately the same as that of the solution generally, which is little different from pure water. However, in a few seconds some of the saponine concentrates at the surface and produces a reduction in the surface tension. This gradual reduction is seen in the slow sinking of the column to its final height.

From this experiment we get a very important result. **When a substance, on being dissolved in water, reduces its surface tension, there is a concentration of the substance in the surface layer.**

This conclusion, indeed, is predicted from theoretical considerations based on the laws of thermo-dynamics, and it can be verified by many other experiments. A system always endeavours to change so as to have the least possible potential energy. When such a substance goes into the surface it reduces the surface energy, thus contributing to a reduction in the total potential energy.

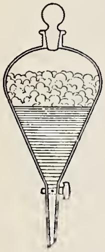


Fig. 331. — Froth of a solution of methyl violet above the liquid.

It is to be observed that the surface layer is excessively thin, so that the actual amount of matter concentrated there need not be great.

(2) Make a solution of methyl violet (1 gram to 4 litres of water). About one-third fill a large separating funnel (Fig. 331). Shake vigorously, causing froth to gather above the liquid.

Let it stand 4 or 5 minutes, to allow the liquid between the bubbles to run down. Then drain off all the liquid which has collected. Call this solution *A*.

Next, let it stand for 4 or 5 minutes more, to allow the froth to settle, and then draw off the liquid formed from it. Call this solution *B*.

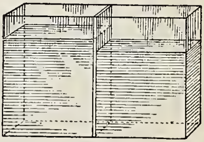


Fig. 332.—The solution from the froth is of a deeper colour than the other.

Now make a solution of 1 c.c. of *A* to 20 c.c. of water, and pour in one side of a double glass vessel with plane sides (Fig. 332).

Make a solution of 1 c.c. of *B* to 20 c.c. of water, and pour in the other side of the vessel.

Place the vessel in the lantern, and project on the screen, or hold it in front of a piece of white paper or up to the window.

It will be found that the second solution is of a slightly deeper colour than the first.

This result is explained as follows:—Methyl violet, when dissolved in water, reduces the surface tension of the water, and any substance which does that concentrates at the surface. The bubbles of froth have much surface compared to their mass, and the methyl violet is concentrated on their surfaces.

Hence, the liquid formed from the bubbles contains more of the dye per c.c. than does the liquid first drained off.

In performing this experiment be sure that the proportions in the two solutions compared are accurately the same as the final difference in colour is only slight. Use a 1 c.c. and a 20 c.c. pipette, previously rinsing out with some of the liquid to be measured.

If froth does not form on the solution, make a new one with fresh water.

(3) Place a drop of a weak solution of red ink on white filter or blotting paper, and observe how it spreads. When the action has ceased it will be found that the red colouring matter has spread a certain distance, but the water in the solution has gone a considerable distance farther.

Many solutions of salts or of dyes exhibit this phenomenon, the water diffusing amongst the fibres of the paper and leaving the dissolved substance behind upon the fibres.

Still more striking and beautiful effects are obtained with solutions of two dyes. Make a dilute solution of picric acid and crocein scarlet, and put several drops on white filter paper (supported on the top of a beaker). When the spreading has ceased there will be seen a large spot of red with a yellow fringe, and this surrounded by clear water. The picric acid diffuses more freely than the scarlet.

A solution of acid magenta and indigo sulphate of soda will give an indigo spot fringed with magenta.

Instead of putting drops on the paper, a strip of filter paper may be suspended with its lower end in the solution. The clear liquid rises highest, and usually one colour higher than the other, if two are present.

These experiments are easy to perform, and the results are beautiful and suggestive.

The action illustrated in the above experiments is almost certainly present in some cases of dyeing. **The coloured solution comes in contact with the surface of the material to be dyed; the tension of the surface there is reduced and the colouring matter concentrates at the surface and is deposited on the material.** Probably, with some materials, the water of the coloured solution passes freely through the capillary spaces, leaving the particles of the dye behind on the material.

### III.—*In Filtration*

**261. The Action of Filters.** Filters can be divided into two classes.

In filtering solid impurities, or a precipitate, from a liquid, the filtering material (paper, cloth, sand, etc.) has interstices through which the liquid can pass but the solid particles cannot. Surface tension does not enter here.

It has been known for many years that neutral filters, such as sand in layers, will remove colouring matter and, to some extent, salts in solution. This filtering action is undoubtedly intimately connected with the large amount of surface of the particles presented to the liquid, the greater the surface the stronger being the action.

In Experiment 4, above, the red matter in the ink was held back while the pure water flowed on.

The action in these cases is certainly a surface phenomenon, probably explainable in the same manner as the phenomena of dyeing just described above.

It may be well to remark, however, that in the purification of water by filtration other considerations enter. For a long time this was looked upon as a mechanical process of straining out the solid particles and thus rendering turbid water clear. But now it has been shown that in sand-filtration of water on a large scale an essential feature is the presence, in the upper surface layer of the sand, of colonies of bacteria forming jelly-like masses. Not until a fine film of mud and microbes has been formed upon the surface of the sand are the best results obtained.

### IV.—*Effect on Waves*

**262.** Everyone has noticed the brilliant colours produced by the spreading of a drop of oil over the surface of water. The oil spreads because its surface tension is less than that of water.

In a storm, the waves may be calmed to a certain extent by pouring oil on the water. The wind acting on a portion of this oil layer tends to drive it forward and so to expose a comparatively pure water surface, which has a stronger surface tension. As a result there is a backward pull tending to neutralize the forward motion of the wave. The persistence of the smooth lane of water showing the track of a steam-ship is probably due to the effect of small quantities of oil and other matter left on the surface by the passage of the ship.

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### *Solution for Soap-Films and Bubbles*

A solution of Castile soap and rainwater, with some Price's glycerine added to make the film last longer, will probably answer all purposes; but for the very best results a specially prepared solution is desirable.

The following is the recipe recommended by Reinold and Rücker and by Boys. Fill a stoppered bottle three-fourths full with distilled water, add one-fortieth by weight of fresh oleate of soda, and leave for a day to dissolve. Nearly fill the bottle with Price's glycerine, and shake well. Leave the bottle a week in a dark place, and then with a siphon draw off the clear liquid from under the scum into a clean bottle, add a drop or two of strong ammonia solution to each pint, and keep carefully in the stoppered bottle in a dark place, filling a small working bottle from it when required, but keeping the stock bottle undisturbed and never putting any back into it. Do not warm or filter the solution and never leave the stopper out or expose the liquid to the air.

# CHAPTER XXVII

## The Flow of Fluids

**263. Services Obtained from Flowing Fluids.** From an economic point of view the study of the laws of flowing fluids is of great importance. Immense stores of energy are present in the waters of our rapid rivers, and in order to utilize it we must know the laws according to which they move. In the systems of waterworks in our cities and towns the water is pumped into iron pipes, from which it is drawn for domestic use, for running elevators and water motors, and for other industrial purposes.

Air and steam, forced through pipes, are used for actuating drills, for driving turbine and ordinary engines, for applying the brakes on railway trains and street cars, for heating buildings, and for numerous other purposes.

Again, our winds are currents in the air, their motion being shown in the swaying of trees, and in the sweeping onward of clouds in the sky or of clouds of dust and smoke at the surface of the earth.

It is, therefore, evident that a knowledge of the laws in accordance with which fluids move is of the highest value. The phenomena, however, are very complicated, and the determination of their laws is a matter of difficulty.

**264. Unsteady and Steady Flow.** Consider the water moving forward in a river with a very irregular channel or flowing in a pipe which has a varying diameter, and which, perhaps, has abrupt changes in direction. If we could colour the particles of water in successive cross-sections, thus rendering it possible to trace their motions, we would probably be surprised to see the extraordinary way in which some of them eddy about instead of simply moving forward. The particles near the shore and bottom of the river, or near the surface of

the containing pipe, are continually being thrown into eddies. Such turbulent motion is called **Unsteady Flow.**

But it is evident that if the source of supply is perfectly constant, and if there are no abrupt changes in the channel, the flow will be continuously steady and successive particles passing any point will follow approximately the same paths. For example, if a vessel is kept constantly full by allowing water to run uniformly into it from a reservoir, and if the water is permitted to escape from an opening anywhere in the vessel, the motion of the particles which pass any fixed point in the vessel will be the same at all times. If a water-sprite could stand in the liquid and mark each particle as it came along in a certain direction to that point, all of these particles would be seen to follow the same curved path.

**By Steady Flow we mean that at any point in the stream the conditions remain constant with respect to time; and the lines imagined to be drawn in the liquid so as to be at each point in the direction of the flow, or, in other words, the lines along which the particles travel, are called Stream Lines.**

Thus, consider steady flow through a pipe with a contrac-

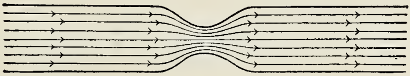


Fig. 333.—Stream lines in a pipe.

tion, or *throat,* in it (Fig. 333). The fine lines indicate the form of the stream lines.

Let us consider the stream lines drawn through a closed curve $a$ (Fig. 334) in the liquid. A particle of the fluid which commences to move along one of these lines will continue to do so. It is evident that these lines of flow taken together form a tube; it is called a **Tube of Flow.**


Fig. 334.—A tube of flow.

Since the line of flow, or stream line, passing through a point indicates the direction of flow at that point it is evident that **two lines of flow cannot cross each other.** If they did the motion at the point of intersection would have two directions, but in steady motion the movement of the particles at a point are continually in a single definite direction.

Such being the case, there can be no flow across the bounding walls of the tube, and the particles which are within the tube at one time will continue within it. The particles composing a cross-sectional layer will continue to be a cross-sectional layer.

Actually perfectly steady flow is seldom, if ever, met with in engineering, but in water mains, water turbines, power canals and hydraulic machinery in general, the flow is made as steady as possible. Unsteady flow means wasted energy.

**265. Height to Which a Jet will Rise.** In the apparatus shown in Fig. 335, the water escapes from a small orifice. The jet rises nearly to the level of the free surface of the liquid in the vessel, and we suspect at once that if there were no losses through friction the jet would rise exactly to that level.

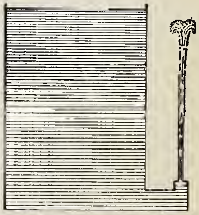


Fig. 335—Height of a jet of water.

Now if a body falls through a height $h$ it attains a velocity $v$ where $v = \sqrt{2gh}$. In the same way, if the body is thrown upward, and rises to a height $h$ the initial velocity $= \sqrt{2gh}$.

In the case of the jet of liquid, if $h$ is the depth of the orifice below the free surface in the vessel, the velocity of efflux $= \sqrt{2gh}$.

This relation is rigidly true only for a perfect liquid, or one which flows without friction.

**266. Flow of Liquid from an Opening in a Vessel.** The result obtained in the last section can be deduced from the principle of energy.

Let the opening be at a distance $h$ cm. below the surface of the liquid (Fig. 336). Let the density of the liquid be $\rho$, the area of the free surface be $A$ sq. cm., and the velocity of the outflowing liquid be $v$ cm. per sec., that is, a small speck of dust in the liquid at the opening would be carried forward at this rate.

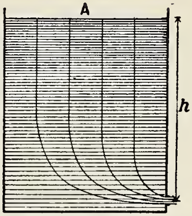


Fig. 336.—Calculation of velocity of outflow.

Suppose that in a very short time the level of the free surface falls a very small distance $x$ cm. Then the

$$\begin{aligned} \text{Volume of escaped liquid} &= Ax \text{ c.c.} \\ \text{Its mass} &= \rho Ax \text{ grams.} \\ \text{And if its velocity} &= v \text{ cm. per sec.,} \\ \text{Its kinetic energy} &= \tfrac{1}{2}\rho Axv^2 \text{ ergs (see Sec. 82).} \end{aligned}$$

This kinetic energy must be gained at the expense of the potential energy of the liquid. Now each layer has fallen through a height $x$ cm., or the entire volume has fallen through this distance. The mass is $\rho Ah$ grams, and its weight is $\rho Ahg$ dynes. Hence the

$$\text{Decrease in potential energy} = \rho Ahgx \text{ ergs.}$$

Therefore, $\qquad \tfrac{1}{2}\rho Axv^2 = \rho Ahgx,$

or $\qquad v^2 = 2gh,$

and $\qquad v = \sqrt{2gh};$

that is, **the velocity is the same as that which would be acquired by falling through the distance of the opening below the free surface.**

This is known as **Torricelli's Law.** It was formulated by him in 1643, more than 200 years before the principle of the conservation of energy had been established.

This result was obtained on the assumption that the liquid was perfect, that there was no friction in the passage of one layer over another, or in other words, that it had no *viscosity*. As a matter of fact, water, ether, alcohol, mercury and such liquids possess very little viscosity, and the law is very nearly fulfilled by them. In the case of water the velocity is not quite that given by theory, a small amount of the energy being transformed into heat. The velocity is approximately $\frac{98}{100} \times \sqrt{2gh}$.

**267. The Contracted Vein.** The rate at which liquid is escaping, however, cannot be found from knowing the area of the opening and the velocity $v$ of the efflux. Just outside the opening the jet contracts somewhat, and we must take the area of a cross-section where it is least. The size and shape of the cross-section is modified by the shape of the opening, and the area in general can be determined only by experiment. When the opening is a sharp-edged round orifice in a plane surface the area of the jet is on the average $\frac{62}{100}$ of that of the opening, or the cross-section of the jet is about $\frac{5}{8}$ of the area of the opening.

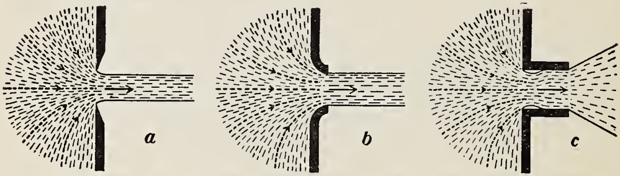


Fig. 337.—Discharge from three different circular orifices: *a*, sharp edged; *b*, curved; *c*, short tubular aperture.

The reason for this contraction will be seen by referring to Fig. 337*a*. The stream lines are converging as they approach the orifice and continue to converge for a short distance past the opening, with the result that the water in the central part of the orifice has difficulty in escaping. The velocity of efflux calculated in the preceding section is the **velocity at the contraction.**

Figs. 337*b* and 337*c* show the form of discharge from other types of circular orifices.

**Experiment.**—Test the rate of flow from orifices of different shapes, circular, square, triangular. This can conveniently be done by making an opening of some size (say $1\frac{1}{2}$ in. in diameter) near the bottom of a tank, and then placing over this plates with orifices of different shapes in them. The experiment in each case should continue only a short time so that the flow may be nearly uniform. The rate of flow can be determined by taking the time and observing the fall of the water in the tank, or better, by catching the water and measuring it.

Also compare the flow through a circular orifice in a thin plate with that through a short tubular orifice of the same internal diameter.

## PROBLEMS

(In the following problems, the density of water is to be taken as 62.5 lb. per cu. ft.; the velocity coefficient as 0.98 and the contraction coefficient as 0.62.)

1. In a water-works system the pressure is maintained by the water in a stand-pipe 100 feet high, situated on a hill, 50 feet above the valley. Find the pressure, in pounds per square inch, on the ground floor of a house in the valley.

2. If the stand-pipe is 30 metres high and the hill 20 metres above the valley, find the pressure in dynes per square cm., and also in kilograms per sq. cm.

3. A large tank, 3 metres high, is kept full by water continually running into it, and a small round opening, 1 cm. in diameter, is made at the base. At what rate will the water escape?

4. A can contains oil to a depth of 18 inches, and a small round hole $\frac{1}{8}$ inch in diameter is punched through it at the base. At what rate will the oil begin to run out?

**268. Energy of a Liquid Under Pressure.** A liquid possesses potential energy by virtue of its being submitted to pressure, and the amount of this energy can be calculated in the following way.

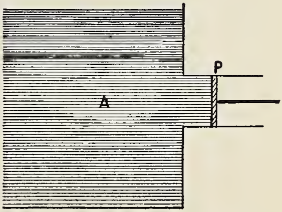


Fig. 338.—Energy due to pressure.

Let $A$ (Fig. 338) be a tank in which is water under a pressure

of $p$ grams, or $pg$ dynes, per sq. cm., and let $P$ be the piston of a pump by which water is forced into the tank. Let $a$ sq. cm. be the area of the piston. The total thrust on the piston is $ap$ grams, or $apg$ dynes, and when it moves inwards through a distance $x$ cm., it does $apx$ gm.-cm., or $apgx$ ergs, of work.

In doing so it forces $ax$ c.c. of water into the tank, which must possess as potential energy the energy expended in placing it where it is. This potential energy due to pressure will be called **pressure energy.**

Hence, $ax$ c.c. have $apx$ gm.-cm., or $apgx$ ergs, of P.E., and 1 c.c. has $p$ gm.-cm., or $pg$ ergs, of P.E.; *i.e.*, **the measure of the pressure energy per unit of volume possessed by a liquid is the same as the measure of the pressure to which it is subjected.**

Thus, if a liquid is under a pressure of 10,000 dynes per sq. cm., each c.c. of it possesses 10,000 ergs of pressure energy. If the pressure is 60 pounds per sq. ft., each cu. ft. possesses 60 ft.-pd. of pressure energy.

Examples of this effect are often seen. When water from the city waterworks, at a pressure of, say, 100 pounds per sq. inch, is admitted to the cylinder of an elevator in a high building, it performs work in raising the car of the elevator to the upper stories of the building. Or, when pumped into the cylinder of a hydrostatic press, immense pressures are produced, which are used in compressing bales, etc.

**269. Energy of a Liquid in Motion.** Let a liquid be flowing with a velocity of $v$ cm. per sec., and let $m$ grams be the mass of 1 c.c. (*i.e.*, the density) of the liquid,

Then the kinetic energy per c.c. $= \frac{1}{2}mv^2$ ergs.

If the velocity is $v$ ft. per sec. and density is $\rho$ lb. per cu. ft., then the kinetic energy per cu. ft. $= \frac{1}{2}\rho v^2$ ft.-poundals,

$$= \frac{\frac{1}{2}\rho v^2}{g} \text{ ft.-pounds,}$$

since 1 pound force $= g$ poundals.

**270. Rate of Flow of a Liquid.** First, consider steady flow in a tunnel or a pipe of uniform cross-section (Fig. 339). Let the area be $a$ sq. cm., and the velocity be $v$ cm. per sec. Then the amount which flows past any point in 1 sec. is $av$ c.c. In practical engineering work the rate of flow is usually stated in cu. ft. or cu. metres per sec.


FIG. 339.—Rate of flow = area multiplied by velocity.

Next, let the pipe have a contracted portion or throat, as in Fig. 340.

Let the area of the cross-section at $A_1$ be $a_1$ sq. cm., the velocity there be $v_1$ cm. per sec., and the pressure there $p_1$ dynes per sq. cm.

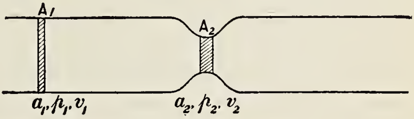



FIG. 340.—The velocity of a liquid is inversely proportional to its cross-section.

At $A_2$ let the corresponding values of these quantities be $a_2$, $v_2$, $p_2$.

Now the same quantity flows past $A_1$ and $A_2$ during 1 sec. Hence $a_1 v_1 = a_2 v_2$.

But $a_1$ is greater than $a_2$; hence, $v_2$ is greater than $v_1$, and we have the law: **The velocity of the liquid is inversely proportional to the area of the cross-section.**

**271. Relation between Pressure and Velocity.** Suppose the tube in which the liquid is flowing is horizontal as in Fig. 340, and that its central axis is at a height of $h$ cm. above the ground. Let us consider the motion of 1 c.c. of the liquid along the axis, from the centre of the section at $A_1$ to the centre of that at $A_2$.

Its energy at $A_1 = p_1 + \frac{1}{2}\rho v_1^2 + \rho g h$ ergs . . . . . . . . . . . . . (1)

Its energy at $A_2 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h$ ergs . . . . . . . . . . . . . (2)

But these must be equal, and therefore

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h \quad \text{. . . . . . . . (3)}$$

$$= \text{the corresponding expression for any section.}$$

Hence, the quantity

$p + \frac{1}{2}\rho v^2 + \rho gh$ is a constant for any section..........(4)

The relation (3) can be written

$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$..............................(5)

But since the area at $A_2$ is smaller than that at $A_1$, the velocity $v_2$ is greater than the velocity $v_1$, and also $v_2^2$ is greater than $v_1^2$. Hence, $p_1$ is greater than $p_2$, and we obtain the law that **when the velocity increases the pressure diminishes.**

**The pressure exerted by the liquid at a contracted portion of the pipe is less than where the pipe is larger.** This is entirely contrary to the view commonly held. Most people think that when the liquid enters a contracted portion its particles are squeezed together and it exerts a greater pressure against the walls of the pipe. This view, however, is quite erroneous.

The relation between pressure and velocity given in (4) above is a simple case of a law of hydraulics known as **Bernoulli's Principle.**

**272. Experimental Illustrations of Bernoulli's Principle.**

(1) Obtain a glass tube, blown as illustrated in Fig. 341, having two

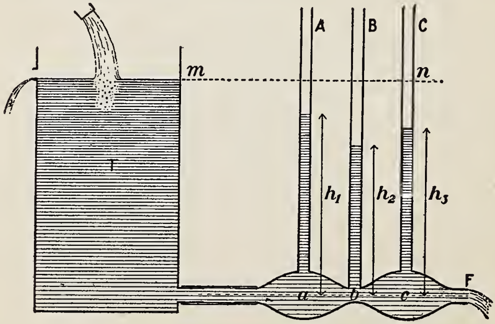



FIG. 341.—Apparatus to illustrate relation between pressure and velocity.

larger portions separated by a smaller neck, with a small tube rising from each of these portions. The large portions should have a diameter as great as possible, and their lengths should be several times as great as their diameters. If the tube is too small, friction considerably affects the flow, and if the expanded portions of the tube are too "bunty" the water is thrown into eddies and the flow is far from being steady.

Connect to a tank $T$ which is kept full of water, or attach directly to a water tap.

First, hold a finger over the end $F$. There will be no flow, and the water in the tubes $A$, $B$, $C$ will rise to the line $mn$, assuming the same level as in $T$.

Next, let the water run freely. Now if the particles of water are crowded together as the sections of the cone get smaller, and are thus subjected to increased pressure, this would be shown in the water level in the tubes. We might expect that in $B$ to be highest and that in $A$ or $C$ lowest; but such is not at all the case. They assume the levels shown in the figure. They are slightly lower than they would be if the water moved entirely without friction.

Observe that the pressures at $a$, $b$, $c$, etc., are those due to a 'head' $h_1$, $h_2$, $h_3$, etc. (cm.) respectively. Then if the corresponding pressures are $p_1$, $p_2$, $p_3$, etc. (dynes per sq. cm.) and if the tube is horizontal, we have the relations

$$p_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \tfrac{1}{2}\rho v_2^2 = \text{(similarly for each section)} = \text{constant.}$$

Note.—If the tube is joined directly to the water tap it is advisable to connect $A$, $B$ and $C$ by short pieces of rubber tubing and a glass $T$-tube in order to prevent the water from overflowing. The differences in the height of the water in the tubes will indicate the differences in the pressures at $a$, $b$ and $c$.

In place of the glass tubes shown in Fig. 341, an apparatus such as illustrated in Fig. 342 may be used. This consists of two zinc or tin cones soldered together, 3 inches in diameter at the common base and tapering to $\frac{1}{2}$ inch at the ends. The shorter is 3 inches, and the other 12 inches long. Three openings are in the longer cone. In these can be inserted corks through which glass tubes pass. The water should flow as shown in the figure.

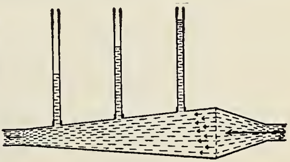


Fig. 342.—Simple apparatus to illustrate Bernoulli's principle.

A third convenient form of the apparatus is shown in Fig. 343. It is made of glass. One end of a **U**-tube is fused into the wide portion and the other end into the narrow portion of the tube. On causing the water to flow through, it rises in both arms of the **U**-tube, but higher in that portion joined to the wide part of the tube. The experiment shows that the pressure of the air within the **U**-tube, exerted upon the surface of the water in the two arms, is greater than an atmosphere, but it is the same in both arms.

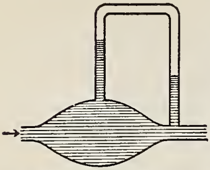


Fig. 343.—Another convenient form of apparatus.

(2) Another interesting experiment is illustrated in Fig. 344. A piece of glass tubing about 6 inches long and one-half inch in diameter is drawn out at the middle to form a constriction. The upper end is connected to the water tap by a piece of pressure tubing and the water is turned on slowly. When a certain rate of flow is reached a hissing sound is heard and the water in the lower half of the tube and in the beaker presents a foggy appearance.

The velocity in the constricted portion of the tube becomes great and the pressure correspondingly small. The reduction in pressure causes the air dissolved in the water to separate out and also causes a certain amount of boiling of the water at the constriction. A similar effect is sometimes noticed at a partially-opened tap.

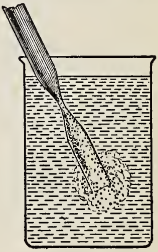


Fig. 344.— Water at room temperature boiling at a constriction in a tube.

**273. The Venturi Water Meter.** The object of this instrument is to measure the rate of flow in a water-main. Its

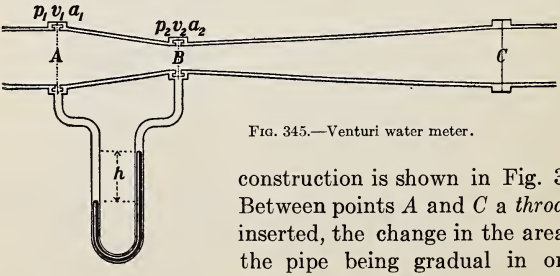


Fig. 345.—Venturi water meter.

construction is shown in Fig. 345. Between points $A$ and $C$ a *throat* is inserted, the change in the area of the pipe being gradual in order to avoid eddies. The areas of the cross-sections at $A$ and

$B$ are carefully measured and connections to pressure gauges or a manometer are inserted at these points. Now if we know the areas of these cross-sections and the difference between the pressures there, we can determine the flow in the pipe. To do so we must use the formula

$$p_1 - p_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2),$$

where $p_1$, $v_1$ are the pressure and velocity at $A$ and $p_2$, $v_2$ those at $B$.

$$\text{Since} \quad p_1 - p_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2),$$

$$p_1 - p_2 = \tfrac{1}{2}\rho v_1^2\left\{\left(\frac{v_2}{v_1}\right)^2 - 1\right\}.$$

$$\text{But} \qquad \frac{v_2}{v_1} = \frac{a_1}{a_2}. \quad \text{(Sec. 270)},$$

$$\text{hence,}\ p_1 - p_2 = \tfrac{1}{2}\rho v_1^2\left\{\left(\frac{a_1}{a_2}\right)^2 - 1\right\}.$$

Now, $p_1 - p_2$ is given by the difference of level $h$ of the mercury in the manometer and $\rho$, $a_1$ and $a_2$ are known. The velocity $v_1$ follows, and this multiplied by $a_1$ gives us the rate of flow in the pipe.

**Example.**—In a small Venturi meter, the diameter at $A$ is 10.5 cm. and the diameter at $B$ is 3.5 cm. Find the rate of flow of the water when $p_1 - p_2 = 50$ cm. of mercury.

$$50 \text{ cm. of mercury} = 50 \times 13.6 \times 980 \text{ dynes per sq. cm.}$$

$$\text{Hence, } 50 \times 13.6 \times 980 = \tfrac{1}{2} \times 1 \times v_1^2\ (9^2 - 1),$$

$$\text{whence} \qquad v_1 = 129.08 \text{ cm. per sec.}$$

$$\text{Rate of flow} = 129.08 \times \tfrac{22}{7} \times \left(\frac{10.5}{2}\right)^2 = 11181.5 \text{ c.c. per sec.}$$

$$= 11.1815 \text{ litres per sec.}$$

This meter was invented in 1887 by Clemens Herschel, an American engineer. He named it after Venturi, an Italian, who in 1797 described an experiment illustrating the principle on which it is constructed. There are other forms of water meters, but this one is especially convenient in the case of very large water-mains. In large meters the pressures are recorded automatically as in the barograph (Sec. 207).

## PROBLEMS

1. At what velocity must the water flow in a canal 30 feet wide and 8 feet deep to discharge 1000 cu. ft. per second?

2. If the canal narrows through a rock cutting to 25 ft. in width, find the velocity through the cut.

3. Find the work done in pumping 20 gallons of water into a boiler in which the pressure is 50 pounds per square inch. (1 gal. = 277.3 cu. in.).

4. Water in a pipe is under a pressure of 60 pounds per square inch and is flowing at the rate of 5 feet per second. Find the energy per cubic inch. (Neglect potential energy due to gravity).

5. If the pressure is 5 kilos. per sq. cm., and the rate of flow is 2 metres per second, find the energy per c.c.

6. In a Venturi meter the diameters at the wide section and at the throat are 30 and 10 cm. respectively. Find the rate of flow when the difference in the pressures is 360 gm. per sq. cm.

7. Taking the diameters to be 12 and 4 in. and the difference of pressure to be that of a head of 12 ft. of water, find the rate of flow.

8. In a glass tube 9 cm. in diameter is a throat 3 cm. in diameter and a **U**-tube is fused in as shown in Fig. 346. The **U**-tube contains mercury and when the water is flowing through the pipe, the difference in the mercury levels is 10 cm. Calculate the flow in c.c. per sec.

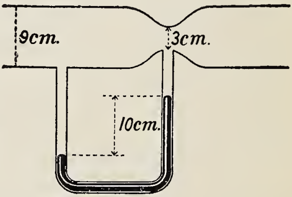


FIG. 346.

9. A large elevated tank supplies water to a house through a pipe of section 10 sq. cm.

(*a*) Find the pressure in the pipe 6 metres below the level of the water in the tank, when no water is flowing.

(*b*) Find the pressure at the same point (neglecting friction) when water is being drawn from the lower end of the pipe at the rate of 400 c.c. per second.

(*c*) Why does the rate of flow from one faucet diminish when a second one is opened?

**274. Examples of the Flow of a Gas.** The laws according to which a compressible fluid, such as a gas, flows are much more complicated; but when the variations in the pressure are not too great the relation between the pressure and the velocity still holds.

(1) Examine a Bunsen burner. The gas escapes from a small hole *A* (Fig. 347) in the base of the burner with a high velocity. The pressure, consequently, is reduced, and air rushes in through the opening *B* in the lower part of the tube, and the mixture of gas and air burns with a non-luminous flame at the top of the tube.

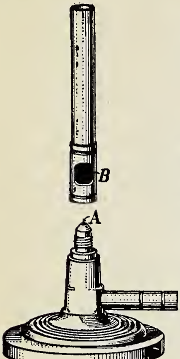



Fig. 347. — Bunsen burner (unscrewed to show construction.)

(2) In Fig. 348 a tube is fixed in a flat disc, the end of the tube being flush with the surface of the disc. A light disc of metal or cardboard is held near it by means of three metal pins which move freely through the lower disc. If a vigorous current of air is blown through the tube when it is held vertically, the lower disc will rise up to the other one. In this case the air spreads out in the space between the discs radially from the tube. As it spreads out its velocity is diminished and the pressure increased. Now at the rim the pressure is approximately that of the atmosphere, and so at the centre it must be less than one atmosphere. Hence, the atmospheric pressure on the lower side pushes the disc upward.

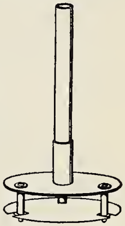


Fig. 348.—On blowing through the tube the lower disc rises.

A very simple form of the apparatus is shown in Fig. 349. A glass tube is pushed through a cork until its end is flush with the lower side. A thin layer of cork, with a pin through it to prevent it moving aside, will be drawn up to the thicker cork when a current of air is blown through the tube.

The above effect was first observed in some iron works in France, about 1826. One of the forge-bellows opened in a flat wall, and it was found that a board presented to the blast was sucked up against the wall.

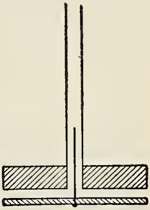


Fig. 349.—Simple form of apparatus.

(3) The simplest way to exhibit the effect, however, is due to Faraday. By means of the palm of the left hand hold snugly up against the palm of the right hand a piece of tissue paper 3 or 4 inches square, and then blow through the opening between the first and second fingers against the middle of the paper. Instead of being blown away,

the paper will be sucked up to the hand. After a few trials the experiment will be easily performed.

(4) Another simple experiment is shown in Fig. 350. *T* is a short, wide glass tube. Through a cork in one end is a glass tube *A* drawn out to a small opening *a*. Through a cork in the other end a wider tube *B* is inserted. At the bottom is a manometer *C* filled with coloured water. On blowing through *A* the liquid in *C* rises. Explain this.

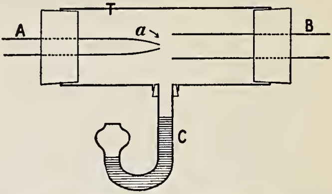



Fig. 350.–Experiment to test Bernoulli's principle.

**275. The Jet Pump.** The principle of the jet pump is illustrated in Fig. 351. Water is led from a reservoir *A* by a pipe *B* which tapers at *C*. The velocity here is great and the

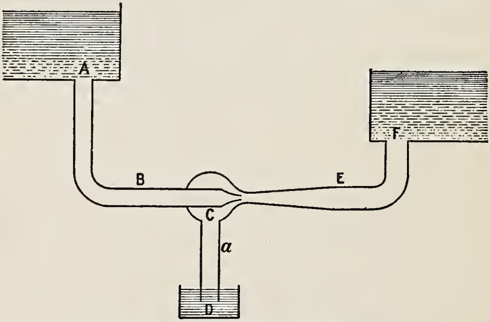



Fig. 351.—The principle of the jet pump.

pressure is reduced until below that of the atmosphere, which, acting upon the surface of the water in *D*, forces it up the pipe *a*. It mixes with the water flowing from C, and the combined stream flows on by the tube *E* to the reservoir *F*, which, however, cannot be higher than *A*. Thus the water is pumped from *D* up to *F*.

A simple apparatus for showing the action of this pump is illustrated in Fig. 352. The tube $B$ is attached to a water tap (or a supply of compressed air), and the tube $A$ is placed in the liquid to be pumped. To start the pump it may be necessary to fill it with water.

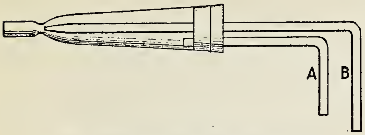



FIG. 352.—A simple form of jet pump.

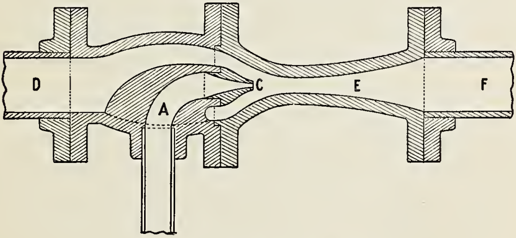



FIG. 353—A practical form of jet pump.

In Fig. 353 is shown a practical form of the pump. The water which supplies the energy for pumping enters at $A$. It discharges at $C$, and the water from $D$ is carried on by the pipe $E$ to the pipe $F$.

**276. The Bunsen Filter Pump.** Appliances for producing a suction current of air are known as aspirators. One of the commonest of these is the Bunsen filter pump, a vertical section of which is shown in Fig. 354. Water is forced through the tube-nozzle $N$, which gradually tapers and then expands again. At the place where its section is least there is joined on an off-set tube $A$, which is connected to the vessel from which the air is to be removed. An explanation often given is

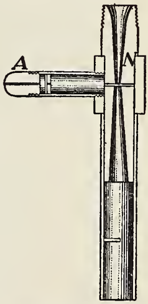



FIG. 354.—The Bunsen filter pump.

that the water rushing with great velocity through the narrow passage reduces the pressure there, causing the air to flow in through $A$ and be carried off by the water. But recent investigations show that Bernoulli's principle alone cannot account for the efficiency of the pump. Moreover the motion below the constriction is quite turbulent while Bernouilli's principle holds only for steady flow.*

In all probability the water stream, because of friction and viscosity, carries along a layer of air on its surface as it passes the constriction. Then the water stream breaks up into drops which act as "pistons" in carrying the air before them. The small baffle near the bottom of the pump helps to set up this action.

**277. The Atomizer.** The atomizer is an instrument for reducing a liquid to a fine spray. Its construction is shown in Fig. 355. On pressing the bulb $B$ an air-blast is forced in a jet from the fine opening $A$. It crosses the top of the tube $C$, and as its velocity is great the pressure just above the top of $C$ is much reduced. The pressure of the atmosphere on the surface of the liquid $D$ forces it up the tube, and as it escapes it is blown into a fine spray.

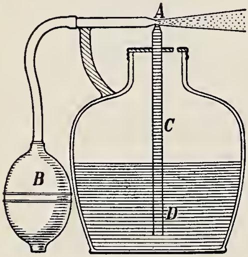



Fig. 355.—The atomizer.

The atomizer has many practical applications. It is used to obtain a fine shower of perfume, or a fine spray of oil in oil-burning engines. Artists render permanent their charcoal or crayon drawings by spraying them with a solution of mastic in alcohol. The alcohol evaporates and leaves the picture covered with a thin transparent varnish of mastic. The atomizer is often used also in medical practice.

*"The Bunsen Aspirating Pump," by W. C. Baker, Queen's University, Kingston, in "Physical Review," September, 1919.

**278. The Steam Injector.** This is an application for supplying steam-boilers with water, especially used with locomotives but not exclusively so. It was invented in 1858 by Giffard, a French engineer. The steam and water within the boiler are under considerable pressure, but by means of the injector the steam from the boiler, or even steam at a lower pressure, is able to force water into the boiler.

In Fig. 356 is shown a longitudinal section of the injector. Steam enters at *A* and blows through the round orifice *C*. Feed water flows in at *B*, and, meeting the steam at *C*, causes it to condense. In this way a vacuum is produced at *C*, and the water rushes in with great velocity down into the cone *D*, its velocity being increased by the steam from *C* striking it from behind. In the lower part of the nozzle *E* the stream expands; in doing so it loses velocity and gains pressure, and at the bottom the pressure is so great that it enters the boiler through a check valve which opens only in the direction of the stream. An overflow pipe *F*, by providing a channel through which steam and water may escape before the stream has acquired sufficient energy to force its way into the boiler, allows the injector to start into action. In the actual instrument there are certain valves for regulating the flow of the steam and the water which are not shown in the diagram.

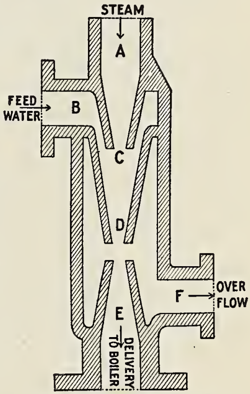


Fig. 356.—The steam injector.

The mechanical efficiency of the injector is much lower than that of the steam pump, but it has the advantage of working when the engine is still and of heating the feed water before delivering it to the boiler.

**279. The Ball Nozzle.** This is illustrated in Fig. 357. At the end of a tube is a hollow cup in which a ball fits snugly. If a vigorous current of air or steam is forced through the pipe its velocity at *a*, where it leaves the pipe, is greater than at the edge of the cup where it escapes into the atmosphere. Hence, the pressure at *a* is less than at the edge of the cup, and the latter is the pressure of the atmosphere. Consequently the atmospheric pressure on the side of the ball opposite *a* will prevent the ball from leaving the cup.


Fig. 357.—Ball nozzle.

**280. Forced Draught.** In order to keep a locomotive moving, steam must be generated rapidly, and to do this a fierce fire must be maintained. To secure such a fire the exhaust steam from the cylinder *C* of the engine is discharged through a contracted nozzle *A*, a little distance below the base of the smoke-stack *B*, which is usually flared out like an inverted funnel (Fig. 358). The steam escapes with high velocity. This reduces the pressure greatly and produces a powerful aspiratory effect, which draws in great quantities of air through the fire-box and the boiler tubes *D*, thus keeping up an intense fire.

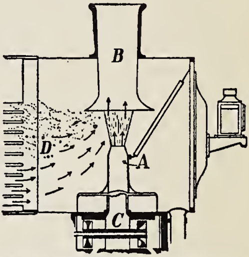



Fig. 358.—Forced draught in a locomotive.

## Other Illustrations of Bernoulli's Principle

**281. Curve of a Ball.** The curve given to a ball by a "cut" in tennis, by a "slice" in golf or by a skilful pitcher in baseball can also be accounted for by Bernoulli's Theorem.

In order to explain the effect it is more convenient to consider the ball as standing still while a current of air is forced past it, than to take the air as standing still and the ball to be rushing through it. From a mechanical point of view the conditions are equivalent—there is a motion of the ball relative to the air.

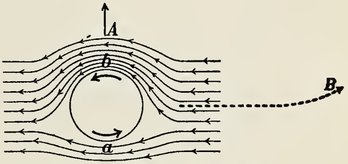


FIG. 359.—The curve of a ball.

The essential requisite to produce a curve is to give the ball a spin as well as a motion forward. Let the ball be spinning about a horizontal axis in the direction shown by the two curved arrows (Fig. 359), and let the air current be in the direction from right to left. The ball in its spinning carries round with it some of the air near its surface. At *b* the air carried round by the ball will unite with the motion of the outer air current, while at *a* it will oppose the outer air current. Consequently the velocity of the air current at *b* will be greater than at *a*, and the pressure at *a* will be greater than that at *b*. Hence, the ball will move across the air current in the direction from *a* to *b*, as shown by the arrow *A*. If now we consider the air to be at rest and the ball to be moving from left to right and having the same spin as before, it will curve up, as shown by the broken line *B*.

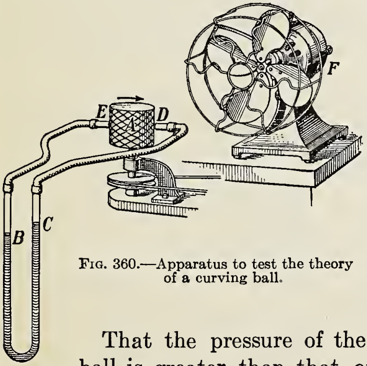


FIG. 360.—Apparatus to test the theory of a curving ball.

That the pressure of the air on one side of the ball is greater than that on the opposite side can be shown by the following experiment.

*A* (Fig. 360) is a wooden cylinder or ball about two inches in diameter

mounted on a rotator. The manometer $BC$ contains coloured water and its arms are connected by rubber tubing to two short tubes $D$ and $E$, held by clamps on opposite sides of $A$ and within about $\frac{1}{8}$ in. of it. The air-blast from the fan $F$ is directed on the cylinder when stationary and the tubes are adjusted until the manometer remains steady with the water at the same level in the two branches. On rotating the cylinder rapidly in the direction shown by the arrow the water at $C$ rises and that at $B$ sinks showing that the pressure at $D$ is less than that at $E$.

A roughened cylinder will be found to produce a greater difference in level than a smooth one since it carries more air with it.

(If the difference in level does not show up well when the manometer is vertical it should be placed in a nearly horizontal position where it will be much more sensitive. The tubes used should not have too great a diameter.)


FIG. 361.—Ball held up by a jet of air.

**282. Light Ball in a Jet of Steam or Air.** A light ball (made of celluloid, or a tennis ball), may be held in equilibrium by a jet of air or steam as illustrated in Fig. 361. The ball is under the action of three forces: its own weight $W$; $I$, the force of impact of the fluid against the ball; and $P$, an excess of atmospheric pressure over the pressure on the other side of the ball, due to the high velocity of the escaping fluid. With a few trials a position can usually be found for the ball where the three forces are in equilibrium, and the ball remains there.

Where a supply of compressed air is not available this action may be shown by using a bent glass tube of about $\frac{1}{4}$ in. internal diameter. Air blown through this by the mouth will support a sphere of cork about 1 cm. in diameter (Fig. 362). A fine wire stuck in the cork will serve as a guide to keep the ball from rolling to the floor when the velocity of the air jet slackens.

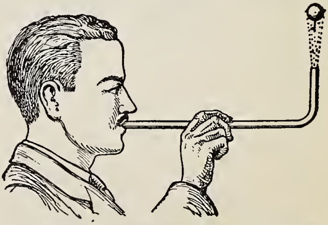


FIG. 362.—Simple apparatus for showing a ball supported by a jet of air.

**283. Two Balls in a Current of Air.** If two light balls are suspended side by side in a current of air from an electric fan (Fig. 363) the wind-current between the balls is greater than that on the other side of them. The air-pressure on the outer sides is therefore greater than that in the space between, and the balls are consequently pushed toward each other.

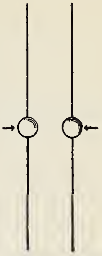


Fig. 363. — Balls pushed together when a current of air is directed upon them.

**284. Two Ships Steaming Side by Side.** If a ship is anchored in a river the water flows past it, the particles moving in definite stream lines. If the vessel is moving forward through still water, there is a similar relative motion between it and the water, and the resulting stream lines are similar to those in the other case.

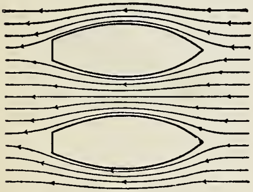


Fig. 364.—Ships drawn together when steaming side by side.

If two ships are steaming side by side (Fig. 364) the water streams past them more swiftly in the space between than on the outer sides. On account of this increased velocity the pressure exerted by the water against the inner sides of the ships is less than that against the outer sides, and the ships are pushed toward each other. One might expect the water between the ships to be heaped up, but such is not the case, its level is *below* the level at other places. Large ships should not manoeuvre too close to each other; accidents have occurred through ships being apparently drawn together in the manner just described.

## QUESTIONS

1. How should a pitcher make a base-ball spin to produce (*a*) an in-curve, (*b*) an out-curve?

2. In the Flettner rotor-ship, large vertical cylinders are made to rotate by an auxiliary engine and the wind acting on these cylinders tends to drive the ship in a direction at right angles to the wind. Explain the action.

Omit

# CHAPTER XXVIII

## The Aeroplane; Hydraulic Power

**285. The Aeroplane.** The aeroplane is justly considered one of the greatest triumphs of modern mechanics and the theory of its action provides an interesting example of the resolution of forces, in addition to many problems involving the flow of air.

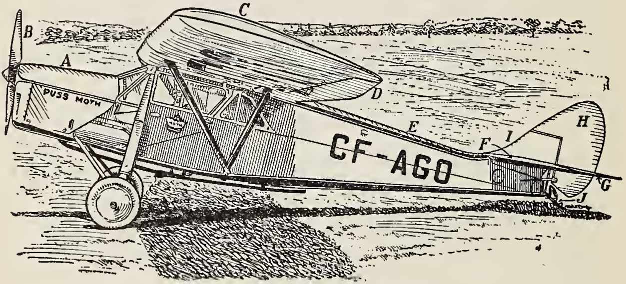



Fig. 365.—A modern monoplane.

Fig. 365 shows the principal parts of a monoplane.

A powerful engine $A$, mounted at the nose of the aeroplane, drives the propeller or air-screw $B$, by which the aeroplane is pulled through the air in much the same way as the rotation of an auger causes it to sink more deeply into the wood. Most of the lifting effect is caused by the action of the resulting air stream against the plane $C$ which has a concave under surface and a convex upper surface. Near the ends of the wings the ailerons $D$ are hinged to the trailing edge of the main plane.

At the rear of the fuselage $E$ the stabilizer $F$ is mounted, and to this are hinged the elevators $G$. The rudder $H$ is

hinged to the tail fin *I*. The tail skid *J* prevents damage to the tail in landing and also acts as a brake.

In taking off, the propeller is made to rotate at a high speed and the reaction of the air on it causes the plane to move forward. The resulting effect on the main plane, stabilizer, elevators and rudder is the same as if there was a strong wind blowing in the opposite direction against them.

While the aeroplane is running along the ground the tail lifts off the ground because of the action of the air stream on the stabilizer and elevators. Usually the latter are depressed to assist in this action. As soon as sufficient speed is attained the pilot raises the elevators which results in a lowering of the tail and a corresponding increase in the angle of incidence of the air stream against the main plane. This causes the aeroplane to climb, while depressing the elevators at any time results in the plane losing elevation.

To turn to the right the rudder is deflected to the right, while at the same time the right aileron is raised and the left one lowered. This action of the ailerons produces banking, that is, a dipping of the right wing and a corresponding raising of the left wing. Banking offsets the bad effects of centrifugal force in the same way that banking a railway track at a curve enables the train to take the curve at a high speed without being thrown off the track.

It is evident, then, that in all parts of the plane, including the propeller, we have examples of the action of an air stream against an inclined plane. This action in the case of the main plane will be considered more in detail in the next section.

In order to decrease "parasitic" air resistance, structural parts such as the fuselage, struts and landing gear, which do not contribute to the lift, are given stream-line form as far as possible.

Fig. 366 shows how great an effect the shape of a body has on the air flow around it. In Fig. 366*a* the body has good stream line form and there is an absence of eddies. In Fig.

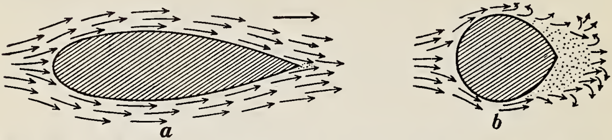



Fig. 366.—Air flow around bodies with, (*a*) good stream-line form, (*b*) poor stream line form. The arrows show the direction of the air stream; the dots represent areas of negative pressure.

366*b* there are many eddies and also an area of partial vacuum or "negative pressure," both of which retard the motion of the body.

**286. Forces Acting on the Main Plane.** Consider a plane surface $AB$ (Fig. 367) inclined to the horizontal at an angle $\theta$ moving from right to

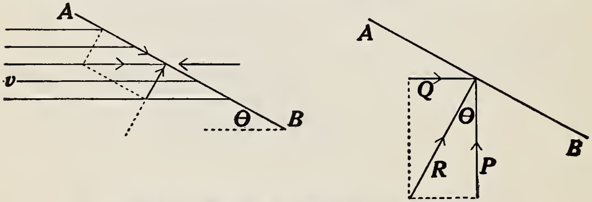



Fig. 367.—Finding the pressure upon an aeroplane.

left with a velocity $v$ ft. per sec. The pressures upon it due to the air are precisely the same as if the plane was held fixed and a current of air was directed against it from left to right with a velocity $v$.

This is equivalent to a stream of air with a velocity

$v \cos \theta$ along the plane,

and $v \sin \theta$ at right angles to the plane.

The former is assumed to slip without friction along the plane and so cannot produce any pressure on its surface. The latter exerts a force

$$R = k\ A\ \rho\ (v \sin \theta)^2 \text{ pdl.} \quad \text{(Sec. 56),}$$
$$= k\ A \times 0.08\ (v \sin \theta)^2 \text{ pdl.,}$$
$$= k\ A \times 0.0025\ (v \sin \theta)^2 \text{ pd.,}$$

where $k$ is a constant depending on the shape of the plane and $A$ is its area in sq. ft.

The pressure $R$ may be resolved into two components (Fig. 367),

$$P = R \cos \theta, \text{ vertically upward,}$$
$$Q = R \sin \theta, \text{ horizontally to the right.}$$

Consequently as the plane is rushing forwards there is developed a lifting force $P$, and a resistance to the motion $Q$ which must be overcome by the engine. These forces are called "lift" and "drag" respectively.

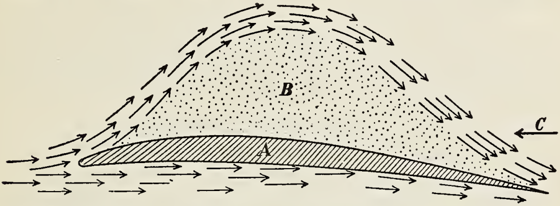



FIG. 368.—The air-flow past a plane. $A$, cambered wing; $B$, area of negative pressure; $C$, direction of motion of plane.

In the actual construction of aeroplanes the wings are made arched, like a bird's wing, and this increases the lifting power (Fig. 368).

Moreover, about seventy-five per cent. of the lifting effect is due to the decreased pressure on the top surface of the plane produced by the air currents in that region. This area of negative pressure, $B$, is indicated by dots in the diagram.

**287. The Hydraulic Ram.** The hydraulic ram (Fig. 369) is an interesting device for utilizing the inertia of a moving column of water. It consists of a reservoir $A$ fed by a natural stream, and from this a pipe $B$ of considerable

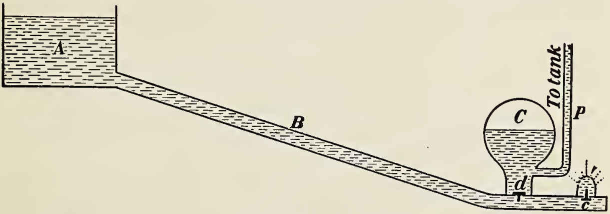



FIG. 369.—The hydraulic ram. Water is raised from $A$ to a considerable height.

length leads the water to a lower level where it rushes against and closes a valve $c$. The inertia of the column carries it onward, and, pushing upward the valve $d$, some of the water enters the chamber $C$ and thence goes into

the pipe $P$, which runs up to a tank in the attic of a house or in some other elevated position. Immediately after coming to rest the water rebounds, and the valve $c$ drops. This allows some water to escape, and the column starts moving in the pipe $B$ again, and the operation is repeated. The pipe $B$ should be comparatively long and straight. The greater part of the water escapes at $c$, but a fall of (say) 4 feet can raise the remaining portion to a height of perhaps 30 feet.

**288. The Hydraulic Air Compressor.** An application of the principles involved in the flow of fluids is to be seen in the great air compressor at Ragged Chutes, on the Montreal River, eight miles south-west from Cobalt, the centre of a great mining region in Northern Ontario.

A cement dam 660 feet long across the river raises the level of the water. By a large tube $A$ (Fig. 370) the water is led into two vertical pipes $P$ (only one shown in the figure), 16 feet in diameter into each of which is fitted a framework holding 66 intake pipes $a$, $a$, 14 inches in diameter. The water-line is about 10 or 12 inches above the top of the nest of intake pipes. In descending the water forms a vortex in the mouth of each pipe through which air is drawn down into the shaft below. Thus air and water are mixed together. At $b$ the pipe is reduced to 9 feet and near the bottom, at $c$, is enlarged to $11\frac{1}{2}$ feet in diameter.

The water drops 350 feet, falling on a steel-covered cone $B$, from which it rushes into a horizontal tunnel over 1000 feet long, the farther end $d$ of which is 42 feet high. In this large channel the water loses much of its speed and the air is rapidly set free, collecting in the upper part of the tunnel. At $e$ the tunnel narrows and the water races past and enters the tail-shaft $T$, 300 feet high, from which it flows into the river again.

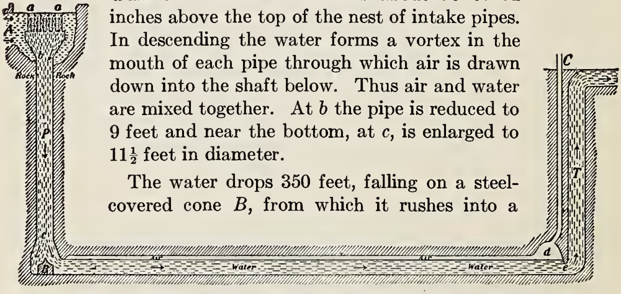



Fig. 370.—Taylor air compressor at Ragged Chutes on Montreal River (section).

The air entrapped in the tunnel is under a pressure due to about 300 feet of water, or about 125 pounds per square inch. From $d$ a 24-inch steel pipe leads to the surface of the earth, and from here the compressed air is piped off to the mines.

Other air compressors on the same principle are to be found at Magog, Quebec; at Ainsworth, B.C.; at the lift-lock at Peterborough and at the

Victoria Mines in Michigan; but the one near Cobalt is the largest in existence.

**289. Water Power.** From early times men have used water-wheels to transform the energy of falling and running water into useful work. Many forms have been invented, the most modern and most efficient being the *Impulse* or *Pelton Wheel* and the *Reaction Turbine.*

**290. The Impulse Wheel.** The small water-motor (Fig. 371) used for driving washing-machines and other household appliances is an example of the impulse wheel. The water, under considerable pressure, comes to the motor by the pipe *A* and issues from the small nozzle with high velocity. The impact of the water on the cup-shaped buckets of the wheel cause it to rotate with great speed. Having done its work the water leaves the motor by the pipe *C*.

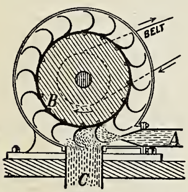


Fig. 371.—The Pelton water-wheel.

Pelton wheels are generally used where the fall of the water is very great, say above 1000 feet. They have been constructed with diameters as great as 10 feet, and sometimes the buckets are arranged in pairs about the periphery of the wheel so that two jets of water side by side may add their effects. An impulse wheel using a single jet has been made to develop over 20,000 horse-power.

**291. The Reaction Turbine.** This type of water-wheel is now being almost universally installed in large power plants where only a moderate head of water is available. Some of the finest examples are to be found in the neighbourhood of Niagara Falls, among the largest being those of the Hydro-Electric Power Commission of Ontario.

Fig. 372 shows the general arrangement of the Commission's power plant at Queenston. Water from the Niagara River several miles above the Falls is conducted by a canal 13 miles

long to the top of the cliff at Queenston where it is delivered through a steel penstock *A* to the 60,000 horse power turbine *B* which is directly connected by a vertical shaft 30 inches in diameter, to the 45,000 kilowatt generator *C* immediately above it. The electricity is generated at a pressure of 12,000 volts and is "stepped up" to a pressure of 110,000 volts by the transformer *D* from which leads off the transmission line *E*.

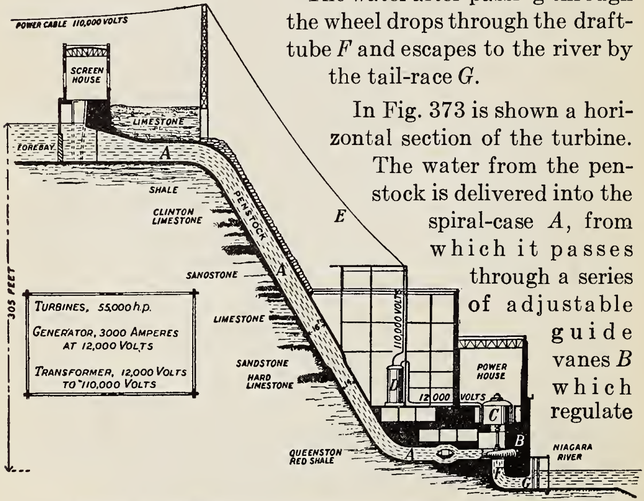



FIG. 372.—Arrangement of Hydro-Electric power plant at Queenston.

The water after passing through the wheel drops through the draft-tube *F* and escapes to the river by the tail-race *G*.

In Fig. 373 is shown a horizontal section of the turbine. The water from the penstock is delivered into the spiral-case *A*, from which it passes through a series of adjustable guide vanes *B* which regulate the inward flow of the water and also direct it against the blades of the "runner" *C* in a direction best adapted to produce rotation. *D* is the shaft of the runner. The water moves through the runner inwards and downwards and the blades are curved to take advantage of both motions. On leaving the runner the water passes out through the draft-tube into the tail-race.

Figure 374 shows the guide vanes and the mechanism by which they are controlled. Fig. 375 gives a good idea of

the enormous size of the runner and also shows how the blades are curved. The runner is of cast steel. Its outside diameter is 10 ft. 5 in., its weight is 42,000 lbs. and it ro-

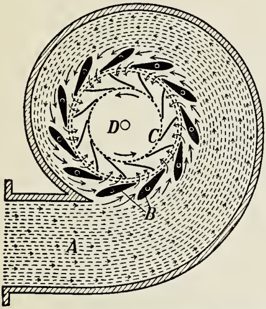



FIG. 373.—Horizontal section through the turbine showing spiral-case *A*, guide vanes *B* and runner *C*.

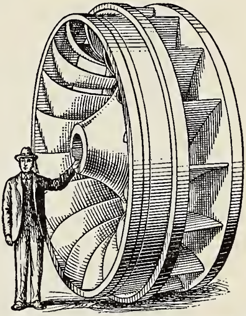


FIG. 375.—The runner or rotating part of the turbine. Note its great size and the curvature of the blades.

tates 187½ times per minute. The power house at Queenston contains 10 turbines similar to the one just described, developing a total of nearly 600,000 horse power.

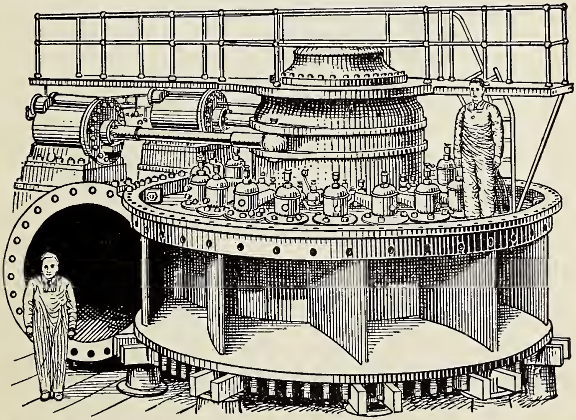
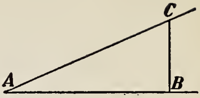


FIG. 374.—Part of the spiral case is removed showing the guide vanes. The mechanism for controlling them is seen above.

## Sines, Cosines and Tangents

Sine of angle $CAB = BC/AC$,
Cosine of angle $CAB = AB/AC$,
Tangent of angle $CAB = BC/AB$.
$\text{Cos } A = \text{Sin } (90° - A)$.
For example, $\text{Cos } 20° = \text{Sin } 70° = 0.9397$




| Angle | Sine | Cosine | Tangent | Angle | Sine | Cosine | Tangent |
|---|---|---|---|---|---|---|---|
| 0° | 0.0000 | 1.0000 | 0.0000 | 30° | 0.5000 | 0.8660 | 0.5774 |
| 1 | 0.0175 | 0.9998 | 0.0175 | 31 | 0.5150 | 0.8572 | 0.6009 |
| 2 | 0.0349 | 0.9994 | 0.0349 | 32 | 0.5299 | 0.8480 | 0.6249 |
| 3 | 0.0523 | 0.9986 | 0.0524 | 33 | 0.5446 | 0.8387 | 0.6494 |
| 4 | 0.0698 | 0.9976 | 0.0699 | 34 | 0.5592 | 0.8290 | 0.6745 |
| 5 | 0.0872 | 0.9962 | 0.0875 | 35 | 0.5736 | 0.8192 | 0.7002 |
| 6 | 0.1045 | 0.9945 | 0.1051 | 36 | 0.5878 | 0.8090 | 0.7265 |
| 7 | 0.1219 | 0.9925 | 0.1228 | 37 | 0.6018 | 0.7986 | 0.7536 |
| 8 | 0.1392 | 0.9903 | 0.1405 | 38 | 0.6157 | 0.7880 | 0.7813 |
| 9 | 0.1564 | 0.9877 | 0.1584 | 39 | 0.6293 | 0.7771 | 0.8098 |
| 10 | 0.1736 | 0.9848 | 0.1763 | 40 | 0.6428 | 0.7660 | 0.8391 |
| 11 | 0.1908 | 0.9816 | 0.1944 | 41 | 0.6561 | 0.7547 | 0.8693 |
| 12 | 0.2079 | 0.9781 | 0.2126 | 42 | 0.6691 | 0.7431 | 0.9004 |
| 13 | 0.2250 | 0.9744 | 0.2309 | 43 | 0.6820 | 0.7313 | 0.9325 |
| 14 | 0.2419 | 0.9703 | 0.2493 | 44 | 0.6947 | 0.7193 | 0.9657 |
| 15 | 0.2588 | 0.9659 | 0.2679 | 45 | 0.7071 | 0.7071 | 1.0000 |
| 16 | 0.2756 | 0.9613 | 0.2867 | 46 | 0.7193 | 0.6947 | 1.0355 |
| 17 | 0.2924 | 0.9563 | 0.3057 | 47 | 0.7313 | 0.6820 | 1.0724 |
| 18 | 0.3090 | 0.9511 | 0.3249 | 48 | 0.7431 | 0.6691 | 1.1106 |
| 19 | 0.3256 | 0.9455 | 0.3443 | 49 | 0.7547 | 0.6561 | 1.1504 |
| 20 | 0.3420 | 0.9397 | 0.3640 | 50 | 0.7660 | 0.6428 | 1.1918 |
| 21 | 0.3584 | 0.9336 | 0.3839 | 51 | 0.7771 | 0.6293 | 1.2349 |
| 22 | 0.3746 | 0.9272 | 0.4040 | 52 | 0.7880 | 0.6157 | 1.2799 |
| 23 | 0.3907 | 0.9205 | 0.4245 | 53 | 0.7986 | 0.6018 | 1.3270 |
| 24 | 0.4067 | 0.9135 | 0.4452 | 54 | 0.8090 | 0.5878 | 1.3764 |
| 25 | 0.4226 | 0.9063 | 0.4663 | 55 | 0.8192 | 0.5736 | 1.4281 |
| 26 | 0.4384 | 0.8988 | 0.4877 | 56 | 0.8290 | 0.5592 | 1.4826 |
| 27 | 0.4540 | 0.8910 | 0.5095 | 57 | 0.8387 | 0.5446 | 1.5399 |
| 28 | 0.4695 | 0.8829 | 0.5317 | 58 | 0.8480 | 0.5299 | 1.6003 |
| 29 | 0.4848 | 0.8746 | 0.5543 | 59 | 0.8572 | 0.5150 | 1.6643 |
| 30 | 0.5000 | 0.8660 | 0.5774 | 60 | 0.8660 | 0.5000 | 1.7321 |

Sines, Cosines and Tangents—*Continued.*

| Angle | Sine | Cosine | Tangent | Angle | Sine | Cosine | Tangent |
|---|---|---|---|---|---|---|---|
| 61° | 0.8746 | 0.4848 | 1.8040 | 76° | 0.9703 | 0.2419 | 4.0108 |
| 62 | 0.8829 | 0.4695 | 1.8807 | 77 | 0.9744 | 0.2249 | 4.3315 |
| 63 | 0.8910 | 0.4540 | 1.9626 | 78 | 0.9781 | 0.2079 | 4.7046 |
| 64 | 0.8988 | 0.4384 | 2.0503 | 79 | 0.9816 | 0.1908 | 5.1446 |
| 65 | 0.9063 | 0.4226 | 2.1445 | 80 | 0.9848 | 0.1736 | 5.6713 |
| 66 | 0.9135 | 0.4067 | 2.2460 | 81 | 0.9877 | 0.1564 | 6.3138 |
| 67 | 0.9205 | 0.3907 | 2.3559 | 82 | 0.9903 | 0.1392 | 7.1154 |
| 68 | 0.9272 | 0.3746 | 2.4751 | 83 | 0.9925 | 0.1219 | 8.1443 |
| 69 | 0.9336 | 0.3584 | 2.6051 | 84 | 0.9945 | 0.1045 | 9.5144 |
| 70 | 0.9397 | 0.3420 | 2.7475 | 85 | 0.9962 | 0.0872 | 11.4301 |
| 71 | 0.9455 | 0.3256 | 2.9042 | 86 | 0.9976 | 0.0698 | 14.3007 |
| 72 | 0.9511 | 0.3090 | 3.0777 | 87 | 0.9986 | 0.0523 | 19.0811 |
| 73 | 0.9563 | 0.2924 | 3.2709 | 88 | 0.9994 | 0.0349 | 28.6363 |
| 74 | 0.9613 | 0.2756 | 3.4874 | 89 | 0.9998 | 0.0175 | 57.2900 |
| 75 | 0.9659 | 0.2588 | 3.7321 | 90 | 1.0000 | 0.0000 | Infinity |

Densities of Substances, in Grams per Cubic Centimetre

| | | | |
|---|---|---|---|
| Alcohol, ethyl | 0.791 | Lead, cast or wrought | 11.34 |
| Alcohol, methyl | 0.810 | Maple (average) | 0.68 |
| Aluminium, cast | 2.56 | Marble | 2.65 |
| Aluminium, wrought | 2.72 | Mercury | 13.60 |
| Benzine | 0.90 | Nickel | 8.60 |
| Bismuth | 9.80 | Oak (average) | 0.75 |
| Brass wire (70Cu+30Zn) | 8.70 | Paraffin | 0.89 |
| Cadmium, cast | 8.56 | Petroleum | 0.878 |
| Cedar (average) | 0.53 | Pine, white (average) | 0.42 |
| Cobalt, cast | 8.60 | Pine, red (average) | 0.59 |
| Cork (average) | 0.24 | Platinum | 21.45 |
| Copper, cast | 8.88 | Sea-water | 1.025 |
| Copper, wrought | 8.90 | Silver, cast | 10.45 |
| Diamond | 3.5 | Silver, wrought | 10.56 |
| Glycerine | 1.26 | Steel, wire | 7.85 |
| Gold, wrought | 19.34 | Sulphuric acid | 1.84 |
| Ice | 0.90 | Tin, cast | 7.29 |
| Iridium | 22.10 | Tungsten | 19.12 |
| Iron, gray cast | 7.08 | Uranium | 18.49 |
| Iron, wrought | 7.85 | Zinc, cast | 7.10 |

For further information on Mechanics the student is referred to the following bibliography.


R. W. Angus, *Hydraulics for Engineers.*

C. V. Boys, *Soap Bubbles.*

Briggs and Bryan, *Matriculation Mechanics and Hydrostatics.*

John Cox, *Mechanics.*

C. V. Durell, *A School Mechanics (in 3 parts).*

J. Duncan, *Applied Mechanics for Beginners.*

J. Duncan, *Steam and Other Engines.*

R. L. Daugherty, *Hydraulics.*

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W. D. Eggar, *Mechanics.*

R. T. Glazebrook, *Mechanics and Hydrostatics.*

W. D. Hills, *Mechanics and Applied Mathematics (in 2 parts)*

Sir O. Lodge, *Pioneers of Science.*

L. S. Marks, *Mechanical Engineers Handbook.*

E. Nightingale, *Experimental Hydrostatics and Mechanics.*

V. W. Pagé, *Everybody's Aviation Guide.*

*Encyclopedia Brittanica;* 14th Edition, Various Articles.

# ANSWERS

**Page 9.** **1.** 2,500,000 mm. **2.** 299,730.96 km. **3.** 29.921 in. **4.** 183.49 m. **5.** 4.80 mm. **6.** 16.535. **7.** 535.797. **8.** 2.37 in. **9.** 76.390 cm.; 30.088 in. **10.** 17.4; 0.166. **11.** 8° 44′. **12.** 4.59 mm.

**Page 17.** **3.** 88. **4.** 108. **5.** (1) $27\frac{3}{11}$, (2) $3\frac{9}{22}$. **6.** $48\frac{8}{9}$. **7.** (1) 2:1, (2) 11:6. **8.** 5:56. **9.** $\frac{44}{45}$. **10.** 60 miles. **11.** 7200. **12.** $\frac{15}{22}\,ab$. **13.** $36\,\frac{ch}{s}$ m. **14.** (1) $\frac{5}{44}$, (2) $1\frac{3}{22}$. **15.** $\frac{22}{15}\frac{bc}{a}$. **16.** 11 miles/day. **17.** $2\frac{9}{11}$ miles/day. **18.** (1) 10.5 cm./sec.; (2) 10 cm./sec.; (3) 11cm./sec. **19.** (1) 1 cm./sec.; (2) 3 cm./sec.; (3) 1 cm./sec.; (4) 1 cm./sec.

**Page 20.** **1.** 13 m.p.h. **2.** 14 ft./sec.; 8 ft./sec.; 11.4 ft./sec. **3.** 12.30 m.p.h. **4.** 62.45 cm./sec.; 68.06 cm./sec. **5.** 7.55 ft./sec.

**Page 33.** **1.** $2\frac{17}{54}$cm./sec./sec. **2.** $-\frac{1}{12}$ ft./sec./sec. **3.** 600 ft./sec.; 600. **4.** (1) 300 cm./sec., (2) 18,000 cm./sec. **5.** (1) $\frac{1}{5}$ ft./sec.; (2) $\frac{1}{300}$. **6.** (1) 0.5 ft./sec.; (2) $\frac{1}{120}$. **7.** 10 min. **8.** 1 sec. **9.** (1) 6; (2) 2; (3) $\frac{1}{10}$; (4) $\frac{1}{30}$. **10.** (1) 50, (2) 5000, (3) $\frac{5}{6}$, (4) $83\frac{1}{3}$. **11.** (1) 1, (2) 3600, (3) 3600, (4) 60. **12.** (1) 1200, (2) 72,000, (3) 720, (4) 12, (5) $\frac{1}{5}$. **13.** (1) 30, (2) 108,000.

**Page 39.** **1.** 62.5 cm./sec./sec.; .08 sec. **2.** 90.06 cm./sec./sec.; .06 sec. **3.** 314 cm./sec./sec.

**Page 42.** **1.** 100 cm./sec. **2.** 20. **3.** −185 cm./sec. **4.** (1) 5, (2) 165 cm./sec., (3) 20 sec. before its velocity was 100 cm./sec. **5.** (1) 10 sec., (2) $3\frac{1}{8}$ sec. **6.** (1) 550 cm., (2) 1 sec. **7.** (1) 1.5 sec., (2) 11.25 cm. from starting point. **8.** (1) 160 ft., (2) 250 ft., (3) 90 ft. **9.** 156 ft. **10.** 20 ft./sec. /sec. **11.** (1) 12, (2) 78 ft. **12.** 6 ft./sec./sec. **13.** −32 ft./sec./sec. **14.** 2 sec.; $\frac{7}{8}$ sec.

**Page 45.** **1.** 81.25 cm./sec./sec.; 76.33 cm./sec./sec.; 78.08 cm./sec./sec. **2.** 80.65 cm./sec./sec.; 78.55 cm./sec./sec.; 80.50 cm./sec./sec.

**Page 50.** **1.** 192 ft./sec.; 576 ft. **2.** 190 cm./sec. **3.** 128 ft./sec. **4.** 7.82 sec.; 4.37 sec. **5.** $759\frac{3}{8}$ ft.; $33\frac{3}{4}$ sec. **6.** $\frac{1}{2}$ ft./sec./sec. **7.** 4 sec.; 1 sec.; 78.4 m. **8.** 144 ft. or 44.1 m. **9.** 15 sec. **10.** Yes; $29\frac{1}{3}$ ft. to spare. **11.** (1) 160 ft./sec., (2) 320 ft./sec. **12.** (1) 420 ft./sec., (2) 260 ft./sec. **13.** (1) 1960 cm./sec., (2) 980 cm./sec. **14.** (1) 256 ft., (2) 112 ft., (3) $156\frac{1}{4}$ ft. **15.** 49 m. **16.** $156\frac{1}{4}$ ft. **17.** (1) 400 ft., (2) 16 ft. **18.** 25 ft. **19.** 100 m. **20.** (1) $1\frac{1}{2}$ sec. and $4\frac{1}{2}$ sec., (2) 3 sec. **21.** (1) $2\frac{1}{2}$ sec., (2) $4\frac{1}{2}$ sec. **22.** (1) 6 sec., (2) 5 sec. **23.** (1) 96 feet/sec., (2) 126 ft./sec., (3) 80 ft./sec. **24.** (1) 36 ft./sec., (2) 20 ft./sec. **25.** (1) $39\frac{1}{16}$ ft., (2) 116.49 ft./sec.

**Page 55.** **1.** 50 ft. from the house. **2.** 1250 m./sec. **3.** 12 sec.; 480 ft. from point on earth directly below the balloon. **4.** 5776 ft.; 1862 ft. **5.** 18.606 sec.; 1823.39 ft. **6.** 792 ft.

**Page 63.** **1.** 625. **2.** 15 lb. **3.** (1) 200 dynes, (2) 25,000 dynes, (3) 30,000 dynes, (4) $55\frac{5}{9}$ dynes, (5) 30 dynes, (6) $\frac{mv}{t}$ dynes. **4.** (1) 1 cm./sec./sec., (2) $\frac{3}{1000}$ cm./sec./sec., (3) 1960 cm./sec./sec. **5.** (1) $\frac{1}{2}$ gm., (2) $2\frac{1}{2}$ gm., (3) 216 gm., (4) 3920 kg. **6.** 5 cm./sec./sec.; 25 cm./sec.; 10,000 units. **7.** 5 gm.; 2 cm./ sec./sec. **8.** 200 dynes. **9.** 75 ft./sec.; 5 ft./sec./sec.; 750 units. **10.** 7 : 15. **11.** (1) $n:1$, (2) $1:n$. **12.** $c:1$.

**Page 68.** **1.** (1) 9,800,000, (2) $\frac{1}{98}$, (3) 384, (4) 10. **2.** (*a*) 25, (*b*) 800. **3.** (*a*) 500,000, (*b*) 510.2. **4.** 3750 : 49. **5.** $\frac{ma}{g}$ gm. **6.** 10 min. **7.** 37.5 dynes. **8.** 785 cm./sec./sec. **9.** 7,350,000 units. **10.** 80 cm./sec./sec.; 144,000 dynes. **11.** 2520 cm.; 1,680,000 dynes. **12.** $\frac{4}{7}$ sec.; 56 cm./sec. **13.** (1) 1750 cm., (2) 1050 cm., (3) 700 cm./sec. **14.** 712 cm./sec.; 106,800 dynes. **15.** 8960 cm.; 1120 cm. **16.** (1) 7750 cm., (2) 2790 cm. **17.** 985 gm. **18.** 5 : 3.

**Page 73.** **1.** $2 \times 10^7$ dynes. **2.** $41\frac{2}{3}$ pd. **3.** 944.64 pd. **4.** 8.856 pd. **5.** 24.6 pd.

**Page 78.** **1.** Doubled. **4.** $44\frac{4}{9}$, 25, 16 kg. **5.** 0.37 pd.

**Page 82.** **1.** 68.03 gm. **2.** $\frac{1}{60}$ dyne.

**Page 88.** **4.** 3 : 1. **5.** 11.72 ft./sec. **6.** $2\frac{1}{4}$ ft./sec. **7.** 1900 ft./sec. **8.** 5 ft./sec. **9.** 31.25 F.P.S. units; 24.4 pd. **10.** 10 tons; 32 ft./sec.

**Page 93.** **1.** 1040.5 m.p.h. **2.** 23,025.1 m.p.h.

**Page 97.** **1.** 8224.5. **2.** 7254.

**Page 104.** **1.** (*a*) 100 ft.-pd.; (*b*) 3200 ft.-pd. **2.** (*a*) 640 ft.-pd.; (*b*) 20,480 ft.-pd. **3.** 100,000 ergs. **4.** 1800 ft.-pd. **5.** 50,000 ft.-pd. **6.** $\frac{1}{245}$ kg.-m. **7.** 150,000 ft.-pd. **8.** 528,000 ft.-pd. **9.** 98,000 joules. **10.** 98,000 joules. **11.** 3920 joules. **12.** 144 joules. **13.** 1,886,500 joules. **14.** 1509.2 joules.

**Page 113.** **1.** (1) 3200 ft.-pdl.; (2) 100 ft.-pd. **2.** (1) 4,802,000 ergs, (2) 1,200,500 ergs, (3) 0, (4) 4,802,000 ergs. **3.** 112,500 ft.-pdl. **4.** 7203 joules. **5.** 8000 joules. **6.** 4000 ft.-pd.; 4000 ft.-pd. **7.** 200 ft.-pdl.; 50 pd. **8.** 20 joules. **9.** 9.8 joules. **10.** 42.14 joules. **11.** (1) 25,600 ft.-pd.; (2) 14,400 ft.-pd. **12.** (1) 10,240 ft.-pdl.; (2) 32 ft./sec. **13.** 1482 ft./sec. (approx.). **14.** 598,950 ft.-pd.; 136,125 ft.-pd.

**Page 121.** **1.** 200. **2.** 400. **3.** 187.5. **4.** 678,787.$\dot{8}\dot{7}$. **5.** 1600. **6.** 60 erg/sec. **7.** 100 erg/sec. **8.** 10,000 erg/sec. **9.** 100. **10.** 1000. **11.** 20. **12.** 39.2. **13.** 0.28 h.p. **14.** $65\frac{1}{3}$. **15.** 784 h.p. **16.** 7,800,000 ft.-pd.; 2.44 h.p. **17.** 9.6. **18.** 980. **19.** 70 watts. **20.** 735. **21.** 196. **22.** 990. **23.** 600 litres. **24.** 80.

**Page 129.** **1.** 1169.4 cal. **2.** 0.213° F. **3.** 2.68° C. **4.** 62,211 B.T.U. (nearly). **5.** 116.6 B.T.U. **6.** 750; 3.6% low. **7.** 8,402,400 ft.-pd.; 441.8 lb. **8.** 11,394.9. **9.** 0.838 k.w. **10.** 4.03 joules per cal. **11.** 5.36° C.

**Page 138.** **1.** 82.2%. **2.** 302.5 ohms; 125. **3.** $2\frac{1}{2}$ h.p. (allowing for losses). **4.** 95,465 cal. **5.** 28.6° C. **6.** 31.5° C. **7.** 11.43%. **8.** 40.3 ohms. **9.** 94.1%. **10.** 63.3%.

**Page 140.** **1.** 3,600,000. **2.** 22 c. **3.** 0.2 k.w.h. **4.** 11 c. **5.** $\frac{1}{3}$ amp.; 1 c.

**Page 142.** **2.** 97 c. **3.** $3.83; 75 c. **5.** $2.03. **6.** $2.27. **7.** $11.14; 90 c; $1.22. **9.** $62.97. **10.** $106.42; $37.54.

**Page 147.** **3.** (1) 180 gm.; (2) 120 gm.; (3) 150 gm.; (4) 210 gm.; (5) 30 gm. **4.** (1) 12 pd.; (2) 36 pd.; (3) 42 pd.; (4) 30 pd.; (5) 6 pd. **5.** (1) 6 kg.; (2) 18 kg.; (3) 21 kg.; (4) 12 kg.; (5) 15 kg. **6.** 4.5 gm.; 7.5 gm.; 6 gm. **7.** 2.2 pd.; 2.5 pd.; 2 pd. **8.** 35 gm.; 5 gm. **9.** $2P$; $2Q$. **10.** 39 pd. **11.** 37 kg. **12.** 18 pd. **13.** $12P$. **14.** 13 pd. **15.** 15 pd. **16.** 400 pd.

**Page 151.** **1.** (1) Yes; (2) No; (3) Yes; (4) Yes. **4.** 120° apart. **6.** 5.77 pd.; 11.55 pd. **7.** 17.32 pd.; 20 pd.; No. **8.** 20 pd.; 15 pd.; 25 pd.

**Page 153.** **1.** (1) 84 pd.; (2) 18.477 pd.; (3) 5.176 pd.; (4) 70 pd.; (5) 8.789 pd.; (6) 2.125 pd.; (7) 18.915 pd.; (8) 12.64 pd.; (9) $P$ pd. north. **3.** $\frac{1}{2}\sqrt{7}$ times force represented by side of triangle. **4.** 50 pd. acting towards centre. **5.** $\sqrt{6}$ pd. **6.** 8 grams. **7.** 12 pd. **8.** $5\sqrt{2}$ kg. at 135° with first force.

**Page 156.** **1.** (1) $5\sqrt{3}$ pd., (2) $5\sqrt{2}$ pd., (3) 2.58 pd. **2.** $10\sqrt{3}$ and 10 pd. **3.** $6\sqrt{2}$ pd. **4.** $50\sqrt{2}$ pd. **5.** $8\sqrt{3}$ and 8 pd. **6.** $\frac{4}{3}\sqrt{3}$ pd. **7.** 199.23 pd. **8.** 17.32 pd. **9.** 3.42 pd. **10.** 12.68 pd.; 27.19 pd.

**Page 163.** **1.** At 70-cm. **2.** 240 units; $120\sqrt{3}$ units. **3.** 125 gm. **4.** 6 ft. from fulcrum. **5.** 2.5 pd. **6.** 0; 108; $-108$. **7.** $30\sqrt{3}$. **8.** 0; 160; 0; $-160$. **9.** 1:2. **10.** (1) 0, $-6$; (2) 18, 18; (3) 0, 0; (4) 0, $-11\sqrt{2}$; (5) $\sqrt{2}$, 0; (6) 40, 0. **11.** (1) $-201.47$; (2) $-62\frac{1}{2}$. **12.** $25\sqrt{2}$ ft. from ground.

**Page 169.** **1.** 5 dynes acting 3 cm. from smaller force. **2.** 70 pd.; 50 pd. **3.** $37\frac{1}{3}$ pd.; $74\frac{2}{3}$ pd. **4.** 24 and 16 pd. **5.** 2 ft. from stronger man. **6.** $6\frac{1}{4}$ pd.; $3\frac{3}{4}$ pd. **7.** 42 and 21 pd. **8.** 8 dynes acting 25 cm. from smaller force. **9.** (1) 8 pd. 7.5 ft. from smaller force; (2) 22 pd. $2\frac{8}{11}$ ft. from 7-pound force. **10.** 5 pd. 8 ft. from larger force.

**Page 173.** **1.** 6.6 metres from 20 kg. mass. **2.** $1\frac{1}{2}$ ft. from fulcrum. **3.** $1\frac{3}{8}$ ft. from 7-lb. mass. **4.** 2 gm. **5.** $267\frac{2}{3}$ pd.; $624\frac{1}{3}$ pd. **6.** 27 dynes at a point $1\frac{23}{27}$ cm. from end. **7.** 6 dynes. **8.** $11\frac{1}{17}$ cm. from 3-dyne force. **9.** 5 lb. **10.** 35 lb.; 40 lb. **11.** One-quarter of the length of the beam. **12.** 11 ft. from smaller end.

**Page 179.** **1.** 10 pd.; 20 pd. **2.** 2:1. **3.** $\frac{10\sqrt{2}}{1+\sqrt{3}}$ pd.; $\frac{20}{1+\sqrt{3}}$ pd. **4.** $\frac{2}{\sqrt{3}}$. **5.** $4\sqrt{3}$ pd. **6.** $3\sqrt{2}$ pd. **7.** $W(\sqrt{2}-1)$. **8.** 100 pd. **9.** $10\sqrt{3}$ pd.; 10 pd. **10.** (1) $20\sqrt{3}$ pd., (2) 40 pd. **11.** 1600 pd.; 2000 pd. **12.** 4000 pd.; $2000\sqrt{3}$ pd.

**Page 182.** **1.** (1) $\frac{100}{\sqrt{3}}$ pd., (2) $\frac{100}{\sqrt{3}}$ pd., (3) 200 pd. **2.** $10\sqrt{3}$ pd, **3.** $30\sqrt{3}$ pd.; $30\sqrt{39}$ pd. **4.** (1) $22\frac{1}{2}$ pd., (2) 54.8 pd. **5.** $46\frac{2}{3}$ pd.; 68.4 pd. **6.** $W$; $P$.

**Page 183.** **1.** $\frac{1}{2}$ kg.; $\frac{\sqrt{3}}{2}$ kg. **2.** $10\sqrt{3}$ pd. **3.** 120 pd. **4.** $\frac{\sqrt{13}\ W}{2\sqrt{3}}$; $\frac{W}{2\sqrt{3}}$. **5.** 12 pd.; $6\sqrt{3}$ pd. **7.** $\frac{100}{\sqrt{3}}$ pd.; $\frac{100}{\sqrt{3}}$ or $\frac{200}{\sqrt{3}}$ pd. **8.** (1) $\sqrt{3}$ pd.; (2) 1 pd.; $\sqrt{3}$ pd.; (3) 45°. **9.** $3\sqrt{3}$ pd.

**Page 194.** **3.** 45 pd. **6.** 2 pd.; 10.198 pd. **7.** $\frac{1}{15}$. **8.** 4.714. **9.** $\frac{1}{\sqrt{3}}$. **10.** $\sqrt{3}$; 1; $\frac{1}{\sqrt{3}}$. **11.** $\frac{1}{\sqrt{3}}$. **12.** 11.732 pd. **13.** 36 pd. **14.** 10 tons; $42\frac{2}{3}$ ft. / sec. **15.** 0.732. **16.** $24\frac{4}{9}$ pd. **17.** $\frac{4}{3}\sqrt{3}$ pd. **18.** 0.268. **19.** 30°. **20.** 42.77 h.p.

**Page 200.** **1.** 6 in. **2.** 10 in. from the 12-lb. mass. **3.** $4\frac{2}{3}$ in. from the end. **4.** $8\frac{1}{2}$ in. from the 7-lb. mass. **5.** 15 in. from end. **6.** $28\frac{4}{5}$ ft. from first man. **7.** $6\frac{2}{5}$ ft. from 12-lb. mass. **8.** $3\frac{1}{4}$ ft. from 1-lb. mass. **9.** 3.26 in. from the top. **10.** 3.3 in. from the base. **11.** 5 ft.

**Page 202.** **1.** $\frac{2}{5}$ of diagonal from 2-lb. mass. **2.** $OG=\frac{1}{4}\,OD$. **3.** 4.34 in. **4.** $\frac{1}{4}$ of the side of the square. **5.** 3.6 ft. (nearly). **6.** 7.8 in. (nearly). **7.** $8\frac{1}{3}$ in.; $11\frac{1}{3}$ in.

**Page 204.** **1.** $2\frac{2}{3}$ cm.; 3.283 cm. **3.** 2 ft.; $3\frac{1}{5}$ ft.; $3\frac{1}{5}$ ft. **4.** $\frac{4}{3}$ ft.; 1 ft.; $\frac{4}{5}$ ft. **5.** $3\frac{1}{3}$ ft.; 5.696 ft.; 4.807 ft. **6.** 10 in. **7.** 18.04 in. **8.** At the centre of the base of the triangle. **9.** $7\frac{3}{5}$ in. **10.** $\frac{11}{30}$ of the side of the square from the middle point of the base. **11.** $\sqrt{3}:1$. **12.** $\frac{a}{9}$ from $E$. **13.** $\frac{2}{9}$ height from base. **14.** $OG = \frac{2}{21}\,OC$. **15.** In the straight line drawn parallel to $BC$ from the middle point of $AB$ and at a distance $\frac{26}{45}$ of the side of the square from this point. **16.** Distances from $AD$ and $AB$ are $\frac{11}{18}\,AB$ and $\frac{4}{9}\,AD$.

**Page 209.** **7.** $1\frac{12}{13}$ ft. **8.** 120. **9.** 10. **10.** 10 kg. **11.** 50 lb.

**Page 215.** **1.** $1\frac{1}{4}$ lb. **2.** 90 pd., 120 pd. **3.** $37\frac{1}{2}$ pd. **4.** 225 pd. **5.** 22.5 pd. **6.** 90 pd. **7.** (1) 30 gm.; (2) 12 cm.; 15 cm.

**Page 220.** **1.** 1:2; 1:2. **2.** 4. **3.** $\frac{1}{3}$. **4.** 60 pd. **6.** 250 pd.; 12,500 ft.-pd.

**Page 224.** **1.** $26\frac{2}{3}$ pd.; **2.** $53\frac{1}{3}$; 6,400. **3.** $16\frac{2}{3}$ pd.

**Page 228.** **2.** $20\frac{5}{9}$ pd. **3.** $4\frac{17}{27}$ lb. **4.** $7\frac{1}{7}$ pd.; 6.186 pd.; $25\frac{2}{3}$ ft. **5.** $\frac{1}{126}$ pd.; $\frac{1}{252}$ pd. **6.** 14.58 pd. **7.** (1) 95 pd.; 200 pd.; (2) 95 pd.; 105 pd. **8.** $653\frac{5}{7}$.

**Page 234.** **1.** 11:40. **2.** 1221.82 (nearly).

**Page 241.** **1.** (1) 4; (2) 576. **2.** (1) 6; (2) 54. **3.** (1) 50; (2) 500,000. **4.** (1) 0.2; (2) 0.8. **5.** 80 pd. **6.** $312\frac{1}{2}$ gm. **7.** $\frac{9}{49}a$. **8.** 800 kg. **9.** 11,550 pd. **10.** $30\frac{1}{4}$ pd. **11.** 20 kg. **12.** $31\frac{1}{14}$ gm. **13.** $110\frac{1}{4}$; 18.14 pd.

**Page 251.** **1.** 9.122 pd. **2.** 0.0375 gm. **3.** 11.5 pd. **4.** 10,000. **5.** 36 kg. **6.** 184.87 ft. **7.** $31\frac{1}{4}$. **8.** 9 kg. **9.** $230\frac{2}{5}$ ft. **10.** 37,500 pd. **11.** (1) 2.4 gm., (2) 0.64 gm., (3) 0.48 gm. **12.** (1) 416 kg., (2) 1104 kg., (3) 276 kg., (4) 319.8 kg., (5) 96.2 kg.

**Page 257.** **1.** 62.5 pd.; 97.5 pd. **2.** 4.57 pd. **3.** 2.5 kg. **4.** 4.9 gm. **5.** 295 pd. **6.** 7 kg. **7.** 600 gm. **8.** $\frac{1}{4}$. **9.** 0.413 oz. **10.** $\frac{n-m}{n}$ gm. **11.** 0.881. **12.** 5 c.dm. **13.** $133\frac{1}{3}$ c.c. **14.** $4\frac{4}{5}$ cu. ft. **15.** $32\frac{1}{2}$ pd. **16.** 520 gm. **17.** 42 gm. **18.** 9 pd. **19.** 1562.5 lb. **20.** $3906\frac{1}{4}$ pd.

**Page 260.** **1.** 1.47 kg. **2.** 54.05 c.c. **3.** 2.7 gm. per c.c. **4.** 12 kg. **5.** 0.77 gm. per c.c. **6.** 1.072 (nearly) gm. per c.c. **7.** 1.2 gm. per c.c.

**Page 265.** **1.** $\frac{3}{7}$ gm. per c.c. **2.** $\frac{1}{4}$ gm. per c.c. **3.** 25 cm. **4.** $\frac{2}{3}$ gm. per c.c. **5.** 20 c.c.; 6 gm. per c.c.; 0.8 gm. per c.c. **6.** $\frac{8}{9}$ gm. per c.c. **7.** $\frac{5}{8}$; $\frac{5}{6}$; $6\frac{1}{4}$ inches. **8.** 20 c.c. **9.** Gold, 386.4 gm.; silver, 21.04 gm. **10.** 159.14 gm. **11.** 311.9 gm. **12.** 28.5 gm. **13.** $40\frac{5}{8}$ lb. **14.** $13\frac{1}{3}$ lb. **15.** (1) 30 gm.; (2) 20 gm. **16.** 1000. **17.** 0.514. **18.** 15 oz. **19.** 0.64 in. **20.** 6 gm. **21.** (1) None, (2) 30 gm. increase, (3) None.

**Page 267.** **1.** 10 in. **2.** 68 cm.; 170 cm.; 255 cm. **3.** 2.427 ft. **4.** 13.619. **5.** 11 : 7. **6.** 1.4.

**Page 272.** **9.** 1.291 gm.

**Page 279.** **1.** 14.756 pd. **2.** 1033.6 gm. **3.** (1) 15 pd., (2) $14\frac{1}{2}$ pd., (3) 14.72 pd. **4.** (1) 952 gm., (2) 1034 gm., (3) 1030 gm. **5.** $7066\frac{2}{3}$ pd. **6.** 999,600. **7.** (*a*) 27.05 pd.; (*b*) 27.54 pd. per sq. in.

**Page 284.** **5.** 2907.75 kg. **6.** 2683 kg. **7.** Yes; 1334.5 kg. **8.** 4967 pd. **9.** 2360 pd.

**Page 290.** **1.** $6\frac{2}{3}$ cu. ft. **2.** 22.85 litres. **3.** 75,314.7 cu. in. **4.** $483\frac{1}{3}$ in. of mercury. **5.** $562\frac{1}{2}$ mm. **6.** 174 in. of mercury. **7.** 0.00125 gm. (nearly) per c.c. **8.** 101.34 gm. **9.** $3.60. **10.** (*a*) 30 atmospheres; (*b*) 292.148 m.

**Page 294.** **1.** 492.97 m./sec.; 461.13 m./sec.; 393.24 m./sec. **2.** 1299.82 m./sec.

**Page 302.** **1.** (*a*) $\frac{2}{3}$, (*b*) $\frac{8}{27}$. **2.** $\frac{4}{5}$. **3.** $\frac{3}{2}$. **4.** $\frac{16}{25}$. **5.** 2:1.

**Page 312.** **1.** 10.336 m. **2.** 17 ft. **3.** 12.92 m. **4.** 75.19 pd.; 12.53 pd. **6.** (1) $104\frac{1}{6}$ pd.; (2) $156\frac{1}{4}$ pd. **7.** 100. **8.** 40 pd. **9.** 4000 pd.

**Page 315.** **2.** (*b*) 13.6 times height of mercury barometer. **3.** $219\frac{1}{9}$ inches. 9. $\frac{h\rho}{\rho_1}$.

**Page 326.** **1.** 280 dynes; 1680 ergs. **2.** 17,600 ergs. **3.** 16,016 ergs.

**Page 332.** **1.** 28.028 dynes/cm. **2.** 73.01 dynes/cm. **3.** (*a*) 2.98 cm.; 1.24 cm.; 1.33 cm. (*b*) 14.90 cm.; 6.20 cm.; 6.65 cm. **4.** 1.20 cm.; 5.98 cm. **5.** 2.57 cm. **6.** Height in tube twice height between plates.

**Page 337.** **3.** $2:1:\frac{1}{1.04}$. **4.** 28 dynes/sq. cm.; $74\frac{2}{3}$ dynes/sq. cm. **5.** $\frac{2\pi}{3}(x^2+2\sqrt{3}xr)T$ ergs.

**Page 353.** **1.** 65.1 pd./sq. in. **2.** 4,900,000 dynes/sq. cm.; 5 kg./sq. cm. **3.** 366.07 c.c./sec. **4.** 0.877 cu. in./sec.

**Page 360.** **1.** $4\frac{1}{6}$ ft./sec. **2.** 5 ft./sec. **3.** $23,108\frac{1}{3}$ ft.-pd. **4.** 5.014 ft.-pd. (nearly). **5.** 4,920,000 ergs. **6.** 66.41 litres/sec. **7.** 2.43 cu. ft./sec. **8.** 3.674 litres/sec. **9.** (*a*) 600 gm./sq. cm.; (*b*) 587,200 dynes = 599.2 gm./sq. cm. (g = 980).

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K



L



M



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